## Presuppositions, probabilities and belief

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LSA Annual Meeting, Minneapolis, January 2–5, 2014

- **1. Introduction**. Lassiter (2012) provides an elegant solution to the strengthening of conditional presuppositions (Geurts 1996) by relativizing the contextual constraints introduced by presuppositions triggers to probabilistic information states. Utterance presuppositions are felicitous on this account if the probability of semantic presuppositions exceeds some thresh-hold value  $\theta$ , a contextual parameter taking on a value in the interval [0,1]. Crucially, probabilistic information states may be local, taking into account antecedent information. The *locus classicus* of the proviso problem are presuppositions triggered in consequent clauses of indicative conditionals, as in (1) and (2).
- (1) If Theo's wife hates sonnets then his manager does too.
  - a.  $\not \rightarrow$  If Theo's wife hates sonnets then he has a manager.
  - b.  $\rightsquigarrow$  Theo has a manager.
- (2) If John is a diver, he'll bring his wetsuit on vacation.
  - a.  $\rightsquigarrow$  If John is a diver, he has a wetsuit.

Strengthening in such examples, according to Lassiter, arises when the the antecedent proposition is probabilistically independent of the semantic presupposition, viz. that Theo has a manager. These independence assumptions are assumed to be part of the background epistemic model and reflect causal relations between propositions.

Lassiter's (2012) approach accounts for these cases elegantly, even providing simple solutions to problematic examples (e.g., so-called semi-conditional presuppositions) and making correct predictions for new kinds of examples that are problematic for previous approaches. However, he stops short of extending his analysis to belief reports. Lassiter expresses skepticism about the standard semantics of belief reports, suggesting that their incorporation into a solution to the proviso problem depends on a proper analysis of their semantics, and sets the issue aside.

We propose here an extension of Lassiter's approach to belief reports that *does* adopts a standard semantics for belief (cf. Hintikka 1962 and Heim 1992). We add belief operators to a propositional fragment, provide a standard semantics, and explore the adequacy of the extension within Lassiter's general framework. We argue that the extension makes accurate predictions with respect to a range of examples involving presuppositions triggered in the complements of belief reports.

**2. Formal details.** We enrich the propositional fragment in Lassiter 2012 (which is based in turn on Klinedinst and Rothschild 2012) with indexed belief operators  $Bel_a$ , where a ranges over individuals. Models for the fragment are furnished with accessibility relations  $\mathcal{B}_a^w$  that provide the possible worlds consistent with what is presupposed by the speaker about a's beliefs in w. The semantics of belief operators is shown in (3).

$$(3) \qquad \llbracket Bel_a \ \varphi \rrbracket^{c,pr,w} = 1 \text{ iff for every } w' \in \mathcal{B}_a^w, \llbracket \varphi \rrbracket^{c,Rev(pr,\mathcal{B}_a^{pr}),w'} = 1$$

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- a.  $\mathcal{B}_a^w$  = the possible worlds consistent with a's beliefs in w.
- b.  $\mathcal{B}_a^{pr}$  = the possible worlds consistent with what is presupposed about a's beliefs in the information state and is defined formally as follows (cf. van Rooij 2005):

$$\mathcal{B}_a^{pr} = \bigcup \{ \mathcal{B}_a^w \mid w \in W \text{ and } pr(\{w\}) \ge \theta \}$$

c. Rev(pr, p) is the **revision** of pr with the information that p.

One difference from the case of indicative conditionals is that for belief (as well as for counterfactuals, if they were to be implemented in a similar system), we need not just probabilistic update, which corresponds to probabilistic conditionalization, but a probabilistic belief *revision* function which can retract certain beliefs as well as adding them.

We do not give a constructive definition for the *Rev* function (cf. Gärdenfors 1988), but it must meet the following conditions, which are sufficient to show how it behaves for our purposes:

- (4) a. If the information about a's beliefs in pr is consistent with the information in pr, then revision simply amounts to conditionalization, i.e. if  $pr(\mathcal{B}_a^{pr}) > 0$ , then  $Rev(pr, \mathcal{B}_a^{pr})(\varphi) = pr(\varphi|\mathcal{B}_a^{pr})$ 
  - b. Otherwise, pr entails the information that  $\neg \mathcal{B}_a^{pr}$ , i.e. the proposition that a's beliefs are false. In this case, we first **contract** pr by  $\neg \mathcal{B}_a^{pr}$ , that is, remove from pr the information that  $\neg \mathcal{B}_a^{pr}$ , and then conditionalize on  $\mathcal{B}_a^{pr}$ :

$$Rev(pr, \mathcal{B}_a^{pr})(\varphi) = (Contract(pr, \mathcal{B}_a^{pr}))(\varphi|\mathcal{B}_a^{pr})$$

- (i) Contracting by  $\varphi$  requires that  $Contract(pr, \varphi)(\varphi) < 1$
- (ii) This contraction should be **minimal**; if  $pr(\varphi) = 1$ , then  $(Contract(pr, \varphi))(\cdot|\varphi) = pr$ . In other words, it should minimize loss of the information contained in pr.
- (iii) Contracted propositions are the *least* epistemically **entrenched**, with entrenchment defined based on a partial ordering of the propositions accepted by the original state.

The semantics is static but the information state parameter gets updated incrementally along standard dynamic semantic lines. For example, the semantics for conjunction is shown in (5).

$$(5) \qquad \llbracket\varphi\wedge\psi\rrbracket^{c,pr,w}=1 \text{ iff } \llbracket\varphi\rrbracket^{c,pr,w}=1 \text{ and } \llbracket\psi\rrbracket^{c,pr_{\varphi},w}=1.$$

(3) is similar in spirit to the analysis in Heim 1992 where complement clauses are evaluated relative the subjects' doxastic alternatives, though the two accounts are not empirically equivalent (as discussed in more detail in that paper).

Following Lassiter (2012), semantic presuppositions associated with atomic formula are subject to the usage constraint in (6a). Briefly, a speaker should not use a presupposition trigger unless she has a relatively high degree of belief that the presupposition is true. (6b) requires presuppositions triggered by atomic formula in complex sentences to satisfy (6a) relative to their *local* information states.

 $<sup>^{1}</sup>$ In the case where a's beliefs are consistent with pr, the latter definition reduces to the former (conditionalization) anyway.

## (6) Usage constraints:

- a. For an atomic expression p presupposing q, the use of p is felicitous relative to information state pr iff  $pr(q) > \theta$ .
- b. If the atomic sentences of  $\varphi$  are  $p_1, \ldots, p_n$  presupposing  $q_1, \ldots, q_n$  and  $pr_1, \ldots, pr_n$  are information states, where  $pr_i$  is the local information state for  $p_i$ , then the use of  $\varphi$  is appropriate relative to a global information state pr iff  $pr_i(q_i) \ge \theta$  for  $1 \le i \le n$ .

It is our contention that the semantics in (3) together with Lassiter's usage constraints, (6a) and (6b), yield an adequate account of presuppositions triggered in the complement clauses of belief reports. In the remaining space we sketch how the analysis accounts for the strengthening of presuppositions triggered in the complement clause of belief reports.

**3. Examples.** The contrast between (1) and (2) in Lassiter's system is the result of specific assumptions about the background epistemic models. For example, it is assumed that having a manager is probabilistically independent of having a spouse who hates sonnets, while the probability of having a wetsuit is not independent of being a diver.  $\varphi$  and  $\psi$  are independent according to a probability measure pr just in case  $pr(\psi|\varphi) = pr(\psi)$  (and  $pr(\varphi|\psi) = pr(\varphi)$ ).

Let W= Theo's wife hates sonnets; H= Theo's manager hates sonnets; M= Theo has a manager. The usage constraint associated with (1) is  $pr(M|W) \geq \theta$ . Given plausible background assumptions, pr(M|W) = pr(M) and hence the usage constraint reduces to  $pr(M) \geq \theta$ . In other words, the speaker has relatively high degree of belief in the proposition that Theo has a manager and believes her audience does, as well.

As with conditionals, strengthening of presuppositions triggered in belief reports depends on specific assumptions about the background epistemic model. The background assumptions are formally similar to those assumed in the analysis of conditional sentences; however, in the present case they relate to the stance of the interlocutor(s) toward the agent of the belief report regarding some specific proposition, rather than causal dependencies or law-like generalizations.

There are three logically possible background assumptions regarding the interlocutors' attitudes towards the relation between the truth of a proposition  $\chi$  and a's beliefs about  $\chi$ :

- $\begin{array}{ll} \text{(7)} & \text{ a. } & Rev(pr,\mathcal{B}_a^{pr})(\chi) > pr(\chi) \\ & \text{ b. } & Rev(pr,\mathcal{B}_a^{pr})(\chi) = pr(\chi) \\ & \text{ c. } & Rev(pr,\mathcal{B}_a^{pr})(\chi) < pr(\chi) \end{array}$
- (7a) corresponds to a situation of skepticism on the part of the interlocutors about *a*'s beliefs, while (7b) and (7c) correspond to situations of relative confidence in *a*'s beliefs. Note that these background conditions are, of necessity since they deal with the propositional attitudes of a particular agent, less general and law-like than the corresponding conditions that Lassiter uses to derive strengthening or lack thereof in conditionals. This means that they are more malleable and

In situation in which the background assumption in (7a) holds, for example, revising pr with a's beliefs yields a new probability measure that assigns a higher probability to  $\chi$  than does pr. This could be the case if (i) the interlocutors' beliefs about  $\chi$  are inconsistent with those of the attitude holder or (ii) are merely compatible (with no more specific information holding in the epistemic model). Such a situation might arise whenever the interlocutors are skeptical of (certain of) the agent's beliefs. A case close to (i) above is the 'conspiracy theorist' scenario in (8), where

less resistant to change in the wake of some kinds of new information.

the interlocutors may believe the agent's specific beliefs about the CIA are false (or unlikely to be true).

- (8) The conspiracy theorist believes that the CIA finally stopped monitoring his thoughts.
  - a.  $\mapsto$  The conspiracy theorist believes that the CIA was monitoring his thoughts.
  - b. 

    → The CIA was reading the conspiracy theorist's thoughts.

Here there is no strengthening since there is no guarantee that the interlocutors' credence in  $\chi$  exceeds the threshold  $\theta$ .

When the background assumption in (7b) holds, revising pr with contextual information regarding a's beliefs yields a probability measure that assigns the same probability to  $\chi$  as the global information state pr. What this means is that a's beliefs with respect to  $\chi$  are unlikely to be so incongruous with those of the interlocutors that the revision operator would have much of an effect on the likelihood of  $\chi$ . This could be the case if the interlocutors are confident in a's knowledge of particular facts, such that they are likely to adopt as their own beliefs on the matter anything that a believes. The second of these conditions applies to (9), for example.

- (9) Michelle believes that Denise stopped writing at noon.
  - a.  $\mapsto$  Michelle believes that Denise was writing before noon.
  - b.  $\mapsto$  Denise was writing before noon.
- **4. Conclusion**. We show that with plausible extensions, a probabilistic system of presupposition projection solves the proviso problem as applies to presuppositions embedded in attitude reports, while still maintaining the advantageous results of satisfaction systems. This suggests strongly that the probabilistic approach is on the right track, and restores the close parallel between attitude contexts and conditional sentences modeled in Heim 1992.

## References

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