

Collective quantification and the homogeneity constraint¹

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Abstract The main theoretical claim of the paper is that a slightly revised version of the analysis of mass quantifiers proposed in Roeper 1983, Lønning 1987 and Higginbotham 1994 extends to *collective* quantifiers: such quantifiers denote *relations between sums of entities* (type *e*), rather than *relations between sets of sums* (type $\langle e, t \rangle$). Against this background I will explain a puzzle observed by Dowty (1986) for *all* and generalized to all quantifiers by Winter 2002: plural quantification is not allowed with all the predicates that are traditionally classified as “collective”. The Homogeneity Constraint will be shown to be too strong and it will be replaced with the weaker requirement of Divisiveness (for both collective and mass quantifiers). Non-divisive predicates such as *form a mafia* are also allowed, but they will be argued to induce a different type of quantifier, which denotes a relation between entities and sets of entities.

Keywords: collective quantification, mass quantification, homogeneous, cumulative, divisive, groups, sums, maximality operator, plural logic

1 Introduction

The contrast in (1) illustrates Winter's (2002) generalized version of Dowty's (1986) puzzle regarding *all*:

- (1) a. All the/most of the students are meeting in the hall.
b. *All the/most of the students are a good team.

(1a-b) show that only a subset of the predicates that select pluralities (“collective predicates” henceforth) allow quantificational DPs, in particular *most of DPs*, on which I will concentrate. The central empirical claim of the paper will be that the contrast (1a) vs. (1b) is parallel to (2a) vs. (2b), built with mass Q(uantifiers):

- (2) a. All the/most of the water is liquid/dirty.

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b. *All the/most of the water is heavy/weights one ton.

The main theoretical claim will be that a revised version of the Roeper 1983- Lønning 1987-Higginbotham 1994 analysis of mass Qs extends to *collective* Qs (throughout this paper I will use the term “quantifier” as meaning “quantificational determiner”).

(3) Collective Qs (on a par with mass Qs) denote relations between sums (type *e*).

Crucially, sums are not to be equated with sets; sets have the semantic type of predicates (type $\langle e, t \rangle$), whereas sums have the semantic type of arguments (type *e*). In functional terms, collective and mass Qs denote functions from entities into sets (type $\langle e, et \rangle$) rather than functions from sets into sets of sets (type $\langle et, \langle et, t \rangle \rangle$). The proposal to be made in this paper implicitly assumes an algebraic mereological approach to pluralities. Although there are well-known correspondences between mereological and set-theoretical frameworks (due to the correspondence between the part of and the subset relations), the set-theoretical approach cannot account for mass Qs, and as such does not offer an adequate background for the present paper, which is interested in pointing out similarities between plural and mass Qs.²

The semantic computation of the type of quantifier defined in (3) necessarily involves a nominalizing operator, which must apply to the nuclear scope in order to derive an entity from a one-place predicate.

The rest of this article is organized as follows. Section 2 proposes a revision of the Roeper-Lønning-Higginbotham analysis of mass Qs (in both generic and particular contexts) and explains the Homogeneity Constraint (HC) on mass Qs (Bunt 1979, Lønning 1987 and Higginbotham 1994) as a consequence of that analysis. Section 3 extends the analysis to collective Qs, shows that the Homogeneity Constraint (HC) is too strong for both mass and collective Qs and proposes that such Qs are subject to a weaker constraint that requires divisiveness. Section 4 concludes and Section 5 reviews alternative accounts of collective Qs.

2 Mass Quantifiers

Most theoreticians explicitly or implicitly assume that mass Qs can be analyzed as denoting relations between sets of quantities of stuff (Gillon 1992), which amounts to subsuming mass Qs under the analysis of count Qs. See also Moltmann (1997), according to whom mass quantification is possible only in those contexts in which the situation provides a packaging of stuff (e.g., buckets of water are contextually salient).

This widely assumed type of analysis cannot account for the contrast in (2a-

² For a brief discussion of the set-theoretical analysis of collective Qs see § 4.1 below.

b).³ I will instead revive a largely ignored analysis of mass Qs, due to Roeper 1983, Lønning 1987 and Higginbotham 1994, which I will slightly revise in order to increase both its descriptive adequacy and its explanatory power.

2.1 Mass Quantifiers as relations between sums

According to Roeper 1983, Lønning 1987 and Higginbotham 1994 (in what follows, “R-L-H” refers to the common part of these proposals), mass Qs do not denote relations between two sets ($Q_{\langle et, \langle et, t \rangle \rangle}$) but rather relations between two entities ($Q_{\langle e, et \rangle}$), more precisely the maximal sums obtained by applying the generalized join operator notated Σ to the predicates in the restrictor and the nuclear scope, as shown in (5) for the examples in (4):

- (4) a. All gold is yellow.
 b. Most water is liquid.
- (5) a. $ALL_{mass}(\Sigma x. gold(x), \Sigma x. yellow(x))$
 b. $MOST_{mass}(\Sigma x. water(x), \Sigma x. liquid(x))$

According to Higginbotham (1994: 456), Σ is a nominalizing operator that yields the supremum of P provided that any non-zero part M of the supremum also satisfies P and P only holds of parts of the supremum; Σ is undefined otherwise:

- (6) $(\Sigma x)P(x) = \sup\{x: P(x)\}$
 If $M \neq 0$, then $M \leq \sup\{x: P(x)\} \leftrightarrow P(M)$
 $(\Sigma x)P(x) = \emptyset$ (undefined) otherwise

I will assume the following revisions of this analysis:

- (7) a. No nominalizing operator applies to the restrictor of mass Qs. An entity-denoting restrictor must be supplied by the syntax itself.
 b. The nominalizing operator that applies to the nuclear scope is the Maximality operator.

The revision in (7a) is a more constrained view of the syntax-semantics mapping: entity-denoting restrictors must be supplied by the syntax itself; in other words, a Q that denotes a relation between entities must take a DP restrictor (NP restrictors, which denote sets, necessarily correlate with Qs that denote relations between sets). (7a) is supported by empirical evidence showing that crosslinguistically, QPs of the form [MOST NP_{mass}] (where MOST is the

³ Dobrovie-Sorin 2013 gives evidence against the set-quantificational analysis of mass Qs.

superlative of MUCH) are illegitimate (Dobrovie-Sorin 2013), the only legitimate mass-quantificational QPs being of the form [The largest part/the majority of DP_{mass}]. English examples of the type in (4) are not problematic for (7)a, because in English (but not in Romanian or Hungarian, among many other languages) NP-complements of *most* (or *all*) can be kind-referring DPs⁴ headed by a null Det that has the semantics of an intensional Maximality operator (Dobrovie-Sorin 2013), also called “Down operator” by Chierchia (1998).

Note now that the nominalization of the nuclear scope is a necessary condition on the interpretability of those Qs that denote relations between sums: at the syntactic level of representation, the predicate in the nuclear scope denotes a set (type $\langle e, t \rangle$), like any one-place predicate; but the semantics of mass Qs requires the nuclear scope to denote a sum (type e); this condition can be satisfied by a nominalizing operator that applies at the syntax-semantics interface.

Granting that the nominalization of the nuclear scope is required by the semantics of mass Qs, the revision in (7b) is the “null hypothesis”: the required nominalization operator is exactly the same as the one underlying definite articles, which are currently analyzed as denoting the Max(imality) operator (Link 1983, Sharvy 1980), which applies to a set and picks up the maximal element (the maximal element is the one that all other individuals in the set are part of) of that set if there is one (undefined otherwise):

$$(8) \quad [[\text{Nominalizing Operator}]] = \lambda P. \sigma x. P(x)$$

Depending on whether P denotes a singleton set of atoms (see the denotation of NP_{sg}) or a join semi-lattice (see the denotations of NP_{pl} or NP_{mass}), an application of σ will yield either the unique entity or the maximal sum contained in the set. The Iota operator (the denotation of the singular definite article) is thus a particular case of the Max operator and the uniqueness associated to it also follows: the Max operator is not defined for non-singleton sets of atoms because no element of such a set is maximal wrt the others; Max can only apply to sets of atoms that contain only one element, which is therefore maximal.

According to the revised R-L-H analysis, the examples in (4), repeated in (9), are true iff (10) are satisfied; \cap notates the general lattice-theoretic operation “Meet” (intersection is Meet applied to sets):

- (9) a. All gold is yellow.
 b. Most water is liquid.
 (10) a. $\mu(\sigma x. \text{gold}(x) \cap \sigma x. \text{yellow}(x)) = \mu(\sigma x. \text{gold}(x))$

⁴ See also Matthewson 2001, who assumes a stronger hypothesis, according to which NP-complements of *most* and *all* are necessarily kind-referring.

$$b. \mu(\sigma x. \text{water}(x) \cap \sigma x. \text{liquid}(x)) > 1/2\mu(\sigma x. \text{water}(x))$$

In words, (10a) requires that the measure μ of the yellow gold (i.e., the Meet \cap of the maximal sum of gold ($\sigma x. \text{gold}(x)$) and the maximal sum of the yellow entities “all that is yellow” ($\sigma x. \text{yellow}(x)$)) equals the measure of all the gold; (10b) requires that the measure of the liquid water is larger than half of the measure of all the water.⁵

Because size is a ratio scale, particular choices of measure units (for the same dimension) do not affect truth conditions. This means that not only can we disregard canonical measure units (meters, liters, etc.), but we can also use ratios in order to measure the parts of an object with respect to the object itself: the measure of the whole (in this case the overall water) is 1 and the measure of any part of the whole is a ratio r comprised between 0 and 1. In order to check whether (9b) is true we would need to calculate $r = \text{vol}(\sigma x. \text{water}(x) \cap \sigma x. \text{liquid}(x)) / \text{vol}(\sigma x. \text{water}(x))$; (10b) is true iff $r > 1/2$. The computation relying on ratios is particularly useful for the examples at hand, since they arguably involve indeterminate/infinite entities (kinds), which cannot be measured with measure units.

The revised R-L-H analysis of mass Qs naturally extends to Qs built with DP restrictors that refer to particular mass entities. Thus, an example such as (11) is true iff the condition in (12) is satisfied:

(11) Most of this milk is sour.

$$(12) \mu([\text{this milk}] \cap \sigma x. \text{sour}(x)) > 1/2\mu([\text{this milk}])$$

2.2 The Homogeneity Constraint

According to Bunt 1979, Lønning 1987 and Higginbotham 1994, the contrast in (13) illustrates the so-called “Homogeneity Constraint” (HC) on Mass Quantification stated in (14):

- (13) a. All/most water is liquid/dirty.
 b. *All/most water is heavy/weights one ton.

(14) The predicate in the nuclear scope of a mass Q must be homogeneous.
 (Bunt 1979, 1985, Lønning 1987, Higginbotham 1994)

⁵ In order to avoid problems related to *most* meaning “more than half” (Hackl 2009, Solt 2011), the part of (10b) that follows $>$ should be $\mu(\sigma x. \text{water}(x) - \sigma x. \text{water}(x) \cap \sigma x. \text{liquid}(x))$, i.e., (10b) is true iff the measure of the liquid water is larger than the measure of its complement wrt. to all the water. Because the choice between the two representations is not relevant for our present purposes, here and elsewhere I use “ $> 1/2$ ” for conciseness.

- (15) Homogeneity (Higginbotham 1994:453)
- A predicate is homogeneous iff it is both cumulative and divisive.
 - A predicate is cumulative iff it applies to the sum of two things whenever it applies to each. (P is cumulative iff $P(x)$ and $P(y)$ implies $P(x + y)$)
 - A predicate is divisive iff it applies to the parts of the things to which it applies. (P is divisive iff $P(x)$ and $y \leq x$ implies $P(y)$ provided $y \neq 0$)

The definition of divisiveness is confronted with the well-known minimal-parts problem: '[. . .] there are parts of water, sugar, and furniture too small to count as water, sugar, furniture' (Quine 1960). This problem can be avoided by assuming that divisiveness is a property of predicates that does not depend on the actual state of affairs in the real world,⁶ but can instead be defined on the basis of inference patterns, e.g., (16) and (17), inspired by Lønning 1987:

- (16) The white gold is liquid. Cumulativity
The non-white gold is liquid.
 The gold is liquid.
- (17) The gold is liquid. Divisiveness
There is some white gold.
 The white gold is liquid.

On the basis of such inference patterns, predicates such as *liquid*, *dirty* or *yellow* are both cumulative and divisive, hence [+homog], whereas *heavy*, *tall* or *cover a large space* qualify as [+cum, -div], hence [-homog]. The generalization in (14) thus seems to be descriptively adequate wrt. the contrast in (13).

But why should the HC hold? According to Higginbotham (1994), the HC on mass Qs is due to the second line of (6), which is a definedness constraint on the nominalization operator analyzed as the supremum operator; Higginbotham shows that this definedness condition is equivalent to "P is cumulative and divisive", i.e., homogeneous. It seems fair to say that this is not an explanatory account: homogeneity is not derived as a consequence of the proposed analysis of mass Qs, but rather stipulated as a condition on the application of the supremum operator that is assumed to apply to the nuclear scope.

The revision in terms of the Max operator proposed above allows us to derive the HC from the definition of Max itself. Since homogeneous predicates (*yellow*) denote join semi-lattices, which have a maximal element, the Max operator can

⁶ Champollion's (2010) "Stratified Reference" is an attempt at solving the minimal-parts problem by adding a "granularity parameter" that prevents it from applying to parts that are lower than a certain threshold.

apply, and the computation of ALL/MOST, which crucially relies on Max applying to the nuclear scope, can go through. Non-homogeneous (*heavy*) predicates, on the other hand, denote sets of atomic objects, which do not have a maximal element. As a consequence, Max can only apply to a singleton set, yielding a unique heavy entity. However, *heavy* can denote a singleton set of heavy objects only if we relativize the application of Max to a sufficiently small situation, such that there is only one heavy object in that situation. The problem is that examples of the type in (9) are generic, which means that they are situation-independent; and if no situation is there to allow *heavy* to denote a singleton set, we are left with a non-singleton set of atoms, to which Max cannot apply. The semantic computation of (13b) cannot go through, hence the observed unacceptability. In sum, the HC on mass Qs in generic examples of the type in (9) follows without stipulation from the hypothesis that the nominalizing operator that needs to apply to the nuclear scope is Max, the operator that is currently assumed for definite articles: this operator cannot apply to generic non-homogeneous predicates.

Let us now observe that the HC seems relevant for mass Qs not only in generic contexts, but also in particular contexts:

- (18) a. Most of this water is frozen.
 b. *Most of this water is heavy.

In this case, there is in principle no problem for applying Max to *heavy*, yielding the unique heavy entity in some context-determined situation. In order to keep maximal sums distinct from unique entities I will notate the latter with the Iota operator:

- (19) b. $\mu([\text{this water}]) \cap \iota x.\text{heavy}(x) > 1/2\mu([\text{this water}])$

But then, why is it that the example in (18b) is unacceptable, in contrast to (18a)? The answer is that the Meet operation that is crucial for the semantics of mass Qs (see (10), (12) or (19b)) can apply to two entities only if they are structured by the part-of relation, i.e., Meet can only apply to sums (of parts of entities). The computation in (12) can go through because the nominalization of the nuclear scope yields a sum ($\sigma x.\text{sour}(x)$), whereas (19b) crashes because $\iota x.\text{heavy}(x)$ denotes a unique entity that is not structured by the part-of relation.

In sum, I have proposed a revision of the R-L-H analysis of mass Qs and I have shown that the HC is relevant for mass Qs in both generic and particular contexts. I have also proposed to explain the HC as resulting from the fact that by applying the Max operator to the predicate in the nuclear scope – as required by my revision of the R-L-H analysis – we get a unique entity or a sum of entities

depending on whether the predicate is non-homogeneous or homogeneous. Since the semantic computation depends on applying Meet, which can apply to sums (of parts of entities) but not to entities that are not structured by the part-of relation, homogeneous predicates are allowed and non-homogeneous predicates disallowed.

3 Collective Quantifiers

Let us now go back to the main goal of this paper, which is to explain Dowty's puzzle illustrated in (1), repeated in (20):

- (20) a. All the/most of the students are meeting in the hall.
 b. *All the/most of the students are a good team.

The basic assumption will be that collective Qs have the same semantic type as mass Qs:

- (21) Collective Qs (on a par with mass Qs) denote relations between sums (type e).

3.1 Plural and Collective Quantifiers

The label "collective quantifiers" designates plural Qs that take collective predicates in the nuclear scope, and these are the only plural Qs that we will be interested in. But before starting to examine collective Qs, let me make two brief remarks regarding those plural Qs that will be left aside, namely those that take atom predicates (in the sense of Winter 2002; also called "distributive" predicates) in the nuclear scope:

- (22) a. All the/most of the students in my class are hard-working.
 b. All/most students in my class are hard-working.

The example in (22a) shows that the HC is trivially satisfied for those plural Qs that take atom/distributive predicates in the nuclear scope: whenever they apply to pluralities, such predicates are necessarily pluralized (by applying Link's star operator) and pluralized predicates are homogeneous. The example in (22b) shows that those plural quantifiers that take atom/distributive predicates in the nuclear scope allow not only DP restrictors (as in (22a)), but also NP restrictors. According to Dobrovie-Sorin (2013), this points to the existence of two types of plural Qs: (i) plural Qs that are of the same semantic type as mass Qs, i.e., they denote relations between (sums of) entities (see (22a)) and (ii) plural Qs that are canonical set-quantificational Qs, i.e., they denote relations between sets of

entities (see (22b)).

Note now that collective Qs cannot take NP restrictors:

(23) *All /most students in my class are meeting in the hall.

Because *meet* is a collective predicate, the quantifiers *all* and *most* that appear in (23) cannot be analyzed as relations between sets of atoms, but need to be analyzed as relations between sets of sums. The observed unacceptability can be explained if we assume that: (i) collective Qs cannot denote relations between sets of sums⁷ (they are not of type $\langle et, \langle et, t \rangle \rangle$); they can only denote relations between sums (they are of type $\langle e, et \rangle$), as stated in (21) and (ii) the nuclear scope of a $Q_{\langle e, et \rangle}$ must be filled with a DP (*not* with an NP, as in (23)).

3.2 From the Homogeneity Constraint to the Divisiveness Constraint

Given the hypothesis of the common semantic type of collective and mass Qs, it is natural to attribute the contrast in (20) to the HC:

(24) The predicate in the nuclear scope of a collective Q must be homogeneous.

If the HC was adequate, the two classes of collective predicates that are responsible for Dowty's contrast would differ wrt homogeneity:

- (25) a. Non-homogeneous collective predicates
Ex: *mafia, team, committee, numerous, elect*
b. Homogeneous collective predicates
Ex: *meet, gather*

The problem is that collective predicates such as *friends* (and reciprocals in general, e.g., *love each other, neighbours, similar*) qualify as non-cumulative (hence non-homogeneous) on the basis of Lønning-inspired inference patterns:

- (26) The French students are friends with each other. Cumulativity
The non-French students are friends with each other.
The students are friends with each other.

On a closer look, even *gather* and *meet* turn out to be non-cumulative as soon as

⁷ An explanation of the impossibility of collective Qs to denote relations between sets of sums is out of the scope of the present paper (the interested reader is referred to Dobrovie-Sorin's (2013) account of why mass Qs cannot be set-quantificational, which may well extend to collective Qs). Note also that the unacceptability of (23) is difficult to explain on the view that collective Qs denote sets of sets (van der Does 1993, Winter 2002). See § 4.1 below.

we leave aside modifying adverbs:⁸

- (27) The French students met. Cumulativity
The non-French students met.
 # The students met.

Note now that the same predicates qualify as divisive:

- (28) The students are friends with each other. Divisiveness
There are some French students among the students.
 The French students are friends with each other.

Based on such inference patterns, Dobrovie-Sorin (2012: 103-106) proposed that inherently plural predicates (reciprocals, symmetric nouns used in the plural, but also *meet*, *gather*, etc.) are [-cum] and yet [+div], and as such they can be distinguished from *mafia* or *elect*,⁹ which are both [-cum] and [-div]:¹⁰

- (29) a. [-cum,+div]: *friends*, *meet*, *work together*
 b. [-cum,-div]: *mafia*, *elect*, *numerous*

The predicates listed in (29b) indeed qualify as [-div] on the basis of our inference patterns (since [-cum] is obvious, we leave it aside):

- (30) The students elected the president. Divisiveness
There are some French students among the students.
 # The French students elected the president.

Because the [-cum, +div] array of features is largely ignored in the current literature, it is worthwhile illustrating it with a toy example given in Dobrovie-Sorin 2012: 103-106. Let us take a situation in which the three maximal sums that

⁸ The cumulativity of *gather* and *meet* is thus not due to the lexical meaning of the verb itself, but rather to location at the same temporal and/or spatial Location.

(i) The French students met yesterday.
The non-French students met yesterday.
 The students met yesterday.

⁹ Dobrovie-Sorin 2012 suggests that the [-cum] feature that characterizes all collective predicates is due to the fact that such predicates express “integrity” conditions (Simons 1987, Moltmann 1997) and integrity conditions imply non-cumulativity.

¹⁰ The two classes distinguished here on the basis of the mereological property of divisiveness correspond to Winter’s (2002) set predicates and impure atom predicates. For further discussion see §4.1. See also Hackl’s (2002) definition of inherently plural predicates.

satisfy *meet* are $a+b+c$, $b+d+e$, $a+f+g$.¹¹ In this situation, the denotation of *meet* contains not only the set of maximal sums $\{a+b+c, b+d+e, a+f+g\}$ but also the set of all the parts of each of these maximal sums.

- (31) $[[\text{meet}]] = \{a+b+c, b+d+e, a+f+g, a+b, b+c, a+c, b+d, d+e, b+e, a+g, f+g, a+g\}$

In (31), there is no part-of relation among the maximal sums that satisfy *meet*, but there is a part-of relation between the parts of each of these maximal sums. In this type of complex structure, the absence of part-of relations between the maximal sums is responsible for [-cum], whereas the part-of relations internal to each of the maximal sums is responsible for the [+div] feature.¹²

Regardless of a more refined analysis, this type of collective predicates shows that the HC is too strong for collective Qs, because they allow [-cum,+div] predicates. Nevertheless, the postulated parallelism between mass and collective Q is not at issue, since the HC turns out to be too strong even for mass Qs:

- (32) Most of the sand was pushed in a corner by the wind.

In this example, mass quantification is allowed although *be pushed in a corner* is [-cum, +div].

Let us then assume a weaker constraint, which requires divisiveness but not cumulativity:

- (33) The Divisiveness Constraint on $Q_{\langle e, et \rangle}$
The predicate in the nuclear scope of a $Q_{\langle e, et \rangle}$ must be divisive.

This constraint captures Dowty's contrast: *gather*, *meet* or *friends* are [-cum, +div] and they allow quantification, as in (20a), whereas *mafia*, *numerous* or *elect* are [-cum, -div] and they block quantification, as in (20b).

But why is it that a $Q_{\langle e, et \rangle}$ allows its nuclear scope to be filled with a [-cum, +div] but not with a [-cum, -div] predicate? To answer this question, let us

¹¹ This is a corrected version of Dobrovie-Sorin's 2012: 105 example (22), which talked about the groups P, Q and R, each of which contained three elements, notated $p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3$, yielding the various sums that are parts of P, Q and R, respectively. Including groups and sums inside the same denotation was an obvious and regrettable mistake. Moreover, the notation need not suggest that the individuals in the denotation of *meet* are somehow marked (by the use of the same letter) as belonging to a certain group. Hence, the more neutral notation a, b, \dots . Furthermore, we must allow the same individual to be part of more than one sum that satisfies the predicate.

¹² Note that each of the relevant sub-domains are mereologically non-contradictory: (if the notions of $\pm\text{cum}$ and $\pm\text{div}$ are applicable not only to predicates, but also to parts of the extensions of predicates) the set of maximal sums in the denotation of *meet* qualifies as [-cum,-div] and the set of the parts of each of the maximal sums is [+cum,+div].

consider the example in (34). According to our analysis, (34) is true iff (35) holds:

(34) Most of my students met yesterday.

(35) $\mu([\text{my students}] \cap \sigma x.\text{met}(x)) > \mu([\text{my students}] - [\text{my students}] \cap \sigma x.\text{met}(x))$

As explained above, the [-cum] feature of *meet* captures the fact that *meet* denotes the set of maximal sums of people that met in the world. This is however irrelevant for collective Qs, which are only interested in particular situations. Thus, in order to assign truth conditions to an example like (34) we need to consider a minimal situation that contains all of my students; at this point, divisiveness becomes relevant: because *meet* is [+div], $\sigma x.\text{met}(x)$ denotes the maximal sum of individuals that met in that minimal situation.¹³ The computation of the truth-condition in (35) can go through, because we can apply the Meet operation to two sums, the sum of my students on the one hand, and the maximal sum of individuals that met (in the minimal situation containing all of my students), on the other hand.

Consider now the unacceptability of examples built with [-cum, -div] predicates:

- (36) a. *Most of my students are a mafia/a circle/a government.
 b. *Most of my students elected their representative.

These examples are built with predicates that denote sets of groups (“impure atoms” in Winter’s (2002) terminology).¹⁴ By picking up a sufficiently small situation, such that there is only one mafia in it, we can apply σ to that singleton set and obtain the unique mafia in the set. But the semantic computation will be blocked, because the Meet operation cannot apply to a sum and a group.¹⁵

3.3 Intensional and extensional groups

Let us finally consider the following type of example, pointed out to me by Manuel Križ and Yoad Winter:

¹³ The unmarked intuitive interpretation refers to only one meeting. Note however that the context may make it clear that there were several meetings (in the minimal situation that contains all the students), in which case $\sigma x.\text{met}(x)$ will denote the maximal sum of individuals that participated to one or the other of the meetings in that minimal situation.

¹⁴ Following Link 1984, I assume a complex ontology, which contains singular individuals and groups as primitive entities. Sums, on the other hand, may be viewed as derived entities, obtained by applying the sum-operator.

¹⁵ *Numerous* would deserve an analysis on its own. For present purposes it is sufficient to assume that $\sigma x.\text{numerous}(x)$ does not denote the “maximal sum of individuals that satisfy the predicate *numerous*”.

- (37) a. Most of my students formed a circle.
 b. Most of the salt formed a square on the floor.

As the reader may check on his/her own, *form a circle* or *form a mafia* are [-cum, -div] and yet they allow *most/all*, with both collective and mass Qs, showing that the Divisiveness Constraint is too strong.

How can we distinguish between *mafia* (or *elect*) and *form a mafia*, and why is it that *form a mafia* does not block collective Qs? Both classes of [-cum,-div] collective predicates denote sets of groups if by “set of group” we mean ‘set of pluralities that are not ordered by the part of relation’. (Note that both of the two classes of predicates qualify as “impure atoms” on Winter’s (2002) definition.) These two classes of group predicates can nevertheless be distinguished on the basis of their conceptual meanings: *mafia* or *elect* describe intensional groups, which cannot be equated with the sums of their members, whereas *form a mafia* or *form a pyramid* describe extensional groups, which are nothing more than the maximal sum of their individual members. Consequently, one might suggest that the computation in (38) can go through:

$$(38) \mu([\text{my students}] \cap \sigma x.\text{formed a circle}(x)) > 1/2 \mu([\text{my students}])$$

As pointed out by Manuel Križ, this analysis is problematic, because (38) ends up meaning ‘most of my students are part of a group that formed a circle’ rather than ‘there is a group that consists of a majority of my students and that group formed a circle’. This latter reading, which is the intuitively adequate one, can be obtained by assuming Matthewson’s 2001 analysis of *most* and *all* (see § 5.2 below):

- (39) a. [[most (of) DP]]: $\lambda P. \exists x.x \leq [\text{DP}] \ \& \ P(x) \ \& \ \mu(x) > \mu([\text{DP}]-x)$
 b. [[all (of) DP]]: $\lambda P. \exists x.x \leq [\text{DP}] \ \& \ P(x) \ \& \ \mu(x) = \mu([\text{DP}])$.

Based on the adequacy between LF representations and intuitive meanings we must conclude that Qs that take *form-a-circle* type of predicates in the nuclear scope cannot be represented as in (38). It seems reasonable to suggest that the reason for this impossibility is that the nominalization of [-cum, -div] predicates necessarily yields a unique entity, in particular a unique group (even if the group can be equated with the sum of its members as is the case with *form-a-circle* predicates). And Meet cannot apply to a sum and a group.

We are thus led to propose that those collective Qs that take [-div] predicates (*form a mafia*) in the nuclear scope have a semantic type that is different from that of collective Qs with [+div] predicates (*gather, meet*) in the nuclear scope:

they denote relations between entities and sets (see §5.2 for further discussion) rather than relations between sums. This accounts for the difference in the corresponding intuitive readings: with examples such as *Most of my students gathered in the hall* other people may have gathered in the hall, whereas *Most of my students formed a circle* corresponds to an interpretation according to which no people other than (some of) my students formed that particular circle.

In order to explain the contrast between *mafia* and *form a mafia* we may use a distinction between \pm partitive predicates, which cuts across the \pm div distinction: [-div, -part(itive)] : *elect, numerous, be a mafia*; [-div, +part(itive)] : *form a circle, form a mafia*. As already explained in previous sections, divisiveness, which characterizes the structure of the denotation of predicates, allows the nuclear scope to be represented as a sum of parts that satisfy P_{div} . Partitivity, on the other hand, characterizes the way in which a predicate P applies to its argument: non-divisive partitive predicates (*form a mafia, a circle, etc.*) may apply to part of the entity denoted by their DP argument, whereas $P_{[-div,-part]}$ (*elect, numerous, a mafia, a circle, etc.*) can only apply to the overall entity denoted by their DP argument. In other words, $P_{[-div,-part]}$ presuppose that no proper part of their argument might satisfy $P_{[-div,-part]}$, and this is incompatible with the semantics of *most/all* ($Q_{\langle e,e \rangle}$). The distinction between [-div, +part] and [-div, -part] correlates with the distinction between extensional and intensional group descriptions: intensional groups cannot be equated with the sum of their members, and therefore intensional-group predicates cannot apply to a proper part of the group denoted by their argument.

4 Conclusions

I have shown that Dowty's puzzle can be explained if we extend to collective Qs a revised version of the analysis that Roeper, Lønning and Higginbotham proposed for mass Qs: collective Qs denote relations between sums ($Q_{\langle e,et \rangle}$) rather than relations between sets. This semantic analysis requires that (i) the predicate in the nuclear scope be nominalized and (ii) the nominalization should denote a maximal sum (as opposed to a unique group). The contrast observed by Dowty follows as a consequence of the requirement in (ii). The Homogeneity Constraint has however been shown to be too strong and a weaker Divisiveness Constraint has been proposed. Those Qs that allow non-divisive predicates such as *form a circle* in their nuclear scope were argued to be of a different semantic type, ($Q_{\langle e,\langle et,t \rangle \rangle}$).

5 Other accounts

The large body of linguistic research on pluralities stemming from Link 1983 and Landman 1989a,b, 2000 contains relatively few accounts of collective quantification (compared to analyses of the ambiguity collective *vs.* distributive readings) and to my knowledge none of them has envisaged that collective Qs denote relations between sums¹⁶ (rather than relations between sets of sums or relations between sets of sets). In this section I will briefly review two set-theoretical analyses and a plural-logic one.

5.1 Winter 2002

Winter 2002 proposes an explanation of Dowty's puzzle based on the semantic analysis of plural Qs in Scha 1981 and van der Does 1993 and on Winter's (2002) own distinction between atom predicates (which split into pure-atom predicates and impure-atom predicates) and set predicates, which cuts across the distinction between distributive and collective predicates.

Winter's classification of a collective predicate as being either an impure-atom predicate or a set predicate is established on the basis of the following diagnostic test, which requires us to consider the acceptability as well as the semantic equivalence between plural Qs and their singular counterparts:

- (40) a. all the/no/at least two/many committees PRED
b. every/no/more than one/many a committee PRED

If the sentences in (40a) and the corresponding sentences in (40b) are equally acceptable and, if acceptable, are furthermore semantically equivalent, then PRED is called an impure atom predicate. If the respective sentences in (40a) and (40b) differ in either acceptability or truth-conditions then PRED is called a set predicate.

On the basis of this diagnostic, Winter obtains the following classes:

- (41) a. Impure atom predicates: *(good) team, numerous, form a pyramid, elect Clinton, constitute a majority*
b. Set predicates: *meet, gather, disperse; be similar, be alike, be together; like each other, look at one another; lift a piano together, write a book together; colleague(s), brother(s), similar student(s), student(s) who met*

¹⁶ Although this analysis is a natural extension of the Roeper-Lønning-Higginbotham analysis of mass Qs, even Lønning (1997) did not propose it.

Winter's distinction between impure atom predicates and set predicates is parallel to, viz. identical to the distinction between [+div] and [-div] collective predicates (recall that both classes qualify as [-cum]) proposed in section § 3.2 above. Note also that *form a pyramid* is in the impure atom class, on a par with *elect* or *mafia*.

Following Scha (1981) and Van der Does (1993), Winter assumes that plural quantificational Det's denote relations between sets of sets¹⁷ and are to be analyzed as lifted variants of their singular counterparts (which denote relations between sets). According to this view, a plural Det is interpretable iff it is semantically equivalent to an LF involving its singular counterpart. In order to increase readability I have replaced Winter's *dfit(D)* notation with D_{pl} , where D_{pl} is a D that takes a DP_{pl} as a complement:

- (42) For any two sets of sets, A and B, the relation $D_{pl}(A,B)$ holds iff the relation $D(\cup A, \cup(A \cap B))$ holds.

According to this definition, the relation D_{pl} holds between two sets of sets iff the relation denoted by D (the singular counterpart of D_{pl}) holds between the union of the sets in A and the union of the sets that are both in A and in B.

Granting that *all* is the plural counterpart of *every*, an example such as (43a) is true iff (44b) is satisfied. Winter attributes the unacceptability of (43b) to the uninterpretability of the truth condition in (44b):¹⁸

- (43) a. All the students are meeting in the hall.
 b. *All the students are a good mafia.
- (44) a. every ($\cup \{x:\text{students}(x)\}, \cup(\{x:\text{students}(x)\} \cap \{x:\text{meet in the hall}(x)\})$)
 b. #every ($\cup \{x:\text{students}(x)\}, \cup(\{x:\text{students}(x)\} \cap \{x:\text{good mafias}(x)\})$)

According to the formula in (44a), the first argument of *every* is the set of atomic students, obtained by applying union to the set of sets of students and its second argument is the set of atomic individuals obtained by applying union to the set of sets obtained by intersecting the set of sets of students with the set of sets of people who are meeting in the hall. The uninterpretability (notated by the diacritic #) of (44b) can be attributed to the fact that we cannot intersect a *set of sets of*

¹⁷ This analysis is based on the possibility of using sets as a model of sums of individuals (Landman 1989, Schwarzschild 1996).

¹⁸ The account of the contrast in (43) sketched here is faithful to what Winter (2002) means, but not to what he explicitly says. In particular, Winter does not explicitly give the conditions in (44) but only the paraphrases that they are supposed to yield, given in (i)-(ii) below, and he attributes the unacceptability of (43b) to the fact that the paraphrase in (ii) is equivalent to the unacceptable sentence **Every student is a good team*.

(i) Every student participated in a set of students that met.

(ii) *Every student participated in a set of students that each of its members is a good team.

individual students (pure atoms) with a set of sets of good teams (impure atoms).

Before turning to the empirical coverage of this account, let me observe that *Meet* \cap can apply in (44)a only if we assume the empty set to be in the denotation of plural predicates. One may wonder whether this assumption is justified.

On the empirical side, note first that this account of the contrast in (43) cannot explain the behavior of predicates like *form a pyramid*: according to Winter's own classification, they are impure atom predicates, on a par with *mafia*, *elect or numerous*, and as such they should block collective quantification, contrary to fact. Indeed, if we replaced the main predicates in (43b) with *form a pyramid*, we would need to check whether *every* ($\cup\{x:\text{students}(x)\}, \cup(\{x:\text{students}(x)\} \cap \{x:\text{form a pyramid}(x)\})$) is satisfied, but this formula is as uninterpretable as (44b), because we cannot intersect a set of students with a set of pyramid forming groups. Winter's proposal is thus comparable to mine in that collective Qs with *form*-predicates in the nuclear scope must be given an analysis different from that of collective Qs that have *mafia* or *elect* in the nuclear scope.

The two proposals nevertheless crucially differ in that Winter's account cannot capture the parallelism between the constraint on (the nuclear scope of) collective Qs and the constraint on (the nuclear scope of) mass Qs.

5.2 Matthewson 2001

According to Matthewson 2001, followed a.o. by Hackl 2002, 2009 and Crnić 2009, the truth conditions of *most* and *all* are assigned by checking conditions of the type in (45):

- (45) a. [[*most* (of) DP]]: $\lambda P. \exists x.x \leq [[DP]] \ \& \ P(x) \ \& \ \mu(x) > \mu([[DP]]-x)$
 b. [[*all* (of) DP]]: $\lambda P. \exists x.x \leq [[DP]] \ \& \ P(x) \ \& \ \mu(x) = \mu[[DP]]$

As shown in §3.3 above, Qs of the type shown in (45) are needed for the analysis of examples built with [-div,-cum] predicates such as *form a circle*. Such quantifiers denote relations between entities (type e) and sets of entities (type $\langle e,t \rangle$), i.e., they denote functions from entities into generalized quantifiers ($Q_{\langle e,\langle et,t \rangle \rangle}$).

In this paper I have argued that those Qs that take [+div,-cum] collective predicates in the nuclear scope are type-theoretically different from the Qs in (45): they denote relations between sum-entities ($Q_{\langle e,\langle et \rangle \rangle}$). Both types of collective Qs take an e-type restrictor, but they differ in that in one case (type $Q_{\langle e,\langle et \rangle \rangle}$) the computation of the truth conditions crucially involves the nominalization of the nuclear scope and a *Meet* operation applying to two sums, and therefore such Qs are constrained by divisiveness (of the predicates in their nuclear scope); Qs of type $\langle e,\langle et,t \rangle \rangle$, on the other hand, involve existential quantification over part of

the entity denoted by the restrictor. Such quantifiers are not constrained by divisiveness but only by “partitiveness”: predicates such as *mafia* or *government* cannot occur in the nuclear scope because they must apply to the restrictor as a whole.

5.3 Plural logic

Plural logic (Boolos 1984, 1985, Rayo 2002, Nicolas 2008) assumes that sums are neither primitive nor derived entities in the ontology: plural definite descriptions do not refer to a plural entity but rather to several singular entities (even when a collective reading is intended). Plural variables are notated with doubled letters, e.g., $\exists xx$, which is to be interpreted as “there are some objects_i such that $x_i < xx_j$ ”, where $x_i < xx_j$ is to be interpreted as “ i is one of them_j”. The plural definite article is analyzed as in (46), where π is the plural counterpart of Russell’s Iota operator:

- (46) a. $\psi(\text{Iota}_x[\varphi(x)]) = \text{def } \exists x[\forall v(\varphi(v) \leftrightarrow x = v) \ \& \ \psi(x)]$.
 b. $\psi(\pi_x[\varphi(x)]) = \text{def } \exists yy[\forall x(x < yy \leftrightarrow \varphi(x) \ \& \ \psi(yy))]$.

Given this definition of the plural definite article, the collective reading of (47) is assigned the LF in (47a) and the semantics in (47b); capitals notate predicates that apply to singular variables and lower case letters notate predicates that apply to plural variables:

- (47) The sailors carried John home.
 a. $\text{CarriedJ}(\pi x[\text{SAILOR}(x)])$ collective reading
 b. $\exists yy[\forall x(x < yy \leftrightarrow \text{SAILOR}(x) \ \& \ \text{CarriedJ}(yy))]$

It is interesting to observe that the only Q that binds plural variables is the existential Q; the universal Q binds only singular variables. Proportional Qs are defined as denoting relations between plural definite descriptions:

- (48) Almost half of the monkeys became infected.
 (49) $\text{AlmostHalfOf}(\pi x [\text{Monkey}(x) \ \wedge \ \text{Inf}(x)]; \pi x [\text{Monkey}(x)])$

To the representation in (49) corresponds the paraphrase in (50):

- (50) The monkeys who became infected are almost half of the monkeys;

Although the quantifiers represented in (49) look similar to those proposed in this paper insofar as the restrictor is filled with a definite description, they are in fact different: no nominalizer applies to the predicate in their nuclear scope, and therefore the divisiveness constraint cannot be explained.

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