Abstract  It has been proposed that the part structures of denotations of plurals ‘project’ to the denotations of expressions including those plurals (e.g., Gawron & Kehler 2004, Kubota & Levine 2016, Schmitt 2019/2020). If such a plural projection is possible, not only plural DPs but also expressions including those plural DPs denote pluralities (e.g., saw the two recipes denotes a plurality \{Saw(recipe1), Saw(recipe2)\} instead of a singularity \{Saw(\{recipe1, recipe2\})\}).

One piece of support for plural projection comes from Schmitt’s (2020) observation about ‘non-local’ cumulativity. In this paper, I further examine when cumulativity is available non-locally, and show that a source of cumulativity in the literature (e.g., Krifka 1989, Kratzer 2007, Harada 2022b) can capture all the relevant non-local cumulativity data without plural projection while an analysis with plural projection can capture only a proper subset of those data. Therefore, this paper concludes that the relevant non-local cumulativity does not support the need of plural projection.

Keywords: cumulativity, plurality, English

1 Introduction

This paper addresses the meanings of expressions including a plural DP (e.g., saw the two recipes), through an investigation into the locality in the semantic derivation of cumulativity. Cumulativity is a phenomenon where sentences with multiple plurals like (1) receive particular ‘weak’ truth conditions (e.g., Kroch 1974, Scha 1981).

(1)  [The two boys]_{PL1} saw [the two recipes]_{PL2}

Sentence (1) has weak truth conditions; it is true if one of the cumulative scenarios in (2) is true in the evaluation world. For example, (1) is true when one of the two boys saw one of the two recipes and the other boy saw the other recipe. Throughout

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this paper, I assume that the two boys refers to boy1 and boy2, and that the two
recipes refers to recipe1 and recipe2.

(2) Cumulative scenarios in which (1) can be true

\[
\begin{array}{cccc}
\text{boy1} & \text{→} & \text{R1} & \text{boy1} \\
\text{boy2} & \text{→} & \text{R2} & \text{boy2} \\
\text{boy1} & \text{→} & \text{R1} & \text{boy1} \\
\text{boy2} & \text{→} & \text{R2} & \text{boy2} \\
\end{array}
\]

(→: saw-relation)

(R: recipe)

I call the relations between the two boys and the two recipes in the cumulative
scenarios in (2) the cumulations or cumulative saw-relations, and I call the weak
truth conditions in question the cumulative truth conditions, which can be defined as
in (3b). For instance, the cumulative truth conditions of (1) are the following: (1) is
true iff each of the two boys saw at least one of the two recipes, and each of the two
recipes was seen by at least one of the two boys.

(3) a. A P B, where A and B are arguments of the predicate P.

b. 1 iff \( \forall x \in S_A \exists y \in S_B [P(y)(x) = 1] \land \forall y \in S_B \exists x \in S_A [P(y)(x) = 1] \),

where \( S_A \) and \( S_B \) are sets of objects that \( \{A\} \) and \( \{B\} \) consist of intuitively.

(adapted from Schmitt 2019: 7)

Cumulative truth conditions are argued to arise from a grammatical, compositional
source such as a covert operator on predicates (Beck & Sauerland 2000). (4) shows
the definition of such an operator Cuml, which encodes cumulative truth conditions.

(4) \text{Cuml}(P)(B)(A) = 1 \text{ iff } \forall x \in S_A \exists y \in S_B [P(y)(x) = 1] \land \forall y \in S_B \exists x \in S_A [P(y)(x) = 1] ,

where \( S_A \) and \( S_B \) are sets of objects that \( \{A\} \) and \( \{B\} \) consist of intuitively.

(adapted from Beck & Sauerland 2000: 351)

Cuml takes a binary predicate P and its co-arguments, returning 1 iff there is a
cumulative P-relation between the co-arguments. For example, under this approach,
sentence (1) has the LF in (5a), and the cumulative truth conditions in (5b).

(5) a. \( \{TP[the two boys][[Cuml saw] [the two recipes]]\} \)

b. \( \{TP\} = 1 \text{ iff } \forall x \in \{\text{boy1,boy2}\} \exists y \in \{\text{recipe1,recipe2}\} [\text{SAW}(y)(x) = 1] \land \forall y \in \{\text{recipe1,recipe2}\} \exists x \in \{\text{boy1,boy2}\} [\text{SAW}(y)(x) = 1] \)

In this way, compositional source can derive cumulative truth conditions of (1).

Beck & Sauerland (2000) also argue that compositional source is compatible
with the lack of ‘non-local’ cumulativity across a finite clause in (6a). Note that (6a)
seems to be false in the given scenario where there is a pronounced-to-be-against-the-law-by-cumulation between the two plurals across a finite clause. The falsity of (6a) is in contrast to the truth of (6b), given that those two sentences differ only in the finiteness of the embedded clause. I call the cumulativity in (6b) non-local as the two plurals that enter into a cumulation are not the arguments of the same predicate.

(6) [Scenario: Lawyer1 has pronounced that proposal1 is against the law. Lawyer2 has pronounced proposal2 is against the law.]
   a. The two lawyers have pronounced that the two proposals are against the law.
   b. The two lawyers have pronounced the two proposals to be against the law.

(Beck & Sauerland 2000: 365. The scenario is added.)

However, Schmitt (2019)/(2020) observes that (6a) can actually allow non-local cumulativity in richer scenarios (see below), further demonstrating that compositional source does not straightforwardly derive such cumulativity on its own. Based on this gap in the analysis of plurals, Schmitt proposes to base the compositional source on the compositional process called the plural projection. However, this paper argues that the non-local cumulativity in question does not need plural projection as another source of cumulativity, which I call the inferential source (e.g., Krifka 1989, Kratzer 2007, Harada 2022a), can capture the non-local cumulativity.

This paper is organized as follows. Section 2 first shows how plural projection enables compositional source to derive some non-local cumulativity. Section 3 then shows that plural projection is not sufficient to enable compositional source to derive some non-local cumulativity. Afterwards, Section 4 shows that the inferential source captures all the kinds of non-finite cumulativity discussed in Section 2-3. Thus, Section 4 concludes that the inferential source is sufficient and non-local cumulativity does not support the need for plural projection. Section 4 also shows how the inferential source analysis differs from Pasternak’s (2018) analysis of cumulativity, which is similar to the inferential source analysis. Section 5 then demonstrates that the inferential source also captures the lack of non-local cumulativity in a sentence, which the compositional source analysis with plural projection cannot straightforwardly capture. Finally, Section 6 concludes.

2 Non-local cumulativity by compositional source with plural projection

This section shows that plural projection enables compositional source to derive some non-local cumulativity, as claimed by Schmitt (2019)/(2020).

As mentioned in Section 1, Beck & Sauerland (2000) observe that cumulativity is generally impossible across a finite clause boundary (6a), and claim that the
observation is compatible with compositional source. First, they argue that sentence (6b) has the LF in (7); the two lawyers has first moved covertly, and then the two proposals has moved covertly and tucked in below the two lawyers. After that, Cuml attaches to CP/a syntactically derived predicate [λx.λy.y have pronounced x to be against the law].

(7) \[[(DP_1\text{the two lawyers})\ [\[DP_2\text{the two proposals}\] \ [\text{Cuml} \ [CP_2 \ [1 \ [t_1 \text{have pronounced } t_2 \text{ to be against the law}]])]])\]

Cuml takes CP, DP_2, and DP_1 in turn, and derives the cumulative truth conditions saying that sentence (6b) is true iff there is a cumulative pronounced-to-be-against-the-law-by-relation between those two DPs.

Since there is such a cumulation in the scenario of (6b), sentence (6b) is correctly predicted true. On the other hand, sentence (6a) should not be able to derive the same truth conditions as the sentence is false in the same scenario. The question concerns how it is possible for (6a) to have different truth conditions from sentence (6b) given their surface similarity. Beck & Sauerland’s answer to this question is that covertly moving a DP across a finite clause boundary is generally difficult, and thus (6a) cannot derive an LF like (7). As a result, the sentence does not have an appropriate predicate like the CP in (7) that Cuml can take to derive the same truth conditions as (6b). In this way, Beck & Sauerland’s compositional source captures the lack of cumulativity across a finite clause in (6a) as well as the its presence in (6b), tracking constraints on covert movements.

Although (6a) does not sound natural in the given scenario, Schmitt (2019) observes that the same sentence is true in a richer cumulative scenario (8).

(8) [[Scenario: The chair of the linguistics department, Dr. Abe, and the chair of the musicology department, Dr. Bert, keep coming up with crazy proposals. Last week, Dr. Abe proposed to expel all teachers that didn’t know Latin and Dr. Bert brought forth a motion excluding any student that didn’t play the piano. Yesterday, there was a meeting with two lawyers, Dr. Kern, who specialises in the rights of faculty members, and Dr. Marten, the legal representative of the student body. Dr. Kern dismissed Dr. Abe’s proposition, and Dr. Marten declared Dr. Bert’s proposal to be untenable, but both said that the chairs could not be fired on the basis of their behavior.] Well, the two lawyers have pronounced that the two proposals are against the law (as was kind of expected) but neither of them supported the dean’s motion of fire Dr. Abe an Dr. Bert immediately.]

(Schmitt 2019: 24)

The crucial difference between the scenario in (8) and the one in (6) seems to be that in (8), which lawyer dismissed which proposals, i.e., the exact relation between the
two plurals, is neither focused nor important; sentence (8) can be paraphrased as “it is confirmed (by the two lawyers) that the two proposals are against the law”.

Relatedly, it is also noteworthy that sentence (6a) is true if have pronounced is replaced by confirmed, and more generally, sentences with clause-embedding predicates like confirm and prove are more likely to show cumulativity across a finite clause than sentences with predicates like pronounced. Take (9) as an example with proved; the sentence is true if there is a cumulation between the two plurals across a finite clause.

(9) The two boys proved that the two recipes are flawless.

Thus, it can be assumed that the lack of cumulativity across a finite clause in (6a) is attributed to some property of the predicate pronounce, which can be diminished by a rich scenario, and non-local cumulativity across a finite clause is possible in principle. Then, we need a way to derive such cumulativity as Beck & Sauerland’s compositional source analysis does not derive it on its own.

Schmitt (2019)/(2020) observes this gap in the analysis of cumulativity, and claims that derivations of non-local cumulativity need a compositional process called plural projection. To introduce plural projection, I first introduce her ontological assumptions; she assumes not only individual pluralities (10a) (e.g., Link 1983), but also higher-order pluralities like predicate pluralities (10b) (e.g., Gawron & Kehler 2004) and proposition pluralities (10c).

(10) a. [the two boys] = {boy1,boy2}
    b. [sing and dance] = {SING,DANCE} (SING := \lambda x.\lambda w.x sings in w)
    c. [This boy sings and that boy dances] = {SING(boy1),DANCE(boy2)}

Now, I introduce plural projection, which is essentially a compositional process of projecting the part structures of the denotations of plurals to the denotations of expressions including those plurals. For instance, if the DP the two recipes denotes a plurality {recipe1,recipe2}, a VP including the DP like saw the two recipes also denotes a plurality \{SAW(recipe1),SAW(recipe2)} instead of a singularity like \{SAW(({recipe1,recipe2}))\} (11)-(12). Note that in (11), the two sets compose in a point-wise fashion, and the part structure of \{recipe1,recipe2\} projects to \{SAW(recipe1),SAW(recipe2)} in the sense that the derived set involves two members as \{recipe1,recipe2\} does.

(11) Schmitt’s derivation: \{SAW(recipe1),SAW(recipe2)}
(12) Non-Schmitt’s derivation: \{SAW(({recipe1,recipe2}))\}
Locality in the derivation of cumulativity

We are now ready to see how plural projection enables compositional source to derive the non-local cumulativity in (9). The two recipes denotes \{recipe1,recipe2\}, which composes with \{IS\_FLAWLESS\} and \{THAT\} via plural projection in turn. As a result, the embedded clause in (9) denotes a doubleton set \{THAT.RECIPE1.IS\_FLAWLESS, THAT.RECIPE2.IS\_FLAWLESS\}. Then, Cuml takes this set, \{PROVED\}, and \{boy1, boy2\}, deriving the cumulative truth conditions of (9) we are after. In this way, plural projection enables compositional source to derive some non-local cumulativity, primarily because the use of plural projection allows us not to use movements to derive appropriate arguments of Cuml. Thus, an approach with plural projection is free from any movement constraints which prevent Beck & Sauerland’s compositional source analysis capturing non-local cumulativity across a finite clause.¹

3 Insufficiency of plural projection to capture non-local cumulativity

While plural projection enables compositional source to derive cumulativity across a finite clause in the above example, I observe that plural projection is not sufficient to derive a version of such cumulativity. Consider (13), which exemplifies the cumulativity in question; the sentence is true if there is a sub-atomic cumulation between the two boys and the ramen recipe across a finite clause, as it is true in the scenario in (13). I call it sub-atomic cumulativity as it seems that \{boy1,boy2\} and \{noodle(.recipe),broth(.recipe)\} enter into cumulativity, and noodle and broth seems to be sub-atomic parts of ramen in \{ramen\} denoted by the ramen recipe. Given this, I call the cumulativity in (13) sub-atomic (non-local) cumulativity across a finite clause as opposed to atomic (non-local) cumulativity across a finite clause as in (9). I also assume in what follows that the noodle recipe and broth recipe constitute the ramen recipe, unless otherwise stated.

(13) [Scenario: Boy1 proved that the noodle recipe is flawless. Boy2 proved that the broth recipe is flawless. The two recipes constitute the ramen recipe.]

The two boys proved that the ramen recipe is flawless.

On the assumption that plural projection and cumulative operation cannot access sub-atomic parts of the denotations of singulars like the ramen recipe (see Harada 2022a, Harada 2022b, and below for arguments of this assumption), plural projection enables (13) to derive

¹ The analysis presented here is the one minimally different from Beck & Sauerland’s analysis with Schmitt’s (2019) idea about plural projection. In Schmitt’s (2019) original analysis, the new compositional rule for plural projection can itself derive cumulativity without Cuml. For instance, in (9), the matrix predicate derives a predicate plurality \{PROVED.\_RECIPE1.IS\_FLAWLESS, PROVED.\_RECIPE2.IS\_FLAWLESS\}, and the compositional rule ‘cumulatively composes’ the set with \{boy1,boy2\}. It is not crucial here which approach we adopt in explaining Schmitt’s analysis.
{boy1,boy2} and {PROVED.THAT.RAMEN.IS.FLAWLESS} (instead of
{PROVED.THAT.NOODLE.IS.FLAWLESS, PROVED.THAT.BROTH.IS.FLAWLESS}).
Then if those two sets cumulatively compose via Cuml, the sentence is wrongly
predicted true only if each of the two boys proved that the whole ramen recipe
is flawless. In this way, plural projection is not sufficient to capture sub-atomic
non-local cumulativity.

It should be noted that if singulars effectively denote pluralities like plurals
and plural projection and cumulative operations can access sub-atomic parts of
the denotations of singulars, contrary to the above assumption, the compositional
source approach with plural projection then has a difficulty in capturing the contrast
between the falsity of (14a) and the truth of (14b).

(14) [Scenario: Boy1 proved that the noodle recipe is circled. Boy2 proved that
the broth recipe is circled. The whole ramen recipe is not circled.]
a. The two boys proved that the ramen recipe is circled.
b. The two boys proved that the two recipes are circled.

The crucial difference between (14a) and (14b) is that descriptively the proposition
denoted by the embedded clause is true in (14b) but it is not in (14a). Given
this difference, compositional source can capture the contrast between (14a) and
(14b) if we assume that only (14a) violates some presupposition; for instance, each
proposition in the complement set of proved has to be true. But such an analysis
is valid only if the ramen recipe denotes {ramen} and the embedded clause
denotes {THAT.RAMEN.IS.CIRCLED} instead of {THAT.NOODLE.IS.CIRCLED,
THAT.BROTH.IS.CIRCLED}, because the noodle recipe and the broth recipe are
circled in (14) while the ramen recipe is not. Therefore, without the assumption in
question, the compositional source approach has a difficulty in capturing the contrast
in (14), and with the assumption, it has a difficulty in deriving cumulativity in (13).

In this way, plural projection is not sufficient to capture all kinds of cumulativity
across a finite clause.

4 Sufficiency of the inferential source to capture non-local cumulativity

This section demonstrates that the inferential source can capture both atomic and sub-
atomic cumulativity across a finite clause. It will also be shown how the proposed
analysis differs from a similar analysis proposed by Pasternak (2018).

To analyze data with an embedded finite clause, I adopt an intensional semantic
framework (e.g., Lewis 1976) where linguistic expressions are directly mapped to

2 See Harada (2022a) and Harada (2022b) for more examples that make the same point as (14).
their intensions except for the words whose extensions are constant across worlds such as proper names and functional items (e.g., the, if, every). For example, the verb prove denotes \( \lambda \rho_{st}. \lambda \chi_{e}. \lambda w_{s}. \text{prove}'_{w}(\chi, \rho) \), which reads “the truth of each proposition in the set \( \rho \) is established in the world \( w \) as a result of each member of \( \chi \) doing some proving. I assume that while the domain of properties involve only singleton sets (15c-d), the domain of individuals and propositions involve both singularities and pluralities (15a-b).

(15) a. \( D_{e} = \{ \{\text{ada}\}, \{\text{bea}\}, \ldots, \{\text{ada,bea}\}, \ldots \} \)

b. \( D_{st} = \{ \{p_{1}\}, \{p_{2}\}, \ldots, \{p_{1}, p_{2}\}, \ldots \} \)

c. \( D_{est} = \{ \{\text{SAW}\}, \{\text{TYPE}\}, \ldots \} \), where \( \text{SAW} = \lambda \chi_{e}. \lambda \psi_{e}. \lambda w_{s}. \text{saw}'_{w}(\psi, \chi) \)

d. \( D_{stest} = \{ \{\text{PROVE}\}, \{\text{CONFIRM}\}, \ldots \} \),

\( \text{where PROVE} = \lambda \rho_{st}. \lambda \chi_{e}. \lambda w_{s}. \text{prove}'_{w}(\chi, \rho) \) (\( \chi \) and \( \psi \): sets of individuals, \( \rho \): a set of propositions)

As mentioned above, I assume weak meanings for clause-embedding predicates like prove partly because such weak meanings can be compatible with collectivity phenomena exemplified in (16)-(17).

(16) [Scenario: Ada wants to know if the ramen recipe is flawless. So she let boy1 make a ramen using the ramen recipe as she knows that boy1 can make food exactly as recipes say. After boy1 made a ramen, Ada let boy2 who has a ‘perfect’ sense of taste check the taste of the ramen. Boy2 said the ramen was flawless.]

The two boys proved that the ramen recipe is flawless.

(17) [Scenario: Boy1 proved that the ramen recipe or the sushi recipe is flawless. Boy2 proved that the sushi recipe has a flaw.]

The two boys proved that the ramen recipe is flawless.

Sentence (16) and (17) are both true in the scenarios where neither boy1 nor boy2 proved alone that any part of the ramen recipe is flawless. In (16), for instance, boy1 just made a ramen and boy2 said that the taste of ramen (instead of the ramen recipe) is flawless. The truths of (16) and (17) are compatible with the weak meanings we assume for the sentences; these sentences are true iff the truth of the proposition that the ramen recipe is flawless was established as a result of boy1 and boy2 each having done some proving.\(^4\)

3 This does not mean that I disagree with the presence of predicate pluralities; in fact, Harada (2022b) argues for the need of predicate pluralities. I do not assume those here simply because they are not necessary here.

4 But it remains to be seen what exactly ‘doing some proving’ means; in (16), while boy1 is involved in the proving, he did not prove anything by himself, and he can still enter into the cumulation. On the other hand, Ada cannot enter into the cumulation even if she also seems to be involved in the proving.
Now, I will explain how the inferential source analysis captures the cumulativity across a finite clause in (18a-c). Note that these sentences are true if there is a cumulation between the matrix subject and the bold expressions, as they are true in the given scenario.

(18) [Scenario: Boy1 proved that the noodle recipe is flawless. Boy2 proved that the broth recipe is flawless.]

a. The two boys proved that the noodle recipe is flawless and that the broth recipe is flawless.

b. The two boys proved that the two recipes are flawless.

c. The two boys proved that the ramen recipe is flawless.

On the assumption that the conjunction of proposition \(\{q_1, q_2\}\) denotes their union \(\{q_1, q_2\}\), the sentences in (18) denote the following propositions. The differences among (19a-c) lie in what the internal arguments of proved’ are.

(19)

a. \(\left[(18a)\right] = \{\lambda w_.\text{proved}'(\{boy1,boy2\}, \{q_1, q_2\})\}\), where
   \(q_1 = \lambda w_.\text{flawless}'(\{noodle\})\)
   \(q_2 = \lambda w_.\text{flawless}'(\{broth\})\)

b. \(\left[(18b)\right] = \{\lambda w_.\text{proved}'(\{boy1,boy2\}, \{p_1\})\}\), where
   \(p_1 = \lambda w_.\text{flawless}'(\{noodle, broth\})\)

c. \(\left[(18c)\right] = \{\lambda w_.\text{proved}'(\{boy1,boy2\}, \{p_2\})\}\), where
   \(p_2 = \lambda w_.\text{flawless}'(\{ramen\})\)

(19a) denotes a proposition saying that sentence (18a) is true iff the truths of \(q_1\) and \(q_2\) are each established in the world \(w\) as a result of boy1 and boy2 each having done some proving, and the propositions in (19b-c) make similar statements, i.e., very weak statements that can be compatible with cumulative and collective scenarios. As shown below, the existential claims in (19a-c) lie in what the internal arguments of proved’ are.

First, in the scenario, the following two propositions are true (20); that is, the truth of \(q_1\) was established as a result of boy1 having done some proving, and the truth of \(q_2\) was established as a result of boy2 having done some proving (to simplify notations, I will omit "\(\lambda w_\)." when discussing propositions that are true in scenarios and inferences).

(20) True propositions in the scenario

a. \(\lambda w_.\text{proved}'(\{boy1\}, \{q_1\})\) \(q_1 = \lambda w_.\text{flawless}'(\{noodle\})\)

b. \(\lambda w_.\text{proved}'(\{boy2\}, \{q_2\})\) \(q_2 = \lambda w_.\text{flawless}'(\{broth\})\)
Locality in the derivation of cumulativity

Given (20a-b), we can naturally assume that the following inference, which I call the *cumulative inference*, is valid (21a); that is, if the truth of \(q_1\) is established as a result of boy1 having done some proving and the truth of \(q_2\) is established as a result of boy2 having done some proving, then the truths of \(q_1\) and \(q_2\) are each established as a result of boy1 and boy2 each having done some proving.

(21) **Cumulative inference**

\[
\begin{align*}
&\text{proved}'^w(\{\text{boy1}, \{q_1\}\}) \land \text{proved}'^w(\{\text{boy2}, \{q_2\}\}) \\
&\implies \text{proved}'^w(\{\text{boy1,boy2}, \{q_1,q_2\}\})
\end{align*}
\]

Notice that the conclusion in (21) serves to confirm that the proposition in (19a) is true in the scenario in (18). Thus, (18a) (i.e., *the two boys proved that the noodle recipe is flawless and that the broth recipe is flawless*) is correctly predicted true in the cumulative scenario.

The truth of (18b) (i.e., *the two boys proved that the two recipes are flawless*) is also correctly predicted with another new inference which can be naturally assumed to be valid in the scenario of (18b). The inference, which I call the *propositional parts-whole inference*, is (22): if (i) the truths of \(q_1\) and \(q_2\) are each established as a result of boy1 and boy2 each having done some proving, (ii) \(q_1\) and \(q_2\) do not entail each other unless they are equivalent, and (iii) \(q_1 \land q_2\) can be assumed to be equivalent to \(p_1\) in the given scenario (\(\equiv_c = \text{contextual equivalence}\)), then we can naturally assume with our world knowledge about proving that the truth of \(p_1\) is established as a result of boy1 and boy2 each having done some proving (the need of the second and third premises will be motivated below.).

(22) **Parts-whole inference**

\[
\begin{align*}
&\text{proved}'^w(\{\text{boy1,boy2}, \{q_1,q_2\}\}) \land \forall q,q' \in \{q_1,q_2\} [q \equiv q' \lor [q \nRightarrow q' \land q \nLeftarrow q']] \\
&\land [q_1 \land q_2 \equiv_c p_1] \implies \text{proved}'^w(\{\text{boy1,boy2}, \{p_1\}\})
\end{align*}
\]

Notice that the conclusion in (22) serves to confirm that the proposition in (19b), repeated below as (23b), is true in the scenario in (18). Thus, (18b)/(23a) is correctly predicted true in the cumulative scenario.

(23) a. The two boys proved that the two recipes are flawless

b. \([18b]\) = \(\{\lambda w_x.\text{proved}'^w(\{\text{boy1,boy2}, \{p_1\}\})\}\), where

\[p_1 = \lambda w_x.\text{flawless}'^w(\{\text{noodle,broth}\})\]

Lastly, the truth of (18c) (i.e., *the two boys proved that the ramen recipe is flawless*) can also be predicted correctly based on the following parts-whole inference, which

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5 The second premise in (22) can be simplified as there are only two propositions \(q_1\) and \(q_2\). But the format of the premise will be necessary in an example to be discussed.
basically reads in the same way as the above parts-whole inference: if (i) the truths of \( q_1 \) and \( q_2 \) are each established as a result of boy1 and boy2 each having done some proving, (ii) \( q_1 \) and \( q_2 \) do not entail each other unless they are equivalent, and (iii) \( q_1 \wedge q_2 \) can be assumed to be equivalent to \( p_2 \) in the given scenario, then we can naturally assume with our world knowledge about proving that the truth of \( p_2 \) was established as a result of boy1 and boy2 each having done some proving.

(24) Parts-whole inference

\[
\text{proved}^w((\text{boy1,boy2}),\{q_1,q_2\}) \land \forall q,q' \in \{q_1,q_2\} [q \equiv q' \lor [q \not\equiv q' \land q \equiv q']] \\
\land [q_1 \wedge q_2 \equiv c \ p_2] \rightarrow \text{proved}^w((\text{boy1,boy2}),\{p_2\})
\]

Notice that the conclusion in (24) serve to confirm that the proposition in (19c), repeated below as (25b), is true in the scenario in (18). Thus, sentence (18c)/(25a) is also correctly predicted true in the cumulative scenario.

(25) a. The two boys proved that the ramen recipe is flawless.

b. \([18c]\) = \(\{\lambda w_s.\text{proved}^w((\text{boy1,boy2}),\{p_2\})\}\), where \( p_2 = \lambda w_s.\text{flawless}^w(\{\text{ramen}\}) \)

In this way, under the inferential source analysis, embedded clauses like *that the two recipes are flawless* and *that the ramen recipe is flawless* both denote singleton sets. Thus, whether or not the embedded clauses involve a plural like *the two recipes* or a singular like *the ramen recipe* is less important in the analysis than the compositional source analysis with plural projection, which derives singleton or non-singleton sets depending of the presence of plurals in the embedded clause. This is why the inferential source can capture sub-atomic cumulativity across a finite clause more easily.\(^6\) In this way, the inferential source is sufficient to capture both atomic and sub-atomic cumulativity across a finite clause. Therefore, cumulativity across a finite clause does not support the need for plural projection which enables compositional source to derive only atomic cumulativity across a finite clause.

In the rest of this section, I will support the need for the second and third premises in parts-whole inference. I will also explain how the inferential source approach

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\(^6\) This is why the inferential source can also easily capture the cumulativity in (1a) with respect to the *three boys* and every *recipe*. 

(i) [Scenario: There are 50 ramen recipes. Boy1 proved that recipe1-recipe20 are flawless. Boy2 proved that recipe21-recipe50 are flawless.]

a. The **two boys** proved that every *ramen recipe* is flawless. (Huilei Wang, p.c.)

b. \([\text{ia}]\) = \(\{\lambda w_s.\text{proved}^w(\{\text{boy1,boy2}\},\{p_3\})\}\), where \( p_3 = \lambda w_s.\text{every recipe is flawless in } w. \)

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differs from Pasternak’s (2018) analysis which can also provide a straightforward explanation about cumulativity across a finite clause in some data. First, I will present the need for the second premise in parts-whole inference. The premise in question is the one about the universal statement, e.g., ‘∀q,q′ ∈ {q₁,q₂} [q⇔q′ ∨ [q≠q’ ∧ q≠q’]]’ in (24). I will argue that the relevant premise is necessary to explain that sentence (18c), repeated below as (26), is false in scenarios like (26a).

(26) a. Scenario where (18c) is false: Boy1 proved that the noodle recipe and the broth recipe are flawless. Boy2 proved that the sushi recipe is flawless, so boy2 proved that some recipe is flawless.⁷

b. (18c): The two boys proved that the ramen recipe is flawless.

c. [(18c)] = {λwₚ.proved”w((boy1,boy2), {p₂})}, where

p₂ = λwₛ.flawless”w({ramen})

In (26a), there are three proved-relations, as shown in (27) as premises of a cumulative inference. The conclusion of the cumulative states that the truths of q₁, q₂, and q₃ were each established in w as a result of boy1 and boy2 each having done some proving.

(27) Cumulative inference

proved”w({boy1},q₁) ∧ proved”w({boy1},q₂) ∧ proved”w({boy2},q₃)

→ proved”w((boy1,boy2),{q₁,q₂,q₃}), where

q₁ = λwₛ.flawless”w({noodle})

---

⁷ Sentence (18c)/(26b) is true in this scenario if we additionally assume boy1 and boy2 were assigned a task to prove the truths of the three recipes as a group. In this extended scenario, the sentence seems to have the same meaning as a sentence with a group noun like the committee proved that the ramen recipe is flawless, where the committee consists of boy1 and boy2. Given this, it is noteworthy that proper parts of the denotations of group nouns are often semantically inaccessible; for instance, (1) cannot mean that each card in the deck had a red mark on it (e.g., see also De Vries (2017)).

(i) #Each of the deck had a red mark on it. (Schwarzschild 1996: 165)

Given data like (i), it can be assumed that the committee consisting of boy1 and boy2 denotes a singularity (committee1)={boy1+boy2} (a singularity/a singleton set of a group consisting of boy1 and boy2) (e.g., Schwarzschild 1996), and the two boys in sentence (18c)/(26b) in the above scenario also denotes a singularity like {boy1+boy2} (the mapping from a set of individuals like {boy1,boy2} to a singleton set of the sum of the individuals like {boy1+boy2} can be enabled, for instance, by Landman’s (2000) group-forming operator ↑). Then, the sentence in question is correctly predicted true since the truth of the proposition that the ramen recipe is flawless is established as a result of a group of boy1 and boy2 having done some proving. Note that this analysis is also compatible with the falsity of (18c)/(26b) in the scenario of (26a) where we do not have enough evidence to assume that boy1 and boy2 form a group.
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\[ q_2 = \lambda w_s.\text{flawless}^w(\{\text{broth}\}) \]
\[ q_3 = \lambda w_s.\exists \chi[\text{recipe}(\chi) \land \text{FLAWLESS}^w(\chi)] \text{ (i.e., ‘some recipe is flawless’) } \]

Crucially, the second premise in the following parts-whole inference is false because each of \( q_1 \) and \( q_2 \) is not equivalent to \( q_3 \) and each of them entails \( q_3 \). Thus, we cannot assume that the truth of \( p_2 \) is established as a result of boy1 and boy2 each having done some proving.

\[
(28) \quad (27) \land \forall q,q' \in \{q_1,q_2,q_3\}\{q \equiv q' \lor [q \not\equiv q' \land q \not\equiv q']\} \land [q_1 \land q_2 \land q_3 \not\Rightarrow c p_2] \rightarrow \text{proved}^w(\{\text{boy1,boy2}\},\{p_2\})
\]

As a result, it is correctly predicted that \((19c)/(26c)\) is false in the given scenario. In this way, the second premise in parts-whole inference is needed to explain why \((18c)/(26b)\) is true in \((18c)\) but not in \((26a)\). More generally, the second premise is needed to prevent cumulativity with respect to the matrix subject and the embedded clause when at least one of individuals denoted by the matrix subject did not prove any part of the proposition denoted by the embedded clause, which was not proved by anyone else; in \((26)\), boy2 proved that some recipe is flawless, but that was also proved by boy1 who proved that the noodle and broth recipe are true.

Next, I will show that the third premise about the contextual equivalence in parts-whole inference (e.g., \( q_1 \land q_2 \not\Rightarrow c p_2 \)’ in \((24a)\)) is needed to explain that \((18c)/(26b)\) is false in scenarios like \((29)\).

\( (29) \quad \text{Scenario where (26b) is false:} \) Boy1 proved that a noodle recipe is flawless. Boy2 proved that a broth recipe is flawless. But those recipes do not constitute any ramen recipe. Boy3 proved that the ramen recipe is flawless.

In \((29)\), the propositions in \((20)\), repeated below as \((30)\), are true.

\( (30) \quad \text{True propositions in the scenario} \)

a. proved\(^w\)(\{boy1\},\{q_1\}) \quad q_1 = \lambda w_s.\text{flawless}^w(\{\text{noodle}\})

b. proved\(^w\)(\{boy2\},\{q_2\}) \quad q_2 = \lambda w_s.\text{flawless}^w(\{\text{broth}\})

\((30a)\) and \((30a)\) serve as premises of the cumulative inference in \((21a)\), repeated below as \((31)\). The conclusion in the inference says the truths of \( q_1 \) and \( q_2 \) were each established in \( w \) as a result of boy1 and boy2 each having done some proving.

\( (31) \quad \text{Cumulative inference} \)

\[
\text{proved}^w(\{\text{boy1}\},\{q_1\}) \land \text{proved}^w(\{\text{boy2}\},\{q_2\}) \rightarrow \text{proved}^w(\{\text{boy1,boy2}\},\{q_1,q_2\})
\]
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The only difference between the scenarios in (18) (repeated below as (32)) and (29) is that in (29) the parts-whole in (24), repeated below as (33), involves a false premise, i.e., the third premise; \( q_1 \land q_2 \) is not contextually equivalent to \( p_2 \) because the noodle and broth recipe in \( q_1 \land q_2 \) do not constitute the ramen recipe in \( p_2 \).

(32) Boy1 proved that the noodle recipe is flawless. Boy2 proved that the broth recipe is flawless.

(33) **Parts-whole inference**

\[
\text{proved}^{w}(\{\text{boy1,boy2}\},\{q_1,q_2\}) \land \forall q,q' \in \{q_1,q_2\} \left[ q \Leftrightarrow q' \lor \left[ q \nRightarrow q' \land q \nLeftarrow q' \right] \right] \\
\land \left[ q_1 \land q_2 \Leftrightarrow c_p^2 \right] \rightarrow \text{proved}^{w}(\{\text{boy1,boy2}\},\{p_2\})
\]

In this way, the third premise in parts-whole inference is needed to explain why (18c)/(26b) (i.e., *the two boys proved that the ramen recipe is flawless*) is true in the scenario in (18)/(32) but not in (29). More generally, the third premise is needed to prevent cumulativity with respect to a matrix subject and an embedded clause when individuals denoted by the matrix subject are not related to any part of the proposition denoted by the embedded clause.

Lastly, I will illustrate a crucial difference between the inferential source analysis and Pasternak’s (2018) analysis which can also capture cumulativity across a finite clause in some data in a similar way. To begin with, his analysis predicts that a sentence *a and b believe p* is true iff the logical conjunction of a’s belief and b’s belief which are about the ‘situation’ under discussion (i.e., partial possible worlds under discussion; c.f., Barwise & Perry 1981, Kratzer 2002) entails p. This analysis can explain, for instance, why (34) is true in the given cumulative scenario.

(34) Scenario: The two boxers will fight in different matches. Coach1 believes boxer1 would win. Coach2 believes boxer2 would win.

The two coaches believe that the two boxers would win.

In the scenario of (34), the situation under discussion is partial possible worlds where boxer1 and boxer2 are going to fight in different matches. Coach1 and coach2 each have a belief about the situation; coach1 has a belief that boxer1 would win, and coach2 has a belief that boxer2 would win. Then, the logical conjunction of these beliefs entails the proposition ‘the two boxers would win’ which is denoted by the complement clause of *believe*. Thus, (34) is correctly predicted true in the given cumulative scenario under Pasternak’s analysis.\(^8\)

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\(^8\) In this analysis, only the beliefs about the situation under discussion are examined in relation to the proposition denoted by the embedded clause. This is important because otherwise sentences like (1) are wrongly predicted true, as the logical conjunction of \( q_1 \) and \( q_2 \) entails \( p_1 \) in (1) in the given scenario. On the other hand, if we take the situation about the ramen recipe into consideration, the
On the other hand, the inferential source can also capture the truth of sentence (34) in the given scenario in a similar way. I will explain how it captures the truth to show the similarity between the inferential source and Pasternak’s approach. First, under the inferential source approach, sentence (34) denotes the following set of a proposition. In general, a believes p expresses a particular epistemic state, i.e., a has some attitude which supports the truth of p. Thus, I assume that the proposition in (35) says: (34) is true iff coach1 and coach2 have some attitudes, which support the truth of p1 in w1, i.e., the proposition asserting that boxer1 and boxer2 both win.

Such truth conditions are satisfied in the given scenario, as follows.

\[
\text{(35)} \quad \mathcal{L}(34) = \{ \lambda w_s.\text{believe}^w({\text{coach1,coach2}, \{p_1\}}) \}, \text{ where } p_1 = \lambda w_s.\text{win}^w({\text{boxer1,boxer2}})
\]

First, in the scenario in (34), due to the following cumulative inference, we can assume that the truth of q1 and q2 are each supported as a result of coach1 and coach2 each having some attitude.

\[
\text{(36)} \quad \text{Cumulative inference}
\]
\[
\text{believe}^w({\text{coach1}, \{q_1\}}) \land \text{believe}^w({\text{coach2}, \{q_2\}}) \rightarrow \text{believe}^w({\text{coach1,coach2}, \{q_1,q_2\}}), \text{ where }
\]
\[
q_1 = \lambda w.\text{win}^w({\text{boxer1}}) \\
q_2 = \lambda w.\text{win}^w({\text{boxer2}})
\]

Based on the above cumulative inference and the following parts-whole inference, we can assume that the truth of p1 is supported as a result of coach1 and coach2 each having some attitude. This/the conclusion in (37) serves to confirm that the proposition in (35) is true in the scenario in (34).

\[
\text{(37)} \quad \text{Parts-whole inference}
\]
\[
(36) \land \forall q,q' \in \{q_1,q_2\} [q \equiv q' \lor [q \not\equiv q' \land q \not\equiv q']] \land [q_1 \land q_2 \not\leftrightarrow c p_1] \\
\rightarrow \text{believe}^w({\text{coach1,coach2}, \{p_1\}}),
\]
\[
\text{where } p_1 = \lambda w.\text{win}^w({\text{boxer1,boxer2}})
\]

In this way, both the inferential source and Pasternak’s analysis can capture the truth of (34) in a similar way but with one crucial difference; Pasternak’s analysis falsity of (1) is expected because boy2 does not have any belief about the situation and thus it is not appropriate to include boy2 in the matrix subject in (1).

(i) [Scenario: Boy1 proved [the ramen recipe is flawless]q1. Boy2 proved [Montreal was sunny]q2.] The two boys proved [that the ramen recipe is flawless]p1.
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examines the presence of contextual *entailment* between beliefs of individuals and the proposition denoted by the embedded clause, but the inferential source analysis examines the presence of contextual *equivalence* between the same elements in parts-whole inference. But this slight difference between the use of entailment and equivalence makes a crucial difference in the analysis of data like (38). As Schmitt (2020) observes, Pasternak’s analysis incorrectly predicts that sentence (38) is true in the scenario. However, the inferential source can capture the falsity of (38).

(38) [Scenario: Ada is looking forward to Sue’s party: She is certain that every man at the party will fall in love with her. Bea is also looking forward to the party: She hates men and is certain that only one man will attend: Roy. Sue tells me: Ada and Bea are really looking forward to the party...] They believe that Roy will fall in love with Ada. (Schmitt 2020: 575)

In (38), in relation to the situation about Sue’s party, Ada believes that every man at the party will fall in love with Ada, and Bea believes that Roy is the only one man who will attend the party. Note that the logical conjunction of those beliefs entails that Roy will fall in love with Ada. Therefore, Pasternak’s analysis wrongly predicts that the sentence is true in the scenario (Schmitt 2020). In contrast, the inferential source can capture the falsity of (38) as follows. First, (38) denotes the proposition in (39).

(39) \[
\lambdaw, \text{believe}'_{w}(\{\text{ada, bea}\}, \{p_1\})\], where \\
p_1 = \lambdaw, \text{love}'_{w}(\{\text{roy}\}, \{\text{ada}\})
\]

In the scenario of (38), the two premises in the cumulative inference in (40) are true. So we can assume that the truth of the propositions q_1 and q_2 are each supported as a result of Ada and Bea each having some attitudes (the details of the denotations of q_1 and q_2 are abbreviated as they are not important).

(40) **Cumulative inference**

\[
\text{believe}'_{w}(\{\text{ada}\}, \{q_1\}) \land \text{believe}'_{w}(\{\text{bea}\}, \{q_2\}) \rightarrow \\
\text{believe}'_{w}(\{\text{ada, bea}\}, \{q_1, q_2\})
\]

q_1 = \lambdaw, \text{every man at the party will fall in love with Ada in } w. \\
q_2 = \lambdaw, \text{Roy is the only man who will attend the party in } w.

Crucially, the third premise in the following parts-whole is false. This is because q_1 \land q_2 is not contextually equivalent to p_1, as p_1 does not entail q_1 \land q_2 in the scenario. In other words, the proposition that Roy will love Ada does not entail the logical conjunction of the proposition that every man at the party will fall in love with Ada and the proposition that Roy is the only man who will attend the party.

9 More specifically, Schmitt (2020) discusses the German counterpart of (38).
In this way, the inferential source captures the falsity of sentence (38) due to the use of the contextual equivalence, whereas Pasternak’s analysis does not.

To sum up, the inferential source can capture sub-atomic cumulativity across a finite clause as well as its atomic counterparts, while plural projection enables compositional source to derive only atomic cumulativity across a finite clause. So the inferential source is sufficient to capture cumulativity across a finite clause. Thus, cumulativity across a finite clause does not support the need for plural projection, and this conclusion is compatible with Beck & Sauerland’s claim that compositional source should be subject to locality of covert movements. This section also demonstrated that the inferential source can capture the lack of cumulativity across a finite clause in some data like (38) unlike Pasternak’s analysis.

5 More support for the inferential source: lack of non-local cumulativity

In Section 4, we saw that (38) is false even though there is a cumulation across a finite clause. There are more data like it, and this section introduces one of them, further showing that adopting the inferential source brings about good results in explaining locality in cumulativity. The example in question is (42). While sentence (42a) with the subject itself being a conjunction is true, sentence (42b) with the subject including a conjunction is false in the cumulative scenario.

(42) [Scenario: That the the earth is round is true. That the earth is flat is false.]

a. That the earth is round and that the earth is flat are true and false.

b. That the earth is round and flat is true and false.

Plural projection, as it stands, enables the subject in both (42a) and (42b) to denote \{THAT.THE.EARTH.IS.ROUND, THAT.THE.EARTH.IS.FLAT\}, so it is not obvious how the compositional source analysis with the assumption of plural projection captures the contrast between (42a) and (42b). On the other hand, the inferential source can capture the contrast. Assuming a version of non-Boolean and, we can assume that the matrix predicate conjunctions in (42a-b) denote a function that takes a set of propositions and returns a set of a proposition that states the presence of a cumulation between the proposition set and the set \{TRUE,FALSE\} (43a). Given this assumption, (42a) and (42b) denote the sets of propositions in (43b) and (43c).

10 There are data that are not discussed here due to the lack of space, but that require the inferential source analysis to be further developed. See Harada (2022a) and Harada (2022b) for those data.
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(43) a. \[ \{\text{true and false}\} = \{\lambda \rho \lambda w. \exists \rho_1, \rho_2[\rho_1 \cup \rho_2 \land \text{TRUE}(\rho_1)(w) \land \text{FALSE}(\rho_2)(w)]\} \]

b. \[ \text{[(42a)]} = \{\lambda w. \exists \rho_1, \rho_2[(\{q_1, q_2\} = \rho_1 \cup \rho_2 \land \text{TRUE}(\rho_1)(w) \land \text{FALSE}(\rho_2)(w))], \quad \text{where } q_1 = \lambda w. \text{round}''(\{\text{earth}\}) \quad q_2 = \lambda w. \text{flat}''(\{\text{earth}\}) \text{]} \]

c. \[ \text{[(42ab)]} = \{\lambda w. \exists \rho_1, \rho_2[(\{p_1\} = \rho_1 \cup \rho_2 \land \text{TRUE}(\rho_1)(w) \land \text{FALSE}(\rho_2)(w))], \quad \text{where } p_1 = \lambda w. \exists \chi_1, \chi_2[\{\text{earth}\} = \chi_1 \cup \chi_2 \land \text{round}''(\chi_1) \land \text{flat}''(\chi_2)] \]

In (43b), TRUE and FALSE can take \{\{q_1\}\} and \{\{q_2\}\} respectively, so sentence (42a) is correctly predicted true in the scenario. In contrast, in (43c), TRUE and FALSE both take \{\{\text{earth}\}\}, and thus sentence (42b) is true only if the earth is both round and flat. Thus, sentence (42b) is also correctly predicted false in the given scenario. In this way, the inferential source explains the contrast between (42a) and (42b).

To sum up, this section discussed another example that disallows cumulativity across a finite clause. It was shown that the inferential source can capture the example while compositional source with plural projection does not capture it straightforwardly. Therefore, the relevant example also supports the need for the inferential source.

6 Conclusion

This paper discussed cumulativity across a finite clause. Such cumulativity seems \textit{prima facie} to support the need for plural projection. This is because descriptively, sentence (44) is true if there is a cumulative \textit{proved} relation between \{\text{boy1, boy2}\} and \{\text{THAT.RECIPE1.IS.FLAWLESS, THAT.RECIPE2.IS.FLAWLESS}\}, and plural projection enables the sentence to derive those two sets.

(44) The two boys proved that the two recipes are flawless.

However, it was shown that the inferential source of cumulativity can also capture such cumulativity data, including the ones that plural projection or Pasternak’s (2018) analysis cannot capture straightforwardly. Therefore, this paper concludes that what is needed to capture cumulativity across a finite clause is the inferential source, and such cumulativity does not support the need for plural projection.

\footnote{See Harada (2022a) and Harada (2022b) for more data like the one discussed in this section.}
References


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