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Homogeneity and identity: A solution to the problem of minimal parts

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Abstract. Plural and non-count nouns have been characterized as having homogeneous reference, the combination of the second-order properties of cumulative and divisive reference. Homogeneous reference is a useful property that captures similarities in the behavior of count plural and non-countable nouns. However, formal semantic definitions of divisive reference—the property that any part of what a noun refers to can also be referred to with that noun—run into the Problem of Minimal Parts. While both a portion of water and another portion of water that it is a part of can be referred to with the noun phrase *water*, both have parts such as hydrogen and oxygen atoms that are not water. This paper introduces a new formulation of homogeneous reference that avoids the Problem of Minimal Parts by making reference to the sum operation, rather than parthood relations.

Keywords. semantics; philosophy of language; mereology; countability

1. Introduction. Nouns in English—as well as many other languages—can be classified as count or non-count depending on their syntactic behavior including acceptability with cardinal numbers, plural morphosyntax, or quantifiers like *many*, *much*, or *several*, as well as their semantic behavior. In both their syntactic and semantic behavior, there are interesting similarities between count plural nouns and non-count (i.e. mass) nouns. Syntactically, non-count nouns behave similarly to plurals in regard to verb agreement while semantically, some argue the countability distinction parallels a distinction between objects and unindividuated matter, and is reflected in the logical structure of these expressions. For example, a noun phrase has the second-order property of cumulative reference Quine (1960) if for any two things that noun refers to, the sum of those two things can also be referred to with that noun—two groups of penguins are also cumulatively referred to as *penguins*, but a group composed of two singular tigers is not called *a tiger*. A noun phrase has the property of divisive reference (Cheng 1973) if any part can also be referred to with the same noun, such as any portion of the water in a glass of water can also be called *water*. Cumulative and divisive reference together are known as the property of homogeneous reference (Bunt 1985). While these properties have been discussed at length in the semantics literature, accounts of homogeneity, specifically divisive reference, have struggled to model them formally without running into problems. Of particular issue is the Problem of Minimal Parts, that nouns with divisive reference have some point at which they are no longer divisive—not all parts of what *water* refers to are actually water.

This paper presents a new proposal that captures the useful aspects of homogeneous reference while avoiding the Problem of Minimal Parts. For predicates like *water*, the formulation of homogeneity should show that they indeed refer to both an entity and some parts of that entity, but that not all parts should count as *water*. Some entities are identical with regard to the application of a predicate while others are not. At its core, the Problem of Minimal Parts is an identity problem: something about the identity of the referents of noun phrases with this property impacts how reference is achieved. Building on Fine (2010), I argue that, rather than focusing on part-

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hood relationships, homogeneity should be defined in terms of identity preservation with respect to sum formation. With reference to one of Fine's principles of composition, Leveling, I rework the second-order property of homogeneous reference.

Section 2 provides further background on homogeneity, cumulativity, divisiveness, and the Problem of Minimal Parts. Section 3 surveys the history of the Problem and some proposed solutions, including weak homogeneity, restricted homogeneity, stratified reference, and vagueness. In section 4, I motivate a new approach, building off the framework in Fine (2010). I then present the formal details of my proposal of homogeneous reference and discuss the predictions this proposal makes and some relevant data. Section 5 concludes with directions for future work.

2. Background. This section provides background and some relevant definitions. I first present and discuss definitions of cumulativity and divisiveness from Krifka (1989) before laying out the Problem of Minimal Parts in detail.

2.1. CUMULATIVITY AND DIVISIVENESS. Cumulative reference is defined as follows:

(1) $\forall P[\mathbf{CUM}(P) \leftrightarrow \forall x \forall y[P(x) \wedge P(y) \rightarrow P(x \cup y)]]$
P has cumulative reference; for any two things that are P, their sum is P. Krifka (1989)

For example, the plural noun phrase *books* has cumulative reference: any two things that can be referred to as *books*, taken together, are *books*. Non-count noun phrases also have cumulative reference, for example if there is *snow* outside, and later that day more *snow* falls on top of it, the sum of that stuff outside still can be referred to as *snow*. Singular count nouns, on the other hand, do not have cumulative reference—*a bagel* and *a bagel*, even when stacked on top of each other, cannot be collectively referred to as *a bagel*.

It is important to note that in Krifka's view, cumulativity and divisiveness are properties of what he calls "expressions," or noun phrases. For example, *beer* has cumulative reference, but *a bottle of beer* does not. Similarly, *penguins* has cumulative reference but *four penguins* does not as the sum of multiple groups of *four penguins* cannot be referred to as *four penguins*. In some expressions, nouns can have cumulative reference, but not in others. This fact is familiar in the singular-plural distinction, where a noun is not cumulative in singular noun phrase constructions, but cumulative in plural noun phrases.

Divisive reference is defined as follows:

(2) $\forall P[\mathbf{DIV}(P) \leftrightarrow \forall x \forall y[P(x) \wedge y \subseteq x \rightarrow P(y)]]$
P has divisive reference; any part of P is also P. Krifka (1989)

Like cumulative reference, divisive reference is a property of a noun phrase. The *water* in a glass can be divided into two or more portions, and each of these can be referred to as *water* but *two gallons of water* cannot be divided up in a way that each portion can be referred to as *two gallons of water*. Most uses of plural and non-count nouns are divisive, while count nouns are not divisive. There is no way to divide *a guitar* into two or more portions that are each *a guitar*. Importantly, divisive reference should be defined in a way to avoid cases of trivial divisiveness, where the two parts are the same, which can be accomplished by restricting the parthood relation to proper parthood, as in (2), or by including criteria such as non-overlap in the definition.

A predicate has homogeneous reference if it is both cumulative and divisive. As formulated, a noun phrase that is cumulative will also be divisive, but not always. For example, the plural noun phrase *books*, when referring to pairs of books, is cumulative since any group of pairs can

also be referred to as *books*, but it is not divisive, since there is no way to divide the pairs into two or more parts that each can be referred to as *books*.

2.2. THE PROBLEM. Divisiveness predicts that all parts of the thing a noun phrase refers to can also be referred to with the same noun phrase. Thus, when applied recursively, this predicts that any part of the thing a divisive noun phrase like *water* or *penguins* refers to is itself divisive. However, not all parts of what *water* refers to can be called *water*, such as hydrogen and oxygen atoms. Not all parts of what *penguins* refers to can be called penguins, such as individual penguins, their eyes or feet, a single feather, and so on. This is the Problem of Minimal Parts. Almost all nouns in English have a level below which they cannot refer divisively—perhaps only nouns like *space* or *time* are the rare exceptions to this—yet, as defined, there is no way to specify where divisive reference no longer applies.

To illustrate the problem another way, Figure 1 represents divisiveness for some portion of water. The nodes represent either individual water molecules, w_1, w_2, and w_3, composed of hydrogen and oxygen atoms, h_1, o_1 and so forth, or sums of these individuals, representing portions of water. The lines between nodes represent the parthood relationships between various sums or individuals. The brackets on the right side I have added to represent the status of the referent, the facts about the real world state of affairs—at the level of the H_2O molecule and above it is water, below that level it is not water. The bracket on the left indicates what divisive reference, as formulated in (2) predicts as to when a divisive predicate will refer: in this case that every state of affairs involving parts of water is referred to as *water*. The mismatch between the brackets is a way to visualize the Problem of Minimal Parts.

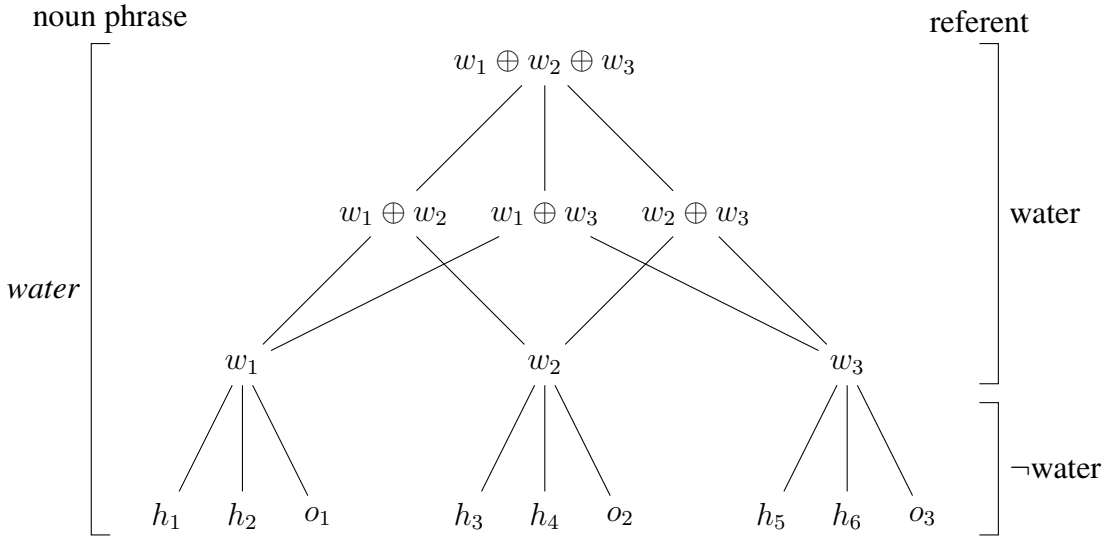

Figure 1. Semiattice representing the Problem of Minimal Parts

While most of the literature on the Problem of Minimal Parts has primarily discussed cases involving non-count noun phrases like *water*, I believe that the problem is most apparent with plural noun phrases. For a noun like *penguin*, some uses can have divisive reference (*penguins, several penguins*) while other uses do not (*a penguin, three penguins, that penguin*). Depending on the objects in the world a speaker wants to refer to, they may switch between a divisive and non-divisive noun phrase. However, divisiveness, as formalized in (2), applies to any part of what

the noun phrase refers to, regardless of its cognitive salience as a minimal part. It is important to clarify that the Problem of Minimal Parts is a problem of the way these properties of reference have been formalized in semantics—it is not a problem about the way things are structured in the world, nor a problem of how we understand or perceive those things. The Problem of Minimal Parts is an issue of where certain terms can be used to refer and where they cannot.

3. Previous Proposals. The first mention of the Problem of Minimal Parts is from Quine (1960), in a passage discussing plurality where he argues that the distinction between singular and plural is superficial as these pairs are in fact the "same term in varying contexts" (p. 90). However, the distinction between what he calls singular and general terms is not similarly superficial. While singular terms refer to only one object, general terms can refer to any number of objects. These general terms posses "built-in modes, however arbitrary, of dividing their reference" (p. 91). Count nouns are general terms In Quine's sense. For example, we know what the noun phrase *a penguin* refers to—part of the knowledge of the meaning of *penguin* is the knowledge of this 'built-in mode' of dividing the reference of *penguin*. On the other hand, non-count nouns like *water* or *oxygen* lack a clear mode for dividing their reference.

Building off of Quine's work, Cheng (1973) proposes two properties. The first one he calls "Quine's Condition," that the sum of things which are x is itself x. The second, "Cheng's Condition," that any part of something which is x is itself x, the first statement of divisive reference in the literature. Bunt (1985) is the first to formulate these properties as homogeneous reference, writing that "mass nouns refer to entities as having a part-whole structure without singling out any particular parts and without making any commitments concerning the existence of minimal parts" (p. 46). Following Bunt, Lønning (1987) introduces what he calls the homogeneous constraint which is a "heuristic principle" to guide analysis. "An intuitive explanation for the Homogeneous Constraint is that a sentence like *Much water* α does not only mean that a quantity of water of a certain size has the property α but also that every subquantity of this quantity has the property α" (p. 9). However, the goal of the homogeneous constraint is to intuitively capture a distributional pattern, not formalize it.

There are two main approaches to formalizing these properties. Link (1983) discusses the "cumulative reference property" shared by plural and non-count noun phrases, noting that "individuals are created by linguistic expressions involving different structures even if the portion of matter making them up is the same" (p. 304), for example *the cards* and *the deck of cards*, despite being formed of the same parts, form two different individuals depending on how they are spoken about. A plurality like *the cards* is a sum while *the deck of cards* is a collection. Link notes that while sums "respect levels of 'linguistic comprehension' [...] By contrast, collections do not, they typically merge those levels" (p. 305). Although he does not use the term divisive reference specifically, this distinction in "levels of comprehension" is equivalent to the natural language patterns that the property of of divisive reference captures. However, Link crucially restricts sum-formation to the domain of atomic individuals. The parthood relationships in this domain are individual part relations. But Link also defines the material part relation, for parthood relationships relative to material fusion, a domain which includes non-atomic entities such as the referents of non-count nouns.

Meanwhile, Krifka (1989) discusses what he calls cumulative and quantized predicates—"if there are two entities to which *beer* applies, this predicate applies to their collection as well" but "if there are two (different) entities to which *a book* applies, this predicate does not apply to their

collection" (p. 75). Unlike Link, Krifka places the source of countability behavior in the predicate, and this his approach does not restrict the domain to atomic individuals. Instead, he defines a number of higher-order predicates that range over first-order predicates "to characterize different reference types" (p. 77). In his formal system the same element in the structure may be described by predicates with homogeneous or non-homogeneous reference, whereas in an approach like Link's, homogeneous and non-homogeneous are in separate domains. One or the other of these approaches are usually picked up by proposals addressing the Problem of Minimal Parts.

3.1. WEAK HOMOGENEITY. One way to address the Problem of Minimal Parts is to argue that homogeneous reference is not a property about the structure of the actual world, weakening the relationship between homogeneity as a property of language and the entities homogeneous predicates refer to. For instance, Quine (1960) argues that the distinction between general and singular terms is a feature of the terms and not a feature of "the stuff they name" (p. 91). Link (1983) warns that conflating these can mislead the discussion towards "reductionist ontological considerations" which are "quite alien to the purpose of logically analyzing the inference structures of natural language." Instead, the "guide in ontological matters has to be language itself" (pp. 303-304), the conditions under which two entities are identical in which statements refer to them. Obviously there is no question about the scientific fact that divisibility does not hold for the referent of nouns like *water*—the actual H_2O molecules in the world—beyond a certain point. Bunt (1985) argues that the focus of linguistic theories "should only account for linguistic facts" and that "nothing in the use of these mass nouns indicates a commitment on the part of the speaker to the existence of minimal parts" (p. 45). Homogeneous noun phrases are simply ways of speaking about the world as if it had no minimal parts, regardless of the existence of those parts. Lønning (1987) writes "our interest lies in the natural language itself, not in the world it describes. This means that the models we build are not necessarily "true" models of the physical world, but means to understand the language, in particular, to give valid forms to intuitively true sentences and inferences" (p. 7). On the Problem of Minimal Parts in particular, he argues that "it is not critical if mass terms really refers *homogeneously*, that is both cumulatively and distributively. Rather what is of importance is whether they behave as if they did and what it means to behave in such a way" (p. 8). Gillon (1992) calls this type of approach the "weak homogeneous reference hypothesis" that "grammar is simply mute on the question of whether or not there are minimal parts" (p. 598). These approaches thus predict divisive nouns like *water* will be used to refer to any amount of water, even when the referent is a single H_2O molecule.

3.2. RESTRICTING HOMOGENEITY. On the other hand, instead of weakening homogeneity, there are reasons to consider a view on which homogeneity can be restricted in some way. Some approaches along this line favor restrictions on the parthood relation itself. Moravcsik (1973) suggests that "two substances F and G are distinct if and only if there is a part of one, say F, that is an F-part but though it may be a part of G it is not a G-part." Thus, for example, "there will be a wood-part of Wood that—though it may be a part of Furniture—is not a furniture-part of Furniture" (p. 281). While Moravcsik's focus is just on substances and non-count nouns, this can be extended to plural count nouns. For example, an individual apple is an apple-part of *apples*, and while its seeds are also a part of *apples*, they are not apple-parts of *apples*. This provides a way to distinguish the minimal parts for something: "a substance with minimal parts is a mereological unit such that not all parts count as proper m-parts" (p. 283). An individual apple is the minimal part of *apples* because no parts of it count as proper *apple*-parts. A similar proposal comes

from Koslicki (1995) who gives a definition of relativized parthood and relativized distributivity. Her proposal requires the notion of relativized atomic parthood, where "For any predicate Φ, x is an *atomic* Φ*-part* of y iff (i) Φ is true of x, (ii) Φ is true of y (iii), x is a part of y, [and] (iv) there is no proper part z of x, such that Φ is true of z" (pp. 112-113). This can be used to define relativized parthood, "For any predicate Φ, x is a Φ-part of y iff (i) either x is an atomic Φ-part of y (ii) or x is the sum of atomic Φ-parts of y" (p. 113). This allows her to revise divisive reference to hold only if a predicate is true of some proper parts which are Φ-parts. However, Koslicki goes on to reject this proposal, as her definition of relativized divisiveness predicts that there are atomic entities in the denotation of non-count nouns, an outcome she finds unsatisfactory.

Meanwhile, Moltmann (1997, 1998) proposes the notion of an "integrated whole" which is a type of "unified part relation" which in turn allows her different ways of restricting the parthood relation. However, what exactly counts an integrated whole is "very difficult" to define and she notes that "no single definition of integrated whole has been developed, and it is not clear that it can be developed at all" (1998: p. 85). It does involve, however, that certain conditions hold among parts relative to situations. For example, John is a part of a group of children, and John's leg is a part of John, but John's leg is not a part of a group of children because only John is an integrated whole relative to the situation. Thus, "an inference from $x \subset y$ and $y \subset z$ to $x \subset z$ is allowed only if y is not an integrated whole" (1998: p. 91). In what cases might one expect such inferences to be allowed? Moltmann argues "it depends ultimately on the nature of the entity *x* itself—that is, on what part structure *x* has essentially—whether some subunit counts as its part or not" (1998: p. 92), which Moltmann calls essential integrity. These types of approaches do not provide clear predictions about minimal parts directly, or in what cases a predicate can refer, but they do allow for context to restrict what parts of an entity are relevant.

3.3. STRATIFIED REFERENCE. Champollion (2015, 2017) proposes stratified reference wherein divisive reference is "approximate" and only holds above a certain level of granularity. "Stratified reference requires a predicate that holds of a certain entity or event to also hold of its parts along a certain dimension [i.e. spatial or temporal] down to a certain level of granularity" (2015: pp. 110-111). Granularity is a parameter that can be set to different values which specify the minimal parts of the predicate in question. Count and non-count noun denotations "are taken to have different formal properties [...] For example, the former entry only applies to mereological atoms, but the latter need not" (2017: p. 42). Singular count nouns denote 'sets of individuals' and plural count nouns are 'sets of 'entities', such that the plural is the algebraic closure of the singular. While Champollion does not formalize a restricted version of divisive reference, he assumes "that any entity in the denotation of a mass noun N that is larger than a certain threshold $\epsilon(C)$ can be divided into parts which are again in the denotation of N" (2017: p. 49) and that this threshold is context-dependent. This proposal predicts a predicate will not refer if the referent is smaller than the specified level of granularity.

3.4. VAGUENESS. Chierchia (2010) argues that the point at which reference picks out minimal parts is vague for non-countable nouns. In this view, "while every noun/concept may in a sense be vague, mass nouns/concepts are vague in a way that systematically impairs their use in counting" (p. 99). While his formal proposal follows the lattice-theoretic approach of Link (1983), he does not distinguish between the atomic and non-atomic domain because "the idea of a 'non atomic or not known to be atomic' domain is obscure. If we construe such a notion literally as 'atomless', the result is counterintuitive" (p. 144). This counterintuitive notion has

its source in the Problem of Minimal Parts. "The idea that [something] can be infinitely subdivided while preserving its quality as [that thing] makes little sense" (p. 144). He rightly identifies that at its core this is a problem of identity—about the contexts in which a noun phrase can correctly refer to something—and his solution is to posit that counting breaks down where identity does. For some noun phrases, like *water*, combination with cardinal numbers or other count morphosyntax is impossible because "counting always involves individuating a level at which to count" which involves reference to "minimal elements that are sufficiently well defined" or "not too vague" (p. 116). On his view, atomicity "becomes a vague and context dependent notion" (p. 120) which can be modeled with a partial join operation on individuals that generates atoms, sums, and "things we don't know whether they are atoms or sums" (p. 120). These are the 'unstable' individuals or sums. For mass nouns, "there will be precisifications of ground contexts in which smaller amounts of the relevant [stuff] will count" so they have "contextually supplied smallest parts" but lack stable atoms (p. 123). This approach predicts speakers will stop using homogeneous predicates past a certain point, though this point is context-dependent, and they will never refer directly to these "smallest" or minimal parts with that predicate.

4. A New Proposal: Composition, Sums, and Identity. This section presents a different approach to a solution to the Problem of Minimal Parts. Section 4.1 presents a framework from Fine (2010) that focuses on composition, rather than the part-whole relation. Section 4.2 presents the details of the proposal, including a formulation of homogeneous reference framed in the sum operation rather than the parthood relation. Section 4.3 provides some corpus data that is relevant to the proposal.

4.1. STARTING FROM COMPOSITION. Mereology by definition takes parthood as primitive (for more on mereology see Simons 1987; Champollion & Krifka 2016). Most formalizations of mereology assume one, unified notion of what it is to be a part and build up their theory around that. Part structures are defined as sets of objects and the relationships between them. These formalizations are extensional: any two things that have the same parts will be identical. However, I argue a different approach is needed, one that focuses on composition, rather than parthood. To my knowledge, this has not been explored in the linguistics literature. In the philosophy literature, however, one related proposal for metaphysical theories has been put forth by Fine (2010). On Fine's ontology, composition is the primitive feature, not parts or wholes. His proposal remains largely theory-neutral about commitments to mereology or set theory: for approaches that assume mereology, composition is a sum or fusion operation, while for approaches that assume set theory, composition involves forming a set. Regardless of the underlying formalism, "the operation of composition will be the characteristic means (summation, set-builder, and so on) by which a given kind of whole is formed from its parts [...] For it is always possible to define the [part-whole] relation in terms of the operation but not always possible to define the operation in terms of the relation" (p. 565). Building a theory of homogeneous reference—and of noun countability more broadly—off of Fine's proposal is advantageous in its flexibility.

Starting from composition, not parthood, also provides other benefits according to Fine. Different kinds of null objects can be defined, based on different composition relations taken of nothing. And there can be distinctions based on the repetition of parts, since multiple of the same part may be involved in composing a whole, even if they all hold the same relationship to the whole. It also allows for a distinction between 'flat' and 'hierarchical' wholes. Fine writes that this distinction depends on "whether repeated applications of the operation are capable of

yielding something new" (p. 566). This is relevant to cumulative reference. The noun phrase *water* has cumulative reference, since for any two things that can be called *water*, their sum can also be called *water*, no matter how many times this process is applied. But for the referents of singular count noun phrases, combining them with another referent of the same singular count noun phrase does not yield something that can also be referred to with the count noun phrase.

After establishing the benefits of defining parts and wholes on the basis of composition, Fine presents four principles which distinguish the different grounds upon which identity of a composed object can hold. These principles conform to what he calls "a general template" of formal and material principles for composition, or the "operational" approach. The formal principles include conditions of application and identity conditions while the material principles concern existence, extension in space and time, and descriptive character. These principles are "the counterpart, within the operational approach, to the standard axioms of mereology" (p. 570) and Fine goes on to define mereological axioms in terms of these principles.

Relevant to my proposal are the identity conditions, which he describes as "a matter of stating when a whole formed in one way by means of the compositional operation is the same as a given object or a whole that has been formed in some other way" (p. 570). An identity condition is regular if the variables appearing in on both sides of it are the same (e.g. *x,y* = *y,x* is regular but *x,y* = *x* is not). From this, he develops four composition principles which are "the different grounds upon which a regular identity may hold" (p. 573). Thus, the four principles are identity statements, which assume the variables (i.e. parts) of the things that occur on either side are the same. Note that Fine uses the sum operator, $\sum$ to denote whatever results from composition.

(3) a. Leveling: $\sum(\sum(w,x),\sum(y,z)) = \sum(w,x,y,z)$
The embedding of components is irrelevant to the identity of the whole
b. Absorption: $\sum(x,x,y,y) = \sum(x,y)$
The repetition of components is irrelevant to the identity of the whole
c. Collapse: $\sum(x) = x$
The whole composed of a single component is identical to that component
d. Permutation: $\sum(x,y,z) = \sum(y,z,x)$
The order of the components is irrelevant to the identity of the whole Fine (2010)

While these composition principles are not rules or restrictions on composition, they are principles that distinguish valid forms of composition and the 'structure' so to speak of the resulting whole. Leveling is the principle relevant to the Problem of Minimal Parts, and the remainder of the paper will focus on it—though Section 5 will briefly discuss the other principles and some ways in which they may be useful for the semantics of countability more broadly.

Leveling captures the intuitions behind cumulativity and divisibility—that some stuff and some stuff together are the same as some stuff, and that part of an entity is identical to that entity with regard to the application of the predicate. For example, (4a) and (4b) show Leveling with respect to the predicates *apples* and *apple*, respectively. While *apples* can refer to either the sum of two apples or the sum of those two apples combined with another two apples, *an apple* cannot refer to any sum of multiple apples.

(4) a. $\sum(apple_1, apple_2, apple_3, apple_4) = \sum(\sum(apple_1, apple_2), \sum(apple_3, apple_4))$
b. $\sum(apple_1, apple_2) \neq \sum(\sum(apple_1), \sum(apple_2))$

(4a) states that one group of apples is equivalent to two groups of two groups of apples, assuming the apples making up the groups are the same apples. Put another way, for a group composed of groups of apples, whether each of those apples are in smaller groups within the group is irrelevant when it comes to reference. On the other hand, (4b) states that one group of two apples is not equivalent to two groups with one apple each—compare this with the distinction between *the cards* and *the deck of cards* from Link (1983) discussed above. A more intuitive and linguistic way to think about this is whether the situations on either side of the identity statements can be referred to with the same noun. In (4a), either situation would be called *apples*, while in (4b) it would not be. These are conditions on identity relative to a property. In a sense, the principle of Leveling can be thought of as a way to induce structure on the domain, picking out entities that accord with Leveling on the one hand from those that do not.

4.2. LEVELED REFERENCE AND HOMOGENEITY. This section motivates an approach to homogeneity building off of the principle of Leveling and the idea that composition, rather than parthood, should be the focus. First, I will adapt the principle of Leveling to apply to predicates. Then, I will incorporate it into a proposal for a second-order property of homogeneous reference that is based upon the sum operation rather than a parthood relation. Even though Fine (2010) calls for a more radical reorientation—doing away with parthood as primitive and moving toward the composition operation instead—my analysis assumes the standard mereological framework used in much work on nominal semantics. As Fine's proposal is a metaphysical one, it is important to clarify how I am adapting it for a linguistic application. Rather than being about the composition of objects in the world, I am adapting it to be about the contexts in which a predicate applies, or rather the structure of the things to which a predicate applies.

(5) $P(x) \wedge P(y) = P(x \oplus y)$
A predicate applying to two entities is equivalent to it applying to the sum of those entities

I am assuming a standard mereological definition of sum (for further discussion see Simons 1987; Champollion & Krifka 2016). The summation, crucially, is not summation of a predicate—it is not $P(x) \oplus P(y) = P(x \oplus y)$—but of the things to which the predicate applies. Parallel to Fine's principle, this adapted form of Leveling is formulated as an identity statement. However, it is about the identity, or equal case of application, of a predicate. The relation of this formulation of Leveling to the definitions of cumulative reference and divisive reference in (1) and (2) may be seen more clearly in the reformulation in (6), which treats it as a conjunct of implication statements rather than an identity statement.

(6) $(P(x) \wedge P(y) \rightarrow P(x \oplus y)) \wedge (P(x \oplus y) \rightarrow P(x) \wedge P(y))$
A predicate applying to two entities implies that it applies to the sum of those entities and it applying to the sum of those entities implies that it applies to each of them individually

However, in what follows, I will retain the formulation in (5) in terms of identity.

Note that the variables in (5) may be atomic individuals or sums. For example, for the count noun phrase *apples*, with variables that are individual apples (represented as a_1, a_2, etc.) this adapted principle of leveling holds when *x* and *y* are sums of apples, as in (7a) where the plural predicate *apples* applies to any sums of individual apples. However, for singular count noun phrases, Leveling does not hold, since *apple* cannot be true of a sum of individuals. Thus, (7b) will be false, the exact outcome that should be expected to occur.

(7) a. $apples(a_1 \oplus a_2) \wedge apples(a_3 \oplus a_4) = apples(a_1 \oplus a_2 \oplus a_3 \oplus a_4)$
b. $apple(a_1) \wedge apple(a_2) = apple(a_1 \oplus a_2)$

For non-count noun phrases like *water* in (8), leveling holds for either sums or individual portions of water, as the noun phrase *water* is true of all of these.

(8) a. $water(w_1 \oplus w_2) \wedge water(w_3 \oplus w_4) = water(w_1 \oplus w_2 \oplus w_3 \oplus w_4)$
b. $water(w_1 \oplus w_2) \wedge water(w_3) = water(w_1 \oplus w_2 \oplus w_3)$

An interesting feature of this adapted principle of Leveling is that for any plural count noun phrase, it only holds for states of affairs involving at least four entities picked out by the counterpart singular noun phrase. There is no way for (7) to be true without at least four individual apples. This creates some interesting consequences for my proposal that I will discuss further momentarily.

Now with this adapted version of the principle of Leveling in hand, the second-order property of homogeneous reference can be defined.

(9) $\forall P[\mathbf{HOMO}(P) \leftrightarrow \forall x \forall y[P(x) \wedge P(y) = P(x \oplus y)]]$
P has homogeneous reference iff for all entities that the predicate applies to, these entities satisfy the principle of Leveling with regard to that predicate

This maintains the intuition behind the properties of cumulative and divisive reference. It also avoids any reference to parthood, although it is recognizable in the analogue of components that are parts of sums, such as portions of water that are components of a larger volume of water, or pluralities of apples that are components of a larger plurality of apples.

This formulation of homogeneous reference in (9) avoids the Problem of Minimal Parts by restricting the domain of a given predicate to cases where it can apply to entities and cases where it can apply to sums of those entities. The inclusion of Leveling in the form of an identity statement achieves the restriction to these two instances simultaneously. Because of this, it avoids situations where reference is made to the minimal parts of an entity.

For example, consider a glass of water. The predicate *water* is homogeneous in cases like in (10a), where each component (w_1, w_2, w_3) or sum of components is itself some portion of water in the domain of *water*. However, in cases like (10b), where the components include some hydrogen atoms (h_1, h_2) which are a part of the glass of water, homogeneous reference will not hold because there is not an identity of application of a predicate—individual hydrogen atoms are not in the domain of *water*, nor are portions of water in the domain of *hydrogen*.

(10) a. $water(w_1) \wedge water(w_2 \oplus w_3) = water(w_1 \oplus w_2 \oplus w_3)$
b. $water(w_1) \wedge water(h_1 \oplus h_2) = water(w_1 \oplus h_1 \oplus h_3)$

Restricting the domain of a predicate in this way also allows for some degree of vagueness, or speaker or contextual variation, about what exactly counts as a case of reference. Since the adapted principle of Leveling is about is about the conditions under which a predicate can apply to entities and sums of entities, the situations in which a noun phrase no longer has homogeneous reference when referring to some entities may vary, which section 4.3 will discuss in further detail.

An illustration helps to explain the solution to the Problem of Minimal parts provided by homogeneous reference in another way. Figure 2 represents a lattice for some portion of water. As in the lattice given in section 2, the nodes w_1, w_2, and w_3 represent water molecules, composed of hydrogen and oxygen atoms, h_1, o_1, and so forth. Each water molecule can combine with other water molecules to form various portions of water. The brackets on the right side rep-

resent the structure of the referent in the real world—at the level of the H_2O molecule and above it is water, below that level it is not water. However, unlike in Figure 1, the brackets on the left indicates what homogeneous reference, rather than divisiveness, predicts, based on which cases it will apply to. When the entities can satisfy the principle of Leveling, as with portions of water like $w_1 \oplus w_2$ or $w_1 \oplus w_2 \oplus w_3$, the noun phrase *water* has homogeneous reference. The threshold where divisive reference stops, however, but the parts still compose to form at least one water molecule, the noun phrase *water* could still refer but would have non-homogeneous reference. Beneath this level, *water* no longer refers, predicates like *hydrogen* or *oxygen* do.

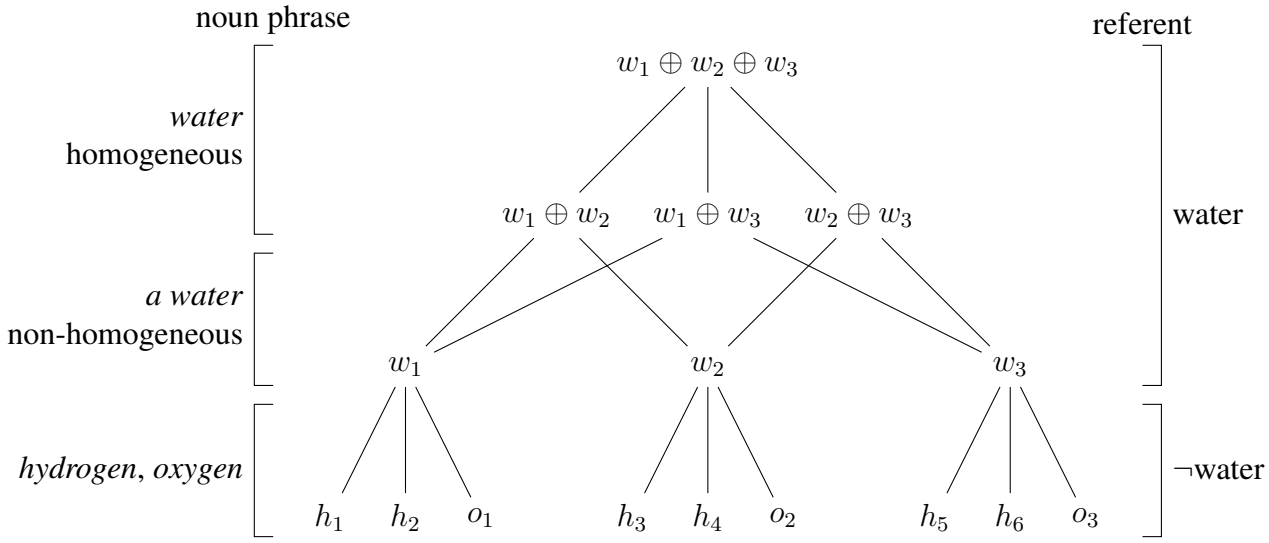

Figure 2. Semilattice representing revised divisiveness

As mentioned above, one result of this proposal is that it predicts noun phrases referring fewer than four entities will not be homogeneous. When *apples* refers to two or three apples, it is not homogeneous. While this might appear to be a flaw, I believe it is actually intuitive. Homogeneity is not the same as plurality. For languages which contain restricted number morphology, such as dual or trial number, this correctly predicts that those noun phrases will be non-homogeneous, which is exactly what it should predict. This approach to homogeneous reference may provide a possible semantics that is reflected by these types of number morphology.

4.3. PREDICTIONS AND SOME RELEVANT DATA. When the proposal I've put forward is worked out for non-count nouns, it raises an interesting question. Does this approach to homogeneous reference in some sense predict that non-count nouns will switch to non-homogeneous reference when no entities remain that are in the domain of reference? Put another way, this proposal predicts that count nouns can shift between homogeneous and non-homogeneous, as is familiar in the singular-plural divide, but does it also predict that non-count noun phrases might shift to count in some contexts?

This possibility has been noted in the literature. In discussing minimal parthood, Cheng (1973) notes that the referent of *water* is made of molecules and atoms, and perhaps if someday we "find it useful" to "speak of water-molecules" we would refer to these as *waters* and "we would mean water-molecules." In any situation where "an organism of type (α) has no part which is (α)" it would necessarily be referred to with a count noun phrase (p. 288). As an example of

this, he notes that counting of individual electrons does occur in particle physics, though he does not consider this further. Meanwhile, discussing the distinction between proposals of homogeneity that are about reference and ones that are about the referents, Bunt (1985) brings up a third possibility: homogeneous reference could instead be a property of the actual use of non-count nouns. This proposal would say that speakers use properties like homogeneous reference to decide between the use of a non-count or count noun. Thus, they are using language in a way that reflects their beliefs about the structure of the world, including its minimal parts. If the referent has minimal parts that are cognitively salient, they will use a count noun and if it does not, they will use a non-count noun. However, Bunt quickly dismisses this possibility, arguing that our contemporary knowledge of molecular structure "has not caused a change in the linguistic role of nouns like *water*" (p. 45). A similar comment comes from Lønning (1987), who notes that this possibility lacks evidence, and if one were to argue for it they would need to "point out where in our use of mass terms the existence of minimal parts is reflected" (p. 8). In short, one would need to argue that competent speakers' language use reflects their knowledge of minimal parts—specifically, that they never use non-count noun phrases like *water* to refer to the substance at a molecular level, where the property of Leveling does not hold of the referent.

Bunt and Lønning dismiss the idea that for non-count nouns like *water* reference could be made to its minimal parts with a non-homogeneous noun phrase. I think this dismissal is too quick, and that it lacks careful consideration of actual language in use. I believe this point is important to clarify because the Problem of Minimal Parts, as formulated in the literature, often relies on the assumption that competent speakers would use homogeneous noun phrases like non-count *water* to refer even at the level when that amount of water cannot be further broken down into parts that are themselves water. For example, Koslicki (1995) discusses in a footnote what she sees as a downside of what her definition of relativized divisiveness predicts, "that *is-water* would fail to be relatively distributive in a world in which there exists only a single H_2O molecule" (p. 113). This comment suggests that she assumes speakers should be able to use homogeneous *water* to refer to a single H_2O molecule—and, given a world with only one H_2O molecule, the noun phrase used to refer would necessarily be distributive. She sees the fact that her formulation of relativized divisiveness predicts this as a flaw, and another reason in favor of rejecting that approach. However, I think this prediction that her relativized divisiveness makes is actually correct, and the linguistic data reflects this, as I will discuss below.

A similar sentiment can be found in Chierchia (2010), who writes that "considering smaller and smaller instances of the property CAT, there is a cut off point such that if you go smaller, you won't have a cat anymore (even though where such a cut off point lies may be somewhat vague); on the other hand in considering ever smaller water samples the cut off point that separates water from non water remains way more elusive" (p. 118). He uses this assumption, that the point that separates water and non-water is unclear, as a supporting argument for his vagueness approach. But again, I would disagree with his assumption—there is a clear line of what separates water from non-water, and many speakers know where that line is. Speakers do in fact use non-homogeneous noun phrases to refer at the level of minimal parts. They know when something is *water* and when it is *a water* and when it is something else entirely.

The examples below are taken from Grimm et al. (2021), with the exception of (12a) and (12d), which were from my own supplemental web searches. Grimm et al. (2021) examined the countability behavior of almost 500 canonical non-count nouns in English and I argue that some of these examples of 'non-countable nouns' in count noun phrases are the exact kind of cases

mentioned in passing as unrealized possibilities by Cheng, Bunt, and Lønning and not acknowledged by Koslicki or Chierchia. In these cases speakers use singular count noun phrases in contexts where the referents no longer compose according to Leveling. The contexts that generate this type of reference are very specialized—primarily in the chemistry and physics literature—but in all cases a count noun is used to refer to individual atoms or molecules of a substance. Despite the infrequency with which these examples occur, I believe they are a case of the shift from homogeneous to non-homogeneous reference for non-count nouns that parallels the shift from homogeneous to non-homogeneous reference for plural and singular count nouns. The examples in (11) show non-homogeneous reference with nouns referring to elements, while the examples in (12) show this for nouns referring to molecules.

(11) a. The two *arsenics* have a collective charge of plus six, and each *arsenic* has a charge of plus three.
b. Calcium chloride has two *chlorines* for each *calcium*.
c. (CH3CH2)4P2O7 molecules contain 8 *carbons*, 20 *hydrogens*, two *phosphoruses*, and seven *oxygens*.
d. If we draw lines through the *titaniums*, every fifth one is missing.
e. As non-bridging *oxygens* are surrounded with *calciums* this energy is initially decreased until the *calciums* begin to crowd each other.

(12) a. Ice XI is a proton-ordered form of ice I_h, where *waters* orient in a repeated manner rather than the more typical random fashion.
b. The chains can be straight or branched, and they can run to thousands of *sugars* in a single chain.
c. Maltose is created by condensation reaction of the two *glucoses*, forming a α-1,4-O-glycosidic linkage.
d. Since there are fewer methanol molecules close to the bilayer than there are *ethanols* the average area per methanol is larger than the average area per *ethanol*.
e. Two *ozones* can combine to form three *oxygens*.

Contrast these examples with typical uses of nouns like *water*, *oxygen*, or *arsenic*: here speakers are shifting between homogeneous non-count noun phrase and non-homogeneous count noun phrase uses of a noun depending on whether the referent in context satisfies the restriction of Leveling or not. In short, their use of nouns with homogeneous reference is selective and reflects the salience of minimal parts, just like with plural nouns, where a speaker will switch to singular count when the minimal part becomes salient.

It is important to note that this data cannot be dismissed as a case of nominal coercion as the nouns refer to individuals, not standard portions or types of a substance. In cases of coercion, a non-count noun is coerced into a count reading by reference to some unit for individuation which is introduced in context; the noun itself is not being counted, but portions or types. There is what Gillon (2012) calls a "difference in construal" (p. 713) between the uses distinguished from from genuine instances of a noun as both non-count and count with no corresponding difference. However, these examples cannot easily be rewritten as *two standard portions of arsenic* or *two types of ozone* or so on. These are clear examples of count noun phrases with non-homogeneous reference in natural sentences produced by competent speakers of English and I argue they show that a shift between homogeneous and non-homogeneous reference is possible for non-count nouns.

5. Further Directions and Conclusion. This way of formalizing homogeneity suggests that language may be highly sensitive to different ways a thing might compose. This proposed approach to homogeneous reference may also play a role in explaining cross-linguistic variations in grammatical number such as the dual and trial, as mentioned above. This proposal supports a view that the way we use language reflects our knowledge of the structure of the world, or at least the knowledge of the ways in which things can compose. A possible area for further research could be to track language use historically and assess when count use of nouns like *water* and *oxygen* first occurred. And while this proposal focuses on nouns, it could be extended to the verbal domain (Krifka 1989; Champollion 2015, 2017).

While the focus of this paper is the Problem of Minimal Parts, there may also be ways to adapt the other composition principles from Fine (2010) to structure domains and account for other countability patterns in natural language. The principle of Absorption may be relevant to the distinction between non-countable and plural nouns. Perhaps while both compose according to Leveling, only non-countable nouns also compose according to Absorption, which prevents them from taking plural morphosyntax—in some sense this is the inverse of ways of talking about count nouns as having some 'unit for individuation' and instead talking about non-countable nouns as having Absorption with regards to reference. Collapse is perhaps loosely the equivalent of what is teased out with distributive predicates as nouns that cannot be collapsed have some internal structure to distribute over. Any portion of water can be the referent of *water*, but no single piece of furniture can be the referent of *furniture* or a single individual the referent of *congress*. And the principle of Permutation might be relevant to the case of collective and group nouns. However, these would need to be worked out further.

This paper has presented a solution to the Problem of Minimal Parts motivated by an approach to mereology built around composition and identity. I adapted the principle of Leveling from Fine (2010), where for all entities a predicate applies to it also applies to the sum of those entities. Incorporating this into the definition of homogeneous reference allows for a solution to the Problem of Minimal Parts.

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