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Modeling Mandarin rhotacization with Recursive Schemes

Ziling Zhu*

Abstract. This paper pursues a novel analysis of Mandarin rhotacization using Sonority Scale and characterizes this phonological process computationally with BOOLEAN MONADIC RECURSIVE SCHEMES (BMRSs, Chandlee & Jardine 2021). Mandarin rhotacization shows an asymmetry between high front vowels (i.e. [i, y] and high back vowels [u] vowels. Diverging from previous phonetics-based accounts to explain this asymmetry (Chao 1968; Duanmu 2007; a. o.), I propose that adjacent segments compete in terms of sonority in order to surface and present a refined, detailed sonority ranking in Mandarin. Finally, the `if...then...else` syntax in BMRSs elegantly formalizes the sonority competition between segments.

Keywords. Mandarin; rhotacization; sonority; phonology; Recursive Schemes

1. Introduction. Beijing Mandarin has a rhotacization marker [ər^{35}]. Morphologically, this morpheme is a diminutive suffix that usually attaches to nouns. Semantically, a rhotacized form expresses the meaning of a slighter degree or a smaller entity or carries a sense of intimacy or endearment. Phonologically, the rhotacization marker affects the realization of the rime that precedes it but leaves onset consonants and glides unchanged. In this project, I provide a novel phonological analysis of Mandarin rhotacization patterns with Sonority Scale (Zec 1989; Kuo 1994) and a computational modeling with Boolean Monadic Recursive Schemes (BMRSs, Chandlee & Jardine 2021).

I first examine the nonrhotacized forms in Mandarin to explore what glide-rime combinations are possible and motivate the underlying forms. Specifically, I make generalizations for glide-rime co-occurrence restrictions and mid vowel harmony. The generalizations are then formalized with Boolean Monadic Recursive Scheme.

Next, I discuss the Mandarin rhotacization data. High front vowels and other vowels are asymmetric in nucleus and coda positions. In rhotacized forms, root vowels /i, y/ are reduced to glides [j, ɥ] and the suffix vowel /ə/ surfaces as the nucleus. However, non-high-front root vowels /u, a, ə/ remain as the nucleus and the suffix vowel is deleted. This contrast extends to coda vowels: /i/ is deleted, but /u/ is not. Existing analysis typically attribute this asymmetry to the articulatory incompatibility between rhotacization and high front features (Chao 1968; Duanmu 2007; a. o.). In contrast, I pursue a novel phonological analysis based on a Sonority Scale that I propose for Mandarin. Adjacent segments compete in terms of sonority to surface. Finally, I use BMRS to computationally model this phonological process.

This paper is organized as follows. Section 2 introduces the dataset of nonrhotacized forms and examines glide-rime combinations and mid vowel harmony. Section 3 models the harmony process with BMRS. Section 4 introduces the rhotacization data and presents my analysis featuring Sonority Scale. Section 5 models the process with BMRS. Section 6 concludes.

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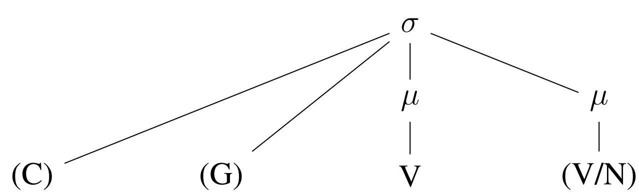
2. Mandarin glides and rimes: data and generalization.

2.1. SYLLABLE STRUCTURE. Beijing Mandarin has a fixed syllable structure with an onset, a glide, a nucleus vowel, and a coda (either a vowel to form a diphthong with the nucleus vowel or a nasal). A nucleus vowel is required, while other positions are optionally filled in.

Structure:



Examples:	(C)	(G)	V	(V/N)	
			a		Tone 35, ‘ah?’
			a	ŋ	Tone 35, ‘raise’
		w	a	ŋ	Tone 35, ‘king’
	t͡ʃ	w	a	ŋ	Tone 35, ‘bed’

Figure 1. Beijing Mandarin syllable structure

I assume that rimes only contain the nucleus vowel and the coda segment, but not the glides, following Duanmu (2007). This assumption is not crucial to our analysis. As Section 4 will show, glides remain unchanged regardless of the presence/absence of the rhotacization morpheme (although high front nucleus vowels may turn into glides in rhotacized forms). I set aside the discussion of which segments can take the onset position, as well as the co-occurrence restrictions between onset segments and glides, since they are irrelevant to the rhotacization patterns.

2.2. GLIDES AND RIMES.

2.2.1. INVENTORY AND FEATURES. In Beijing Mandarin, the glide position can host one of the three segments, [j], [ɥ], and [w], or be left empty. We find minimal pairs in (1), where the rime [an] is shared but the glides are different. Hence, there is a four-way contrast in the glide position.

(1) a. [jan^{55}] ‘smoke’
b. [wan^{55}] ‘curved’
c. [ɥan^{55}] ‘unjust’
d. [an^{55}] ‘safe’

This contrast is not realized for all 19 rimes in this language, grouped by their codas in Table 1.

-∅	-i	-u	-n	-ŋ
i			in	iŋ
y			yn	yŋ
u			un	uŋ
ɤ	ei	ou	ən	əŋ
a	ai	au	an	aŋ

Table 1. Surface forms of rimes

-∅	-i	-u	-n	-ŋ
i			in	iŋ
y			yn	yŋ
u			un	uŋ
ə	əi	əu	ən	əŋ
a	ai	au	an	aŋ

Table 2. Underlying forms of rimes

I make two observations based on Table 1. First, the gaps *[ii], *[iu], *[yi], *[yu], *[ui], and *[uu] suggest a restriction against high vowels in adjacent positions. Second, the four mid vowels are in complementary distribution, so they are allophonic. Their distribution can be predicted based on the preceding glides and succeeding codas. I posit [ə] as the underlying mid vowel, following Chao (1968), Cheng (1973), Xu (1980), Lin (1989), and Duanmu (2007)[1]. Table 2 gives the underlying forms of the rimes.

The inventory of segments in glides and rimes is summarized in (2).

(2) a. onset glide, (G): {j, ɥ, w}
b. nucleus vowel, V: {i, y, u, ə, a}
c. coda vowel, (V): {i, u}
d. coda nasal, (N): {n, ŋ}

The underlying segments can be described with distinctive features in Table 3. Two features, [cons(onantal)] and [syll(abic)], distinguish between vowels ([-cons, +syll]), glides ([-cons, -syll]), and consonants ([+cons, -syll]). Vowels are further distinguished in terms of height: three high vowels [i, y, u], a mid vowel [ə], and a low vowel [a]. Non-high vowels are underspecified for [back] and [round]. Among the high vowels, [i, y] are front, and [y, u] are rounded. The same features apply to glides: all glides are high; [j, ɥ] are front, and [ɥ, w] are rounded.

Segment	[back]	[round]	[high]	[low]	[cons]	[syll]
i	−	−	+	−	−	+
y	−	+	+	−	−	+
u	+	+	+	−	−	+
ə	(−)	(−)	−	−	−	+
a	(−)	(−)	−	+	−	+
j	−	−	+	−	−	−
ɥ	−	+	+	−	−	−
w	+	+	+	−	−	−
n	0	0	0	0	+	−
ŋ	0	0	0	0	+	−

Table 3. Featural representation of underlying segments

2.2.2. CO-OCCURRENCE RESTRICTIONS. Out of the 100 theoretically possible combinations of glides, vowels, and codas (**4** G * **5** V * **5** V/N), only 33 forms are realized in Mandarin. Table 4 lists all combinations grouped by different codas (i.e., all forms in one sub-table share the same coda), with nucleus vowels in rows and glides in columns. From top to bottom and then from left to right, the codas are respectively [∅], [i], [u], [n], [ŋ]. Only forms in black are successfully realized, while the rest are ill-formed. The co-occurrence restrictions are predicted by the generalizations in (3).

[1] Theories differ in the vowels they use for the surface representation. Here we mainly follow Lin (1989), setting aside some optional vowel quality changes documented by Xu (1980) adopted in Duanmu (2007).

Rimes	∅-	j-	ɥ-	w-
i	i	ji	ɥi	wi
y	y	jy	ɥy	wy
u	u	ju	ɥu	wu
ə	ɤ	je	ɥe	wo
a	a	ja	ɥa	wa
ii	ii	jii	ɥii	wii
yi	yi	jyi	ɥyi	wyi
ui	ui	jui	ɥui	wui
əi	ei	jei	ɥei	wei
ai	ai	jai	ɥai	wai
iu	iu	jiu	ɥiu	wiu
yu	yu	jyu	ɥyu	wyu
uu	uu	juu	ɥuu	wuu
əu	ou	jou	ɥou	wou
au	au	jau	ɥau	wau

Rimes	∅-	j-	ɥ-	w-
in	in	jin	ɥin	win
yn	yn	jyn	ɥyn	wyn
un	un	jun	ɥun	wun
ən	ən	jən	ɥən	wən
an	an	jan	ɥan	wan
iŋ	iŋ	jiŋ	ɥiŋ	wiŋ
yŋ	yŋ	jyŋ	ɥyŋ	wyŋ
uŋ	uŋ	juŋ	ɥuŋ	wuŋ
əŋ	əŋ	jəŋ	ɥəŋ	wəŋ
aŋ	aŋ	jaŋ	ɥaŋ	waŋ

Table 4. Glide-rime co-occurrences

(3) a. [+high] segments cannot occur in adjacent positions (*[+hi][+hi]), e.g. *[ji], *[wyn]
b. Glides and vowel codas cannot be identical in [front] or [round], e.g. *[jei]
c. [Gən] is ill-formed (*[Gən]), e.g. *[jən]
d. There are two accidental gaps: *[ɥa], *[ɥaŋ]

2.2.3. Mid vowel harmony. As seen in Table 5, Mandarin shows mid vowel harmony. The mid vowel surfaces with variations in its backness and rounding (blue segments are [+front], while red segments are [+back]; see Table 6 for the feature specification of surface mid vowels). It may also surface as [ɤ] in open syllables without a glide, which is attributed to the different rhyming slots of open and closed syllables in Duanmu (2007).

Rimes	∅-	j-	ɥ-	w-
ə	ɤ	je	ɥe	wo
əi	ei			wei
əu	ou	jou		
ən	ən			
əŋ	əŋ			

Table 5. Mid vowel harmony

Features	[front]	[back]	[round]
e	+	−	−
ə	−	−	−
ɤ	−	+	−
o	−	+	+

Table 6. Surface mid vowel

There are two local backness harmony processes: regressive harmony (e.g. /əi/ → [ei]) and progressive harmony (e.g. /jə/ → [je]). The progressive harmony (V → [α back] / [α back] __) from the glide precedes the regressive harmony (V → [α back] / __ [α back]) from the coda. The opacity lends support to the first rule ordering (Progressive before Regressive) in Table 7.

UR	/jəu/
Progressive	|jeu|
Regressive	|jou|
SR	[jou]

UR	/jəu/
Regressive	|jou|
Progressive	|jeu|
SR	*[jeu]

Table 7. Rule ordering in mid vowel harmony

The dataset also shows rounding harmony, which is parasitic on backness harmony. As I set aside the discussion of [ɤ] here, it is safe to claim that all mid back vowels are rounded in the surface representations.

3. Modeling Mandarin mid vowel harmony with BMRSs. BOOLEAN MONADIC RECURSIVE SCHEMES (BMRSs) is a formal theory designed by Chandlee & Jardine (2021) to explain the computational properties of phonological patterns. BMRSs are definitions of Boolean output values (True or $\top$, False or $\bot$) of monadic predicates defined with an `if` ... `then` ... `else` syntax. For current purposes, I do not restrict the use of predecessor and successor functions (p and s), which should be harmless without any recursion in the definitions.

Relevant phonological features are modeled as functions in the input signature $\mathcal{I}$ as in (4a). The output functions for those features are in the output signature $\mathcal{O}$ as in (4b). A function in the input signature checks the feature value of an input segment, while an output function spells out the output feature value for the input segment.

(4) a. $\mathcal{I} = \{p, s; [\text{FR}], [\text{BA}], [\text{RO}], [\text{HI}], [\text{LO}], [\text{CONS}], [\text{SYLL}]\}$
b. $\mathcal{O} = \{p, s; [\text{FR}]_o, [\text{BA}]_o, [\text{RO}]_o, [\text{HI}]_o, [\text{LO}]_o, [\text{CONS}]_o, [\text{SYLL}]_o\}$

We model the mid vowel harmony in Table 5 by defining the output functions with a series of blocking structures (`if` ... `then` $\bot$) and licensing structures (`if` ... `then` $\top$). They are arranged in the appropriate order to yield the correct output, as shown in (5).

(5)
$$
\begin{aligned}
[\text{FR}]_o(x) &= \texttt{if } [\text{FR}](s(x)) \texttt{ then } \top \texttt{ else}\\
&\quad\ \texttt{if } [\text{BA}](s(x)) \texttt{ then } \bot \texttt{ else}\\
&\quad\ \texttt{if } [\text{FR}](p(x)) \texttt{ then } \top \texttt{ else } [\text{FR}](x)\\
[\text{BA}]_o(x) &= \texttt{if } [\text{FR}](s(x)) \texttt{ then } \bot \texttt{ else}\\
&\quad\ \texttt{if } [\text{BA}](s(x)) \texttt{ then } \top \texttt{ else}\\
&\quad\ \texttt{if } [\text{BA}](p(x)) \texttt{ then } \top \texttt{ else } [\text{BA}](x)\\
[\text{RO}]_o(x) &= \texttt{if } [\text{BA}]_o(x) \texttt{ then } \top \texttt{ else } [\text{RO}](x)\\
[\text{HI}]_o(x) &= [\text{HI}](x)\\
[\text{LO}]_o(x) &= [\text{LO}](x)\\
[\text{CONS}]_o(x) &= [\text{CONS}](x)\\
[\text{SYLL}]_o(x) &= [\text{SYLL}](x)
\end{aligned}
$$

To read these structures, first evaluate the `if`-clause. If the predicate evaluates to $\top$, then evaluate the `then`-clause. If the predicate evaluates to $\bot$, then skip `then` and evaluate the `else`-clause instead. For example, the function $[\text{FR}]_o(x)$ takes an input segment x and obtains the [front] and [back] feature values of the segment's successor $s(x)$. If x's succeeding segment is front, then we proceed to the `then`-clause and outputs $\top$; if x's successor is back, then we proceed to the else-clause (i.e. the second line) and outputs $\bot$. When the segment x has no successor or its successor is underspecified for [front] and [back], both $[\text{FR}](s(x))$ and $[\text{BA}](s(x))$ evaluate to $\bot$ and the function proceeds to the third line. Now the third line says if the predecessor of x is front, then the function outputs $\bot$. Specifically, if x is a /ə/ followed by a nasal (underspecified for [front] and [back]), the final else clause will simply keep the input feature values for the segment x. A similar reasoning applies in the definitions for $[\text{BA}]_o(x)$.

Table 8 demonstrates the mid vowel harmony process of /jəu/ → [jou]. We evaluate the output functions against each input segment and obtains a boolean value. Based on the output values, we can identify the form of the output segments.

input	j	ə	u
$[\text{FR}]_o(x)$	$\top$	$\bot$	$\bot$
$[\text{BA}]_o(x)$	$\bot$	$\top$	$\top$
$[\text{RO}]_o(x)$	$\bot$	$\top$	$\top$
$[\text{HI}]_o(x)$	$\top$	$\bot$	$\top$
$[\text{LO}]_o(x)$	$\bot$	$\bot$	$\bot$
$[\text{CONS}]_o(x)$	$\bot$	$\bot$	$\bot$
$[\text{SYLL}]_o(x)$	$\bot$	$\top$	$\top$
output	j	o	u

Table 8. Derivation: /jəu/ → [jou]

4. Mandarin rhotacization: data and generalization. In this section, I pursue a novel analysis of Mandarin rhotacization patterns using Sonority Scale while maintaining the generalizations from Section 2.

4.1. DATA. The dataset in Table 9 comes from Lin (1989), Shi (2003), and Duanmu (2007).

UR	SR	UR	SR	UR	SR	UR	SR
i-ər	jər	u-ər	ur	ə-ər	ɤr	(w)əi-ər	(w)ər
in-ər	jər	un-ər	ur	jə-ər	jer	(w)ai-ər	(w)ar
iŋ-ər	jə̃r	uŋ-ər	ũr	ɥə-ər	ɥer	(j)əu-ər	(j)our
y-ər	ɥər	(G)a-ər	(G)ar	wə-ər	wor	(j)au-ər	(j)aur
yn-ər	ɥər	(G)an-ər	(G)ar	ən-ər	ər	ʐ̩-ər	ər
yŋ-ər	ɥə̃r	(G)aŋ-ər	(G)ãr	əŋ-ər	ə̃r	z̩-ər	ər

Table 9. Mandarin rhotacization

The asymmetry between high front vowels and other vowels is observed in both nucleus and coda positions. High front vowels ([i, y]) originally in the nucleus position are reduced to glides ([j, ɥ]), and the suffix vowel [] becomes the nucleus vowel. In contrast, back vowels ([u]) and low vowels ([a]) originally in the nucleus position stay as the nucleus, and the suffix vowel is deleted. The vowel quality alternation in the surface forms of mid vowels obtains in rhotacized forms.

However, in diphthongs (the rightmost sub-table in Table 9), the mid vowel no longer undergoes progressive harmony or surfaces as [ɤ].

We abstract away from two observations in the literature: first, the nasal coda [n] does not nasalize the preceding vowel, but the nasal coda [ŋ] does. Zhang (2000) argues that the vowel before [n] is still nasalized, just not as much as the vowel before [ŋ]. Both nasals are deleted in the rhotacized forms. Second, it remains a debate whether [r] following [u] is a secondary articulation or an individual segment.

4.2. ANALYSIS. I argue that adjacent segments compete in terms of their sonority, in order to surface. The competition follows the sonority scale in Table 10, expanded from Kuo's (1994) sonority hierarchy in (6) (cf. Zec 1989). This is in line with Sonority Sequencing Principle (Selkirk 1984; Clements 1990): sonority rises to its peak at the nucleus vowel of any Mandarin syllables.

Most sonorous				$>_s$				**Least sonorous**
VOWEL					GLIDE	CONSONANT		NONE
LOW	MID	HIGH BACK	[ə]	HIGH FRONT		NASAL	ORAL	
a	e, o, ɤ	u	ə	i, y	j, ɥ, w	n, ŋ	r, z̩, ʐ̩	#

Table 10. An expanded sonority scale

(6) Kuo's (1994) sonority hierarchy
non-high vowels $>_s$ high vowels $>_s$ nasals $>_s$ [-son]

To decide which segment survives in the rhotacized form, rime-final segments compete with the suffix vowel /ə/ in sonority. If the preceding segment is less sonorous than the suffix vowel, it is either deleted (if it is a nasal) or reduced to a glide (if it is a high front vowel). In contrast, if the preceding segment is more or equally sonorous than the suffix vowel, the segment is kept and the suffix vowel is deleted. The process repeats itself until the output form converges.

For example, the nasal codas are much less sonorous than [ə] in Table 10. Hence, the suffix vowel wins the competition and surfaces. For an /i/-ending diphthong, since /i/ is also less sonorous than /ə/, [ə] will surface as the nucleus vowel. However, for an [u]-ending diphthong, since /u/ is more sonorous than /ə/, [u] will surface as the nucleus vowel.

Segments	**Sonority compared to [ə]**	**Output**
Oral, Nasal C	$<_s$[ə]	always deleted
High front V	$<_s$[ə]	reduced/deleted
Low, mid, high back V	$>_s$[ə]	survive as nucleus

Table 11. Predictions

As shown in Table 11, our account successfully predicts the following patterns: (i) less sonorous nasal and syllabic consonant codas are always deleted; (ii) high front vowels, being less sonorous than [ə], are reduced to glides in nucleus positions and deleted in coda positions; (iii) other vowels, being more sonorous than [ə], remain unaffected by rhotacization.

5. Modeling Mandarin rhotacization with BMRS.

5.1. FORMAL BACKGROUND. To implement this competition account, I model the competition process with BMRSs (Chandlee & Jardine 2021). I define a binary relation $>_s$ that captures the ordering on the sonority scale, as in (7). To compare segment sonority, I further define two monadic predicates in the input signature: MSTP (i.e. more sonorous than predecessors) in (8a) and MSTS (i.e. more sonorous than successors) in (8b). Each predicate looks for 2 steps in the given direction.

(7) a $>_s$ e, o, ɤ $>_s$ u $>_s$ ə $>_s$ i $>_s$ j, ɥ, w $>_s$ n, ŋ $>_s$ r

(8) a. $\text{MSTP}(x) = x >_s p(x) \wedge x >_s p(p(x))$
b. $\text{MSTS}(x) = x >_s s(x) \wedge x >_s s(s(x))$

The input and output signatures are presented in (9).

(9) a. $\mathcal{I} = \{p, s; \text{[FR]}, \text{[BA]}, \text{[RO]}, \text{[HI]}, \text{[LO]}, \text{[CONS]}, \text{[SYLL]}; \text{MSTP}, \text{MSTS}\}$
b. $\mathcal{O} = \{p, s; \text{[FR]}_o, \text{[BA]}_o, \text{[RO]}_o, \text{[HI]}_o, \text{[LO]}_o, \text{[CONS]}_o, \text{[SYLL]}_o; \texttt{output}\}$

Crucially, an `output` function is defined with the functions from the input signature and added to the output signature. As defined in (10), the predicate first checks if the input segment x is more sonorous than its predecessor and the predecessor of its predecessor. The paths diverge here: (a) if so, $\texttt{output}x$ evaluates to $\top$ and x will surface; (b) if not, the evaluation continues with the `else`-clause and reads the second line. The paths diverge yet again: (b-i) if x is a [ə], then the function evaluates to $\bot$ and x will be deleted; (b-ii) if x is not a [ə], then the function moves onto the third line and checks if x is more sonorous than its successor and the successor of its successor. Then comes the last divergence: (b-ii-1) if so, `output` evaluates to $\top$ and x should surface; (b-ii-2), if not, that means x is less sonorous than any of its surrounding segments (within 2 steps) and hence x fails to surface. In short, `output` decides whether or not a segment should surface.

(10)
$$\begin{array}{llllll} \texttt{output}(x) & = & \texttt{if}\ \text{MSTP}(x) & \texttt{then}\ \top & \texttt{else} \\ & & \texttt{if}\ [\text{ə}](x) & \texttt{then}\ \bot & \texttt{else} \\ & & \texttt{if}\ \text{MSTS}(x) & \texttt{then}\ \top & \texttt{else}\ \bot \end{array}$$

On a side note, $[\text{ə}](x)$ is really just a shorthand notation for a series of licensing conditions that can be defined with functions in the input signature. The syntactic sugar does not change the expressive power of the system.

5.2. DERIVATION. I now demonstrate how this BMRS system captures the vowel asymmetry in Mandarin rhotacization. The key is the evaluation of $\text{MSTP}(x)$ for the input segment /ə/ from the rhotacization suffix. In Section 4, I attribute the asymmetry to sonority: The suffix vowel /ə/ is the most sonorous in /i-ər/, while it is not as sonorous as /u/ in /u-ər/. Hence, [ə] surfaces in the former case but gets deleted in the latter.

As the predicates MSTP and MSTS ask each segment to look for 2 steps in both directions, we place 2 boundary markers on each side of the input strings. The boundary marker has the lowest sonority by default (we can simply insert the # symbol to the bottom of the sonority scale).

Table 12 and Table 13 show that the BMRS system is making the right predictions for high vowels. I will tackle the glide formation process later in this section.

input	#	#	i	ə	r	#	#
MSTP(x)			$\top$	$\top$	$\bot$		
MSTS(x)			$\bot$	$\top$	$\top$		
[ə](x)			$\bot$	$\top$	$\bot$		
`output`(x)			$\top$	$\top$	$\top$		
output			i	ə	r		

Table 12. Derivation: /i-ər/ $\rightarrow$ [jər]

input	#	#	u	ə	r	#	#
MSTP(x)			$\top$	$\bot$	$\bot$		
MSTS(x)			$\top$	$\top$	$\top$		
[ə](x)			$\bot$	$\top$	$\bot$		
`output`(x)			$\top$	$\bot$	$\top$		
output			u		r		

Table 13. Derivation: /u-ər/ $\rightarrow$ [ur]

Mid and low vowels behave alike, so I only demonstrate for the mid vowel in Table 14. Note that I assume that the rhotacization marker affects the *underlying* but not the *surface* forms of rimes (cf. Duanmu 2007; Lin 1989; Shi 2003). This assumption is well motivated since the surface mid vowel alternation can be accounted for with an independent system (see Section 3).

input	#	#	ə	ə	r	#	#
MSTP(x)			$\top$	$\bot$	$\bot$		
MSTS(x)			$\bot$	$\top$	$\top$		
[ə](x)			$\bot$	$\top$	$\bot$		
`output`(x)			$\top$	$\bot$	$\top$		
output			ə		r		

Table 14. Derivation: /ə-ər/ $\rightarrow$ [ər]

Table 15 and 16 demonstrate the derivation for rimes with nasal codas, and Table 17 and 18 are for diphthong rimes with vowel codas. In both cases, the asymmetry between /i/ and /u/ is captured. Notice that in /un-ər/, the deletion of the nasal coda creates the environment for the /ə/ to be deleted, as /ə/ is less sonorous than /u/. This BMRS system captures that by allowing each segment to look at not only its neighbor, but also the neighbor of its neighbor. Therefore, /ə/ evaluates to $\bot$ for MSTP(x) and gets deleted. The same reasoning applies in the diphthong rimes.

input	#	#	i	n	ə	r	#	#
MSTP(x)			$\top$	$\bot$	$\top$	$\bot$		
MSTS(x)			$\bot$	$\bot$	$\top$	$\top$		
[ə](x)			$\bot$	$\bot$	$\top$	$\bot$		
`output`(x)			$\top$	$\bot$	$\top$	$\top$		
output			i		ə	r		

Table 15. Derivation: /in-ər/ $\rightarrow$ [ər]

input	#	#	u	n	ə	r	#	#
MSTP(x)			$\top$	$\bot$	$\bot$	$\bot$		
MSTS(x)			$\top$	$\bot$	$\top$	$\top$		
[ə](x)			$\bot$	$\bot$	$\top$	$\bot$		
`output`(x)			$\top$	$\bot$	$\bot$	$\top$		
output			u			r		

Table 16. Derivation: /un-ər/ $\rightarrow$ [ur]

input	#	#	a	i	ə	r	#	#
MSTP(x)			⊤	⊥	⊥	⊥		
MSTS(x)			⊤	⊥	⊤	⊤		
[ə](x)			⊥	⊥	⊤	⊥		
output(x)			⊤	⊥	⊥	⊤		
output			a			r		

Table 17. Derivation: /ai-ər/ → [ar]

input	#	#	a	u	ə	r	#	#
MSTP(x)			⊤	⊥	⊥	⊥		
MSTS(x)			⊤	⊤	⊤	⊤		
[ə](x)			⊥	⊥	⊤	⊥		
output(x)			⊤	⊤	⊥	⊤		
output			a	u		r		

Table 18. Derivation: /au-ər/ → [aur]

5.3. LOOSE ENDS. I have shown that this BMRS system can capture the sonority competition between segments. There are two loose ends: first, we need to capture glide formation processes ([i, y] reduced into [j, ɥ]). Second, we need to capture the mid vowel alternation in the surface representations, if we assume that rhotacization affects the underlying forms.

5.3.1. GLIDE FORMATION. In fact, the process of glide formation generalizes across the citation forms and the rhotacized forms: if a high vowel is followed by another vowel, the high vowel will be reduced to a glide. Kuo (1994) accounts for this reduction with his sonority ranking. In BMRS, we can tweak our definitions a relevant output feature function, $[\text{SYLL}]_o$. As shown in (11), the function checks if the input segment x is high and followed by another vowel. If both conditions are met, then the predicate evaluates to $\bot$ and x surfaces as a glide. If not, x keeps its [syll] feature value. [2]

(11) $[\text{SYLL}]_o(x) = \texttt{if}\ [\text{HI}](x) \wedge \neg\, [\text{CONS}](s(x)) \wedge [\text{SYLL}](s(x))\ \texttt{then}\ \bot\ \texttt{else}\ [\text{SYLL}](x)$

5.3.2. MID VOWEL ALTERNATION. Table 19 repeats the mid vowel alternation data, in rhotacized forms and in citation forms. There is only one difference between the rhotacized and the citation columns: /(w)əi/ becomes [(w)ər] in the rhotacized form, but [(w)ei] in the citation form.

UR	SR (rho)	SR (cit)
ə-ər	ɤr	ɤ
jə-ər	jer	je
ɥə-ər	ɥer	ɥe
wə-ər	wor	wo
ən-ər	ər	ən
əŋ-ər	ə̃r	ən
(w)əi-ər	(w)ər	(w)ei
(j)əu-ər	(j)our	(j)ou

Table 19. Rhotacization data

[2] A theoretical implication is that the segments in the glide position (G) can be high vowels underlyingly. Although a smaller segment inventory would be nice, there is an uninviting consequence. Originally, we had /wə-ər/ → [wor]; now, assuming [w] is underlyingly /u/, /uə-ər/ → [ur]. This prediction is not borne out. Hence, we need to assume glides in the inventory.

Recall that the BMRS system in Section 3 can capture the opacity in mid vowel harmony by first checking the [front]/[back] feature values of the successor of an input segment. Here we can add a blocking structure in $[\textsc{fr}]_o(x)$ and $[\textsc{ba}]_o(x)$, to check if the succeeding segment (which is not a [ə] from the suffix) is output or not. When /i/ is deleted, the mid vowel harmony disappears. The BMRS system is finalized in (12), ready to capture the mid vowel harmony patterns in both rhotacized and citation forms.

(12)
$$
\begin{array}{lcl}
[\textsc{fr}]_o(x) & = & \texttt{if output}(s(x)) \wedge \neg\, [\text{ə}](s(x))\ \texttt{then} \\
 & & \quad \texttt{if}\ [\textsc{fr}](s(x))\ \texttt{then}\ \top\ \texttt{else} \\
 & & \quad \texttt{if}\ [\textsc{ba}](s(x))\ \texttt{then}\ \bot\ \texttt{else} \\
 & & \quad \texttt{if}\ [\textsc{fr}](p(x))\ \texttt{then}\ \top\ \texttt{else}\ [\textsc{fr}](x) \\
 & & \texttt{else}\ \bot \\
[\textsc{ba}]_o(x) & = & \texttt{if output}(s(x)) \wedge \neg\, [\text{ə}](s(x))\ \texttt{then} \\
 & & \quad \texttt{if}\ [\textsc{fr}](s(x))\ \texttt{then}\ \bot\ \texttt{else} \\
 & & \quad \texttt{if}\ [\textsc{ba}](s(x))\ \texttt{then}\ \top\ \texttt{else} \\
 & & \quad \texttt{if}\ [\textsc{ba}](p(x))\ \texttt{then}\ \top\ \texttt{else}\ [\textsc{ba}](x) \\
 & & \texttt{else}\ \bot \\
[\textsc{ro}]_o(x) & = & \texttt{if}\ [\textsc{ba}]_o(x)\ \texttt{then}\ \top\ \texttt{else}\ [\textsc{ro}](x) \\
[\textsc{hi}]_o(x) & = & [\textsc{hi}](x) \\
[\textsc{lo}]_o(x) & = & [\textsc{lo}](x) \\
[\textsc{cons}]_o(x) & = & [\textsc{cons}](x) \\
[\textsc{syll}]_o(x) & = & \texttt{if}\ [\textsc{hi}](x) \wedge \neg\, [\textsc{cons}](s(x)) \wedge [\textsc{syll}](s(x))\ \texttt{then}\ \bot\ \texttt{else}\ [\textsc{syll}](x)
\end{array}
$$

6. Conclusion. In this project, I revisited Beijing Mandarin mid vowel harmony and rhotacization patterns, proposed a novel analysis with sonority ranking, and modeled the processes with one self-contained BMRS system. Admittedly, this analysis features a sonority scale more detailed than a commonly assumed one (low $>$ mid $>$ high V; de Lacy 2007). Although it is potentially controversial, the hierarchy is motivated for two reasons. First, [ə] is an underspecified mid vowel and behaves phonologically distinct as compared to other mid vowels. This suffix vowel of the rhotacization marker could be special in Mandarin. Second, the asymmetry between [i] and [u] needs to be encoded somewhere, if not the sonority scale. This analysis naturally predicts that nasal codas and front vowel codas are always deleted, but the high back vowels are not.

Future work could extend this analysis to other Mandarin dialects. The vowel asymmetry is also attested in Xi'An and Nanjing Mandarin. Tianjin Mandarin, which differs from Beijing Mandarin primarily in tones (and also slightly in vowel qualities), shows a slightly different pattern that can be captured after tweaking our BMRS definitions. Ideally, we could test this analysis against different Mandarin dialects, to better our understanding of dialectal typology.

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