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A first semantics for *at first* and *at last*

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Abstract. This paper offers the first ever semantic analysis of a puzzling restriction on the distribution of ordinal numbers in English: while the temporal adverbials *at first* and *at last* are felicitous, putting any other ordinal in this environment is degraded (#*at second*, #*at third*). My analysis builds on the notion that assertions are relativized to a salient time interval, known in the literature as reference time or topic time. On my semantics, *at first* and *at last* further relativize an assertion to a salient subinterval of the topic time that shares an infimum (first point) or supremum (last point) with it. On the standard assumption that time-intervals are dense, the infelicity of #*at second*, #*at third*, etc. follows from this semantics. Since *at first* and *at last* invoke the first and last points of a time-interval on my analysis, #*at second* will attempt to invoke the second (i.e. second earliest) point of a time-interval. Invoking the first and last points of a dense interval is coherent, but invoking the second point (i.e. the point closer to the first point than any other) is not. My analysis makes interesting predictions about the interaction of *at first*/*at last* with present tense and frame adverbials, and it opens up several avenues for future research.

Keywords. ordinals; temporal adverbials; density; scale structure

1. Introduction. This paper discusses a puzzling restriction on the distribution of ordinal numbers in English: while the temporal adverbials *at first* and *at last* are felicitous (1), putting any other ordinal in this environment is degraded (2).

(1) a. Harper was jogging at first.
  b. Bobby walked away from the fight at last.

(2) a. Harper was jogging (#at second).
  b. Bobby walked away from the fight (#at eighth).

To my knowledge, there is no literature on *at first*/*at last* or the contrast in (1)-(2). In this paper, I develop a semantic analysis of *at first* and *at last* that critically invokes the notion that assertions are relativized to a salient time interval, known in the literature as a reference time (Reichenbach 1947) or topic time (Klein 1994). On my semantics, *at first* and *at last* further relativize an assertion to a salient subinterval of the topic time that shares an infimum (first point) or supremum (last point) with it. For example, suppose that I ask you what Harper was doing between 8pm and 10pm last night, and you respond with (1a). On my theory, an assertion of (1a) in this context is true if and only if Harper was jogging during a salient initial subinterval of the overall topic time [8pm, 10pm]. This interval could be [8pm, 8:30pm], [8pm, 9pm], etc.

As for the contrast in (1) vs. (2), I suggest that it stems from a mathematical property about ordinal numbers and dense intervals that I will call the Ordinals and Density Principle (ODP):

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(3) **Ordinals and Density Principle (ODP)**:
In a dense interval, one can identify the first element (the infimum) and the last element (the supremum), but it is impossible to identify the second least element, the third least element, the second greatest element, etc.

To start with a mathematical example, consider the interval of rational numbers [0,1],[1] which is dense in the sense that for any two numbers in the interval n and n' such that $n < n'$, there exists an n'' such that $n < n'' < n'$. One can identify the first number in [0,1] (namely, 0) and the last number in the interval (namely, 1), but one can prove by contradiction that, for example, there is no second (i.e. second lowest) number in the interval. Suppose that the interval [0,1] has a second lowest number, call it n. In other words, suppose that n is greater than 0 and closer to 0 than any other rational number in the interval. Regardless of the identity of n, we can always find a number between 0 and n due to the density of the rational numbers, a fact which contradicts the assumption that n is the second lowest number in the interval. Since assuming that there is a second lowest number in [0,1] leads to a contradiction, we can conclude that there is no second lowest number in the interval.

It is standard to treat T, the set of all times, as dense (Taylor 1977, Bennett & Partee 1978, etc.). Armed with this conclusion from the literature, this paper develops an Ordinals and Density Principle-based account of the contrast in (1) vs. (2) by incorporating reference to the first and last points of a time-interval into the definedness conditions of *at first* and *at last*. If one adopts these definedness conditions for *at first* and *at last*, the definedness conditions for #*at second* will invoke the second earliest point of a time-interval, which by ODP is not a coherent notion.

The remainder of this paper is structured as follows. Section 2 defines in sharper terms the puzzles raised by the contrast between (1) vs. (2). In section 3, I develop a proposal for the semantics of *at first*/*at last* and the infelicity of #*at second*, #*at third*, etc. Section 4 discusses some predictions of the analysis and directions for future research.

2. Delineating the puzzle. In this section, I argue that the contrast between (1) and (2) is not self-explanatory but rather constitutes a puzzle that warrants serious inquiry. To do so, I spell out three ways in which the judgments on (1)-(2) are *prima facie* unexpected.

First, the judgments on (1)-(2) are *prima facie* unexpected because generally, substituting ordinals for one another is a matter of mundane lexical choice and does not affect acceptability. For example, one can freely substitute *first*, *last*, and other ordinals for one another when they are in adjectival position (4) and when they are in adverbial position (5). It is not immediately apparent why exercising a similar lexical choice in (1)-(2) leads to an acceptability contrast.

(4) a. The <u>first</u> train arrived at 11am.

b. The <u>second</u> train arrived at 11am.

c. The <u>last</u> train arrived at 11am.

[1] While I use a closed interval in this example, the Ordinals and Density Principle holds for open intervals as well: these intervals also have an infimum and supremum.

(5) a. Harry kissed Vicky first.

b. Harry kissed Vicky second.

c. Harry kissed Vicky last.

Additionally, the judgments on (1)-(2) are *prima facie* unexpected because one can imagine coherent meanings for (2a-b). To see this, consider the fact that one can paraphrase (1a) as "Harper was sleeping in the first stage of the relevant timespan." Since one can use *p + at first* to mean "p holds in the first stage of the relevant timespan," it is unclear why one cannot use *p + at second* to mean "p holds in the second stage of the relevant timespan." A proposition like "Harper was sleeping in the second stage of the relevant timespan" is perfectly coherent, but one cannot use (2a) to express that proposition.

Finally, the judgments on (1)-(2) are puzzling because in languages like Turkish, ordinals can productively combine with locative markers to produce the meaning "in the n-th stage" (Bergül Soykan, p.c.). The examples in (6) show that *first*-LOC, *second*-LOC, and *last*-LOC mean *in the first stage*, *in the second stage*, and *in the last stage*, respectively.[2]

(6) a.

İlk-in-de	Ali-yi	çok	sev-me-di-m.
first-POSS-LOC	Ali-ACC	very	like-NEG-PST-1SG

At first/in the first stage, I didn't like Ali very much.

b.

İkinci-sin-de	on-a	aliş-tım.
second-POSS-LOC	3SG-DAT	get.used.to-PST-1SG

In the second stage, I got used to him. (lit. at second)

c.

Son-un-da	on-dan	nefret	et-ti-m.
last-POSS-LOC	3SG-DAT	hate	make-PST-1SG

At last/in the end, I hated him.

Since one can derive meanings similar to the Turkish *ılk-in-de* 'first-LOC' and *son-un-da* 'last-LOC' in English by attaching the locative preposition *at* to *first* and *last*, it is unclear why one cannot attach *at* to *second* and derive a meaning similar to the Turkish *ıkinci-sin-de* 'second-LOC.'

3. The proposal. Having argued that the data in (1)-(2) are genuinely puzzling, I now develop a semantic analysis of *at first* and *at last* that captures these data. As sketched in section 1, my proposal has two crucial components: (a) a semantics for *at first* and *at last* that incorporates reference to the first and last points of a time-interval into their definedness conditions; and (b) the assumption that time-intervals are dense. Upon adopting (a) and (b), the definedness conditions for, e.g., #*at second* and #*at eighth* will be unsatisfiable because they refer to the second and eighth points of a dense interval, respectively.

I present my proposal in three steps. Section 3.1 lays out my background assumptions about tense and aspect. I then expound the first ever semantic analysis of *at first* and *at last* (section 3.2) and show how this analysis captures the infelicity of (2a-b) (section 3.3).

[2] The possessive marker is (somewhat mysteriously) present in these examples, a fact which I return to in section 4.2.

3.1. BACKGROUND ASSUMPTIONS. Since *at first* and *at last* are temporal adverbials, any analysis of these expressions must be couched within a set of assumptions about tense and aspect. In this paper, I adopt standard assumptions about aspect alongside a version of the referential theory of tense (Partee 1973, Kratzer 1998, *inter alia*) that is most similar to the version presented in the von Fintel & Heim (1997–2023) textbook, which is in turn heavily inspired by von Stechow & Beck (2015).

Following von Fintel & Heim (1997–2023), I will assume the model of the clausal spine schematized in (7). I adopt a type *i* for time-intervals (with points as a special case), and I assume that the semantic denotation function is relativized to a time-interval as well as an assignment function ($[\![\ ]\!]^{t,g}$). Unless bound by a λ operator, this time-interval is the time of utterance, construed as an instant rather than an interval.

(7) $[_{TP}$ T $[_{AspP}$ Asp $[_{vP}$ vP]]]

I assume that vPs denote sets of events (type $\langle$v,t$\rangle$), though nothing in my proposal hinges on using event semantics. I further assume that AspPs denotes propositions, making the simplified assumption that propositions are sets of time-intervals (rather than, e.g., sets of world-time pairs).

In the referential theory of tense adopted here, T heads refer to the salient time-interval under discussion (the topic time), vPs pick out sets of events of a particular type, and Asp heads specify what relation holds between T and vP. To illustrate, consider a simple sentence like *Harper was jogging*, which receives the LF in (8).

(8) $[_{TP}$ PAST$_7$ $[_{AspP}$ IMPV $[_{vP}$ Harper jog]]]

Let us start by giving the meanings for T and vP in (8) before explaining how Asp mediates between them. The vP in (8) denotes the characteristic function of the set of Harper-jogging events, a function which I notate as follows:

(9) $[\![[_{vP}$ Harper jog$]]\!]^{t,g}$ = λe. jog(e, Harper)

A tense operator like PAST$_n$ (10) carries a numerical subscript that the assignment function maps to the time-interval that the speaker has in mind when making the current claim: the topic time. PAST$_n$ is defined only if the topic time g(n) precedes the time of utterance, and when defined, PAST$_n$ refers to g(n).[3] For example, if I ask you what Harper was doing from 8pm to 10pm last night and you respond *Harper was jogging*, the topic time in your response is clearly [8pm, 10pm]. As such, the past tense operator in your response refers directly to the topic time so long as that interval is in the past.

(10) $[\![\text{PAST}_n]\!]^{t,g}$ = g(n), defined iff g(n) $\in$ D$_i$ and g(n) $<$ t

[3] The entry for past tense in von Fintel & Heim 1997–2023 is notationally different from but truth-conditionally equivalent to (10). Their PAST$_n$ takes the proposition denoted by AspP as an argument and asserts that g(n) $\in$ $[\![\text{AspP}]\!]^t$, while g(n) is fed directly to $[\![\text{AspP}]\!]^t$ on my semantics (as shown below). (10) is closer in spirit to Partee (1973)'s original analogy between tenses and pronouns and is used by, e.g., Kratzer (1998).

I will treat present tense as vacuous (von Fintel & Heim 1997–2023) and ignore future tense so as to sidestep the thorny issue of whether it is better thought of as a tense or a modal (see Cariani & Santorio 2018 for an overview of arguments on both sides as well as a synthesis).

Aspect heads (imperfective and perfective) take $[\![\text{vP}]\!]^{t,g}$ and $[\![\text{T}]\!]^{t,g}$ as arguments and return true iff a particular relation holds between the topic time denoted by T and the set of events denoted by vP. The perfective head (11a) returns true iff that the topic time contains the runtime of some event in the set $[\![\text{vP}]\!]^{t}$, while the imperfective head (11b) returns true iff the topic time *is contained in* the runtime of some event in the set $[\![\text{vP}]\!]^{t,g}$ (see Klein 1994, Kratzer 1998, and many others for similar entries).[4]

(11) a. $[\![\text{PFV}]\!]^{t,g} = \lambda \text{P}_{\langle v,t\rangle}.\ \lambda \text{t}'.\ \exists \text{e}\ [\text{P(e)} = 1 \text{ and } \tau(\text{e}) \subseteq \text{t}']$

b. $[\![\text{IMPV}]\!]^{t,g} = \lambda \text{P}_{\langle v,t\rangle}.\ \lambda \text{t}'.\ \exists \text{e}\ [\text{P(e)} = 1 \text{ and } \tau(\text{e}) \supseteq \text{t}']$

If we plug our entries for *Harper jog* (9), PAST$_n$ (10), and IMPV (11b) into the LF for our example *Harper was jogging*, we arrive at the meaning in (12b). (12b) asserts that the topic time g(7) is contained in the runtime of a Harper-jogging event. In other words, Harper jogged throughout g(7), with her jogging possibly continuing beyond the temporal bounds of g(7) on either side.

(12) a. $[\![(8)]\!]^{t,g} = [\![\text{IMPV}]\!]^{t,g}([\![[_{vP} \text{ Harper jog}]]\!]^{t,g})([\![\text{PAST}_7]\!]^{t,g})$

b. $[\![(8)]\!]^{t}$ is defined iff $\text{g}(7) \in \text{D}_i$ and $\text{g}(7) < \text{t}$.
When defined, $[\![(8)]\!]^{t} = 1$ iff $\exists \text{e}\ [\text{jog(e, Harper) and } \tau(\text{e}) \supseteq \text{g}(7)]$

To see that the predicted truth-conditions for *Harper was jogging* are correct, let us once again suppose that *Harper was jogging* is uttered in response to the question *what was Harper doing from 8pm to 10pm last night?* Since the topic time established by the question is [8pm, 10pm], we predict that the response *Harper was jogging* conveys the information that she was jogging throughout the two-hour interval; we also predict that the response leaves it open whether or not her jogging episode extended before 8pm or after 10pm. This predicted meaning accords with our intuitions.

3.2. A SEMANTICS FOR *at first* AND *at last*. This section develops a semantic analysis for *at first* and *at last* using the assumptions laid out in section 3.1. On top of explaining the contrast between (1) and (2), a theory of *at first* and *at last* should capture the following core inferences triggered by these expressions:

(13) Harper was jogging at first.

a. $\rightsquigarrow$ Harper was jogging at a relevant stage treated as "initial"

b. $\rightsquigarrow$ Harper was not jogging later on in the relevant timespan.

[4] $\subseteq$ and $\supseteq$ are the subinterval and superinterval relations. $\tau()$ is the 'temporal trace' function; it takes an event *e* and returns the runtime of that event.

(14) Bobby walked away from the fight at last.

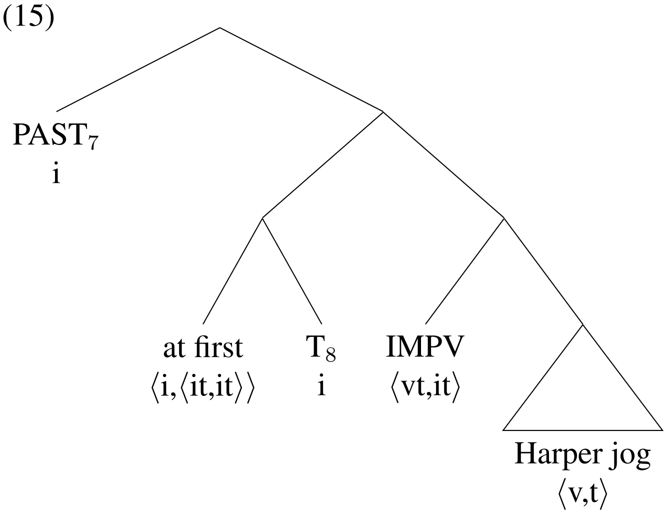
a. $\rightsquigarrow$ Bobby walked away at a relevant stage treated as “final”

b. $\rightsquigarrow$ Bobby did not walk away earlier on in the relevant timespan.

c. $\rightsquigarrow$ Bobby should have/was expected to walk away from the fight earlier than he did.

With these desiderata for a theory of *at first* and *at last* in mind, consider the LF I propose for (13), which is given in (15). The proposed LF for (14), given in (16), is trivially different. In these LFs, T_8 is a time-interval whose value is supplied by the assignment function, and [[at first] T_8]/[[at last] T_8] attach to AspP. For other examples of temporal adverbials that seem to attach to AspP, see Dowty (1979), Vlach (1993), Iatridou et al. (2001), and Newman (2021).

(15)



(16) [PAST$_7$ [[[at last] T$_8$][PFV [Bobby walk away from the fight]]]]

Before presenting the entries for *at first* and *at last* that will make these LFs interpretable, a few comments about my treatment of the internal structure of *at first* and *at last* are in order. I assume that *at first* and *at last* contain versions of *first* and *last* that take a time-interval and return the singleton set consisting of its first and last point, respectively. The entries in (18a-b) formalize this idea; note that *points*() takes a time-interval and returns the set of all points contained within the interval.

(17) $\textit{points}(\mathrm{t}) = \{\mathrm{t}' \colon \mathrm{t}' \subseteq \mathrm{t} \text{ and } \neg\exists \mathrm{t}'' \, [\mathrm{t}'' \subset \mathrm{t}']\}$

(18) a. $[\![\text{first}]\!]^{t,g} = \lambda \mathrm{t}'.\ \lambda \mathrm{t}''.\ \mathrm{t}'' \in \textit{points}(\mathrm{t}')$ and
$\forall \mathrm{t}''' \, [[\mathrm{t}''' \in \textit{points}(\mathrm{t}') \text{ and } \mathrm{t}''' \neq \mathrm{t}''] \rightarrow \mathrm{t}'' < \mathrm{t}''']$

b. $[\![\text{last}]\!]^{t,g} = \lambda \mathrm{t}'.\ \lambda \mathrm{t}''.\ \mathrm{t}'' \in \textit{points}(\mathrm{t}')$ and
$\forall \mathrm{t}''' \, [[\mathrm{t}''' \in \textit{points}(\mathrm{t}') \text{ and } \mathrm{t}''' \neq \mathrm{t}''] \rightarrow \mathrm{t}'' > \mathrm{t}''']$

While the entries in (18a-b) obviously will not work for all uses of *first* and *last*, they are temporal equivalents of the entries for *first* and *last* used in basic cases like *the first/last book*. In *the first/last book*, it is standardly assumed that *first* and *last* take a set of individuals (the set of relevant books) and return the singleton set consisting of the first/last one in the relevant order (Bhatt 2006; Sharvit 2010). Similarly, (18a-b) take a time-interval and return the singleton set consisting of the interval's first/last point in a temporal order. Given the similarity between (18a-b) and the entries for *first*/*last* used in other cases, I anticipate that future research may be able to craft entries for *first* and *last* that work across the board.

Even if a fully unified theory of *first* and *last* turns out to be unattainable, however, the claim that *at first* and *at last* contain atypical entries for *first* and *last* converges with Coppock (2016)'s compositional theory of superlative modifiers like *at least* and *at most*. Coppock (2016) proposes that *at least* and *at most* contain versions of *least* and *most* that take a set of numbers and return the singleton set consisting of its lowest or greatest member, respectively. This proposal involves entries for *least* and *most* that differ from the entries used in, e.g., *John read the most/least books*. If one takes this proposal on board, it would make sense for *first* and *last* to have a similarly atypical meaning in constructions with *at*.

Having discussed my treatment of *first* and *last*, I now introduce the proposed entry for *at first*, which is given in (19). I remain neutral as to whether *at first* is built up from *first* with the help of a non-vacuous *at* or hidden structure.

(19) $[\![\text{at first}]\!]^{t,g} = \lambda t'.\ \lambda p_{\langle i,t \rangle}.\ \lambda t''$:
$t' \subseteq t''$ and $\exists t'''$ $[[\![\text{first}]\!]^{t,g}(t')(t''') = 1$ and $[\![\text{first}]\!]^{t,g}(t'')(t''') = 1]$.
$p(t') = 1$

Informally, *at first* takes a time-interval t', takes a proposition p, takes a second time-interval t'', and returns true iff p holds at t', where t' is presupposed to be a subinterval of t'' that shares a first point with it. In LF (15), t' corresponds to g(8), p corresponds to the denotation of the AspP, and t'' corresponds to the overall topic time g(7), i.e. the denotation of PAST_7. As such, LF (15) is true iff *Harper be jogging* holds at g(8), where g(8) is presupposed to be a subinterval of the overall topic time g(7) that shares a first point with it. As a result of the contribution of PAST_7, g(7) is presupposed to be entirely in the past, meaning that its subinterval g(8) must also be in the past.

(20) a. $[\![(15)]\!]^{t,g} = [\![\text{at first}]\!]^{t,g}([\![\text{T}_8]\!]^{t,g})([\![[_{AspP}\ \text{IMPV [Harper jog]}]]\!]^{t,g})([\![\text{PAST}_7]\!]^{t,g})$

b. $[\![(15)]\!]^{t,g}$ is defined iff $g(7) \in D_i$ and $g(7) < t$ and
$g(8) \subseteq g(7)$ and $\exists t'''$ $[[\![\text{first}]\!]^{t,g}(g(8))(t''') = 1$ and $[\![\text{first}]\!]^{t,g}(g(7))(t''') = 1]$.
When defined, $[\![(15)]\!]^{t,g} = 1$ iff $\exists e$ [jog(e, Harper) and $\tau(e) \supseteq g(8)$]

To see the predicted truth-conditions on a more intuitive level, suppose that when asked the question *what was Harper doing between 8pm and 10pm last night?*, you respond *Harper was jogging at first*. On my semantics, what you are asserting is that *Harper was jogging* holds of a salient subinterval of the overall topic time [8pm, 10pm] that shares a first point with it. [8pm, 8:15pm], [8pm, 8:30pm], and so on are all licit choices for the salient initial subinterval in this context. However, trying to choose something like [8:30pm, 9:30pm] as the salient subinterval leads to presupposition failure on my semantics because [8:30pm, 9:30pm] does not share a first point with the overall topic time [8pm, 10pm]. This semantics captures the intuition that in (13), *Harper be jogging* is claimed to hold in an "initial stage" (13a).

When faced with the task of analyzing two expressions that look as similar as *at first* and *at last* do, it is best to initially consider a unified analysis, even if a unified analysis ends up being untenable in the face of more data. In that spirit, I offer in this short paper a parallel treatment of *at first* and *at last*, discussing below some ways in which the analysis works better for *at first* than *at last*. Assuming an entry for *at last* that mirrors our entry for *at first*, one can derive that the LF in (16) is true iff Bobby walked away from the fight in a salient subinterval of the overall topic time that shares a supremum with the topic time.[5]

(21) a. $[\![(16)]\!]^{t,g}$ = $[\![\text{at last}]\!]^{t,g}([\![\text{T}_8]\!]^{t,g})([\![[_{AspP}\ \text{PFV [Bobby walk away from the fight]]}]\!]^{t,g})$ $([\![\text{PAST}_7]\!]^{t,g})$

b. $[\![(16)]\!]^{t,g}$ is defined iff g(7) $\in$ D_i and g(7) $<$ t and g(8) $\subseteq$ g(7) and $\exists$t‴ [$[\![\text{last}]\!]^{t,g}$(g(8))(t‴) = 1 and $[\![\text{last}]\!]^{t,g}$(g(7))(t‴) = 1]. When defined, $[\![(16)]\!]^{t,g}$ = 1 iff $\exists$e [walk-away-from(e, Bobby, the-fight) and τ(e) $\subseteq$ g(8)]

Prima facie, these truth-conditions for (16) are not obviously on the wrong track (though see section 4.2 for discussion of ways in which the analysis works better for *at first* than for *at last*). For example, a natural candidate for the topic time in (14) is the time between when the fight started and when Bobby walked away from the fight. Suppose that this is the interval [7pm, 8:30pm]. Then, (16) asserts that Bobby walked away in a salient subpart of [7pm, 8:30pm] that shares a supremum with it. This interval could be [8:25pm, 8:30pm], [8:20pm, 8:30pm], etc.

The current analysis captures the fact that *at first* and *at last* trigger the inferences in (13a) and (14a), respectively; after all, the truth-conditional import of *at first*/*at last* on my approach involves relativizing an assertion to a salient initial/final subinterval of the relevant timespan. However, the inferences in (13b) and (14b-c) do not follow from our current semantics for (13) and (14). On the current analysis, (13) is true iff *Harper be jogging* holds at a salient initial subinterval of the relevant timespan; there is no additional truth-conditional requirement that Harper's jogging cease later on in the relevant timespan. But intuitively, (13) does carry such an inference. The current analysis fails to capture the inference from (14) to (14b) for similar reasons, and it fails to capture the inference from (14) to (14c) because the current semantics for *at last* encodes no notion of delay, expectation, or obligation.

In order to capture the inferences in (13b) and (14b), we could add a conjunct to the assertive content of sentences with *at first*/*at last*, as shown for *at last* in (22). Note that the interval $t''\backslash t'$ is the part of t'' that does not overlap with t'.

(22) $[\![\text{at last}]\!]^{t,g}$ = λt′. λp$_{\langle i,t\rangle}$. λt″:
t′ $\subseteq$ t″ and $\exists$t‴ [$[\![\text{last}]\!]^{t,g}$(t′)(t‴) = 1 and $[\![\text{last}]\!]^{t,g}$(t″)(t‴) = 1].
p(t′) = 1 **<u>and p(t″\t′) = 0</u>**

Alternatively, we could stick to entries of the sort shown in (19) and derive the inferences in (13b) and (14b) as implicatures. For example, one could say that *Harper was jogging at first* is in competition with *Harper was jogging throughout*, which is stronger than it. As such, an assertion

[5] "walk-away-from(e, Bobby, the fight)" is obviously not a serious event-semantic analysis of *Bobby walk away from the fight*; I do not pursue a more serious analysis for the sake of expository clarity.

of (13) triggers the inference that Harper was not jogging throughout the topic time. Similar considerations could explain why (14) carries the inference that Bobby did not walk away earlier on in the relevant timespan. I leave the choice between treating the inferences in (13b) and (14b) as truth-conditional content or as implicatures to future research.

While there are some straightforward ways to capture the inferences in (13b) and (14b) on my analysis, the inference in (14c) is more difficult to capture so long we continue to assume a uniform semantics for *at first* and *at last*. The inference from (14) to (14c) calls a unified analysis of *at first* and *at last* into question because there is no parallel inference associated with *at first*: *Harper was jogging at first* does not seem to imply something like "Harper should have jogged/was expected to jog later on in the relevant timespan."

I do not attempt to resolve here the question of whether (14c) necessitates non-uniform entries for *at first* and *at last*. I will merely note that if one allows for non-uniform entries, one can easily capture the inference from (14) to (14c) by adding, e.g., a modal component to *at last* that is absent for *at first*. For example, one could revise our entry for *at last* (22) so that its assertive content has three conjuncts rather than two: (a) *p* holds at a final subinterval of the topic time *t*; (b) *p* does not hold at an earlier subinterval of *t*; (c) in all worlds compatible with obligations/expectations/etc., *p* holds at an earlier subinterval of *t*. On a non-uniform semantics, the modal component of the entry for *at last* would be missing from the entry for *at first*.

3.3. THE INFELICITY OF #*at second*, #*at third*, ETC. Regardless of whether uniform or non-uniform variants of my entries for *at first* and *at last* end up being correct, a major virtue of the general approach to *at first*/*at last* adopted here is that it immediately entails the infelicity of #*at second*, #*at third*, etc. To see this, we need to formulate a general schema for *n-th* modeled on our entries for *first* and *last* (18a-b).

The following entry for *n-th* takes two time-intervals t' and t''; it returns true iff t'' is a point in t' such that there are exactly n - 1 points in t' that precede t''. For example, *second* (i.e. *two-th*) returns true iff t'' is a point in t' such that there is exactly one point in t' that precedes t''.

(23) $[\![\text{n-th}]\!]^{t,g} = \lambda \text{t}'.\ \lambda \text{t}''.\ \text{t}'' \in \textit{points}(\text{t}') \text{ and } |\{\text{t}''' \text{: t}''' \in \textit{points}(\text{t}') \text{ and t}''' < \text{t}''\}| = \text{n - 1}$

By the Ordinals and Density Principle, *second* will return false for all t' and t''. For example, consider the interval [8pm, 10pm] and assume that there is a time t'' such that $[\![\text{second}]\!]^{t,g}$([8pm, 10pm])($t''$) = 1; in other words, 8pm $< t'' \leq$ 11pm and there exists no t''' such that 8pm $< t''' < t''$. Our axiom that time-intervals are dense contradicts our assumption that $[\![\text{second}]\!]^{t,g}$([8pm, 10pm])($t''$) = 1, since the former guarantees that there does exist a t''' such that 8pm $< t''' < t''$. Hence, there is no time t'' such that $[\![\text{second}]\!]^{t,g}$([8pm, 10pm])($t''$) = 1.

Since *second* will return false for all t' and t'', #*at second* ends up having an unsatisfiable presupposition, making it infelicitous in every context. To see this, consider the predicted definedness conditions for #*Harper was jogging at second* (using an entry for *at second* that mirrors the entry for *at first* given in (19)):

(24) $[\![\text{PAST}_7 \text{ [[[at second] T}_8\text{][IMPV [Harper jog]]]}]\!]^{t,g}$ is defined iff g(7) $\in D_i$ and g(7) $<$ t and g(8) $\subseteq$ g(7) and
$\exists \text{t}'''$ [$[\![\text{second}]\!]^{t,g}$(g(8))(t''') = 1 and $[\![\text{second}]\!]^{t,g}$(g(7))(t''') = 1].

We see that just as *at first* presupposes of two time-intervals that they share a first point, #*at second* presupposes of two time-intervals that they share a second point. By the Ordinals and Density Principle, the former presupposition is satisfiable and the latter is not.

4. Conclusion: Directions for future research. In this paper, I have offered a first semantics for *at first* and *at last* that captures many of their core entailments as well as the infelicity of #*at second*, #*at third*, etc. I conclude by highlighting three of the many directions for future research into these expressions.

4.1. MORE ASYMMETRIES BETWEEN *at first* AND *at last*. In section 3.3, I considered the question of whether *at first* and *at last* should receive uniform or non-uniform lexical entries in light of the fact that *at last* carries an inference that *at first* lacks. Future work trying to adjudicate between uniform and non-uniform approaches to *at first* and *at last* should consider not just their different entailments but also asymmetries between the two expressions when they co-occur with present tense and punctual frame adverbials. *At first* is infelicitous with the (non-historical) present and with adverbials like *at 11pm sharp*, while *at last* is felicitous in these environments:

(25) A: How is John feeling right now?
  a. B: #John is tired at first.
  b. B: John is tired at last.

(26) a. #At 11pm sharp, John was tired at first.
  b. At 11pm sharp, John was tired at last.

The theory developed in section 3 can account for the infelicity of (25a) and (26a). The infelicity of (25a) immediately follows from my semantics for *at first* and my assumption that the topic time is an instant in present-tense sentences (see section 3.1). On my account, (25a) asserts that *John be tired* holds at a salient initial subpart of the topic time, which in this case is the present instant. But since the only subpart of an instant is that instant, (25a) makes no semantic contribution above and beyond the simpler *John is tired*, so (25a) is redundant and thus infelicitous. Similar logic would explain the unacceptability of (26a) if we assume that *at 11pm sharp* restricts the topic time to the instant 11pm.

However, the theory as it stands fails to predict the felicity of (25b) and (26b). On the theory presented in section 3.3, *at last* (just like *at first*) is redundant unless the topic time has proper subintervals. So if we adopt the previous paragraph's explanation of the infelicity of (25a) and (26a), we wrongly predict (25b) and (26b) to be equally infelicitous. I leave to future research the question of whether a uniform analysis of *at first* and *at last* can handle the data in (25)-(26) or whether *at first* and *at last* lexically encode different relations to the topic time.

4.2. THE CROSS-LINGUISTIC PICTURE. In section 2.2, I noted that unlike English ordinals, Turkish ordinals can productively combine with a locative marker to produce the meaning "in the n-th stage" (see (6a-c)). Future work should consider more closely this cross-linguistic variation in the ability of ordinals to combine with locative markers. A plausible initial hypothesis is that the relevant difference between English and Turkish relates to the presence of the possessive morpheme in Turkish "locative ordinals"; perhaps the possessive morpheme in (6a-c), which is obligatory (Bergül Soykan, p.c.), betrays the presence of some hidden structure that is absent from the English *at first* and *at last*.

4.3. THE STATUS OF THE ORDINALS AND DENSITY PRINCIPLE. My account of the infelicity of #*at second*, #*at third*, etc. crucially relies on a mathematical property about ordinals and dense intervals that I have called the Ordinals and Density Principle (ODP). Given that the Ordinals and Density Principle is a general mathematical fact, one might wonder whether there exist grammatical reflexes of the ODP beyond the contrast between *at first*/*at last* and #*at second*/#*at third*. Of particular theoretical interest is the question of whether we find ODP effects not just for intuitively dense scales such as times, but also for intuitively discrete scales. If we find apparent ODP effects in sentences that intuitively evoke a discrete scale like the natural numbers, we would have supporting evidence for the Universal Density of Measurement (Fox & Hackl 2006). I leave the task of searching for other ODP effects to future research.

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