

# Directionality in numeral quantifiers: the case of *up to*

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## 1. Introduction

This paper is about modified numerals involving spatial prepositions. The sentences in (1-a) – (1-d) are exemplar.

- (1) a. Our company produced just *under 1000* cars in 2007.
- b. Normal pulse rates vary *between 50 and 100* beats a minute.
- c. Delia's Vegetarian Collection contains *over 250* great recipes.
- d. Jasper is allowed to invite *up to 5* friends to the party.

As far as I know, the semantics of such modified numerals has so far only been discussed in Corver and Zwarts (2006), who focus on the close relation between the locative spatial semantics of prepositions like *under* and their use in numeral quantifiers. In this paper, I will turn to directional prepositions and investigate the role of directionality in numeral modification. I will zoom in on the case of *up to*, as exemplified in (1-d), show that this modifier has a rather quirky distribution and claim that this can be explained by assuming that the use of a spatial preposition in the numeral domain is governed by the same constraints as is its use in the spatial or temporal domain.

The article is structured as follows. First, in Section 2, I will motivate why looking at individual numeral modifiers is an interesting enterprise. Then, I will give some background on prepositional numerals (Section 3), before turning to the crucial data in Section 4. In Section 5, I will propose an analysis in terms of a semantics for *up to* which expresses a homogeneity requirement. Section 6 concludes.

## 2. Modified Numerals

The motivating idea behind this study is that the diverse landscape of modified numerals reflects a diversity in expressed relations. As illustrated in (2), modified

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numerals come in many guises.

- (2)            comparisons — *more than 300, fewer than 300, no fewer than 300*  
                   superlatives — *at least 300, at most 300*  
                   prepositional — *under 300, over 300, between 300 and 500*  
                   coordination — *300 or more*  
                   other — *almost 300, maximally 300, 300 tops*

Intuitively, all such modifiers specify some relation between a cardinality and the amount expressed by the number word. The question then becomes why we find so many different forms, given that, at first sight, many of the modified numerals denote the same quantifier. The answer is two-fold. First of all, on close inspection there appear to be subtle differences between the meanings of modified numerals. Although, at first sight *more than 3, at least 4, over 4, from 4, 4 or more, no fewer than 4*, etc. all express the relation ‘ $\geq 4$ ’, it turns out that there are crucial differences (Geurts and Nouwen 2007, Corblin 2007, Nouwen 2008a,b). Second, there are reasons to believe that it generally makes sense to look beyond what numerical relation is expressed by the modifier, and rather pay attention to the structural decompositional semantics of constructions like those in (2) (Hackl 2000, Takahashi 2006).

Generally, there is a trend not to take quantifier denotations for granted. The research strategy that goes with this trend involves paying close attention to the fine details of quantifier meanings on a case by case basis. (One might say, the trend is to have a particular quantifier theory, next to a generalised one). In this paper, I contribute to this trend by showing that spatial prepositions in modified numerals retain much of their spatial semantics—a point already made by Corver and Zwarts (2006)—and, moreover, showing that the properties of numerals modified by directional prepositions can be derived simply by paying close attention to the general properties of the directional preposition in question.

### 3. Spatial Prepositions in Numerals

It is very easy to find naturally occurring examples of spatial prepositions in numerals. The examples in (3) are the result of a quick google session.

- (3)    a.    The Eskimos have *over 100* words for “snow” but only *around 7* for “sandwich.”  
           b.    Chatham has *under 300* residents.  
           c.    *Between 500 and 600* people came to the exposition.

The literature that focuses specifically on spatial modified numerals is pretty much limited to Corver and Zwarts (2006). The general observations of Corver and Zwarts are that spatial numerals are cross-linguistically very common, that they display a lot of cross-linguistic variation (especially with respect to which prepositions end up modifying numerals and which do not) and that spatial numerals seem

restricted to the vertically oriented prepositions. (This latter observation could be linked to the “more is up; less is down”-metaphor of Lakoff and Johnson (1980), but goes contra the association of numbers with the horizontal dimension as usually found in cognitive psychology; see e.g. Dehaene et al. 1993, Fias and Fischer 2005).

The semantics Corver and Zwarts adopt for spatial numerals is one which takes the spatial meaning of a preposition and maps it to the scale associated with the numeral. In metaphorical terms, the idea is that numerals are ordered vertically and that spatial prepositions can express relative positions on that ordering. Consider, as an example, the case of *boven*, Dutch ‘above’.

- (4) Er zijn al boven de 30 problemen gemeld.  
 There are already above the 30 problems reported.  
 ‘More than 30 problems have already been reported’

The semantics in (5) follows from the fact that the locative function of *boven* is to select a region that is located above its complement, in analogy with (6).

- (5)  $\llbracket \text{boven de } 30 \rrbracket = \lambda n.n > 30$   
 (6) Het schilderij hangt boven de openhaard.  
 The painting hangs above the fire place.  
 ‘The painting hangs over the fire place’

In English, *over* would have a comparable semantics in numerals. Note, however, that the directional sense of *over*, as in *I jumped over the canal*, does not have a use in numerals.<sup>1</sup> One might think that this has to do with directionality. However, given what follows below, it is unlikely that a directional semantics alone prevents the relevant sense of a preposition to be used in numerals. What is a more likely explanation is that, in languages like English, spatial orientation is an important factor in the spatial expression of numeral bounds.<sup>2</sup> That is, since directional *over* is horizontally oriented and numerals are vertically ordered, the two are simply incompatible.

Whatever explanation we might have for the impossibility of using certain prepositions in modified numerals, the examples in (7) make clear that directionality does not generally impede the formation of a spatial numeral. The (complex) prepositions in (7) all lack a locative sense.

- (7) a. I’m allowed to talk for *up to 30* minutes.  
 b. Talks vary *from 10 to 30* minutes.  
 c. We can fit any breed of dog *from 10 lbs and up*.  
 d. The distance between Mars and the earth varies *from 62 million miles down to 34 million miles*.

<sup>1</sup>In Dutch, there exists a preposition *over* which has completely lost its locative sense. Its directional reading is analogous to English. Dutch *over* cannot be used with numerals.

<sup>2</sup>Thanks to Joost Zwarts for discussing the option that orientation plays a role in restricting directional prepositions from occurring in modified numerals.

In the remainder of this paper, I will zoom in on the case of *up to*.

#### 4. The case of ‘up to’: data

In this section, I consider the English complex *up to*, and its use as a numeral modifier.

##### 4.1. Limited distribution

*Up to* in numerals is severely constrained. First of all, it cannot indicate a definite cardinality:

- (8) a. #The total number of visitors last year was up to 100.  
 b. The total number of visitors last year was less than 100.  
 c. The total number of visitors last year was under 100.

A similar observation is visible in (9), which, on the assumption that the speaker knows how many children s/he has, is infelicitous. If acceptable, (9) suggests that the speaker doesn’t know how many children s/he has exactly, but that s/he does know that there aren’t more than 5.

- (9) ??I have up to 5 children.

In the same vein, (10) does not express that the speaker has invited some number of people between 0 and 50. That is, (10) does not have a meaning which is comparable to that of (11). If acceptable at all, (10) rather suggests that the speaker is not certain about the number.

- (10) ??I invited up to 50 people to my birthday party, this year.  
 (11) I invited fewer than 50 people to my birthday party, this year.

The example in (10) contrast with (12). That is, *up to*-numerals are perfectly acceptable in the scope of a weak modal. However, as (13) illustrates, they are not so happy in the scope of strong modals.

- (12) Jasper is allowed to invite up to 10 children to his party.  
 (13) ??Jasper is required to invite up to 10 children to his party.

The data above suggests that *up to*-numerals are somehow sensitive to weak modality. Not only do explicit weak modals license such numerals, in the absence of any existential modal operator, a modal reading seems to be accommodated. That is, the only way to interpret *I have up to 5 children* seems to be with respect to what the speaker considers possible. This intuition of a close relation between *up to* and modality is corroborated by a small corpus study I conducted. I compared a random 56 occurrences of *up to*-numerals with a random 60 occurrences of *less*

*than*-numerals and counted the number of times the modified numeral in question was in the scope of an (adverbial or auxiliary) modal operator.<sup>3</sup> The results, as shown in (14), confirm the intuition that *up to* is particularly associated with weak modal contexts.

(14)

	up to NUM (n = 56)	less than NUM (n = 60)
explicitly modal – weak	32%	2%
explicitly modal – strong	2%	2%
non-modal	66%	96%

There is an interesting parallel between the above data and the use of *up to* in the spatial domain. There, the use of *up to* is also constrained. The intuitive generalisation here, illustrated in (15), seems to be that *up to* needs to combine with some sense of a path (Piñón 1994): *up to* expresses an object's traversal of a path. Notice, however, that weak modals once more lift this constraint.

- (15) a. #Jasper is standing up to the edge of the lake.  
b. Jasper ran up to the edge of the lake.
- (16) a. #Jasper is standing up to here.  
b. Jasper is allowed to stand up to here.

In certain other languages, a preposition exists which behaves exactly like *up to* in the numeral and spatial domain, but which moreover has a comparable usage in the temporal domain. One example is Dutch *tot*:

- (17) ??Jasper heeft tot 10 vriendjes uitgenodigd.  
J. has TOT 10 friends-DIM invited  
'??Jasper invited up to 10 friends'
- (18) Jasper mag tot 10 vriendjes uitnodigen.  
J. may TOT 10 friends-DIM uitnodigen  
'Jasper is allowed to invite up to 10 friends.'
- (19) #Jasper staat tot hier.  
J. stands TOT here.  
'#Jasper is standing up to hier.'
- (20) Jasper is tot hier gerend.  
J. is TOT here ran.  
'Jasper ran up to here.'
- (21) #Jasper kwam tot middernacht de kamer binnengelopen.  
J. came TOT midnight the room inside-walked  
'#Jasper entered the room until midnight'
- (22) Jasper sprak tot middernacht met zijn moeder.  
J. spoke TOT midnight with his mother

<sup>3</sup>I used the free service of the BNC corpus at <http://sara.natcorp.ox.ac.uk/lookup.html>.

‘Jasper spoke with his mother until midnight’

Again, the infelicitous example in (21) contrasts minimally with (23), where it is embedded under a weak modal.

- (23) Jasper mag tot middernacht de kamer binnen komen lopen.  
 J. may *tot* midnight the room inside come walk.  
 ‘Jasper is allowed to walk into the room until midnight.’

Similar data is available for prepositions in at least the following languages: German ‘bis (zu)’, Hebrew ‘ad’, Catalan ‘fins a’, Spanish ‘hasta’. In English, one can observe a parallel between *until* and *up to*. Bare plurals can license the use of an *up to*-numeral, just like they license durative *until*.

- (24) a. #I arrived until 1AM.  
 b. People arrived until 1AM.  
 (25) a. #My computer has up to 2GB of memory.  
 b. Computers of this type have up to 2GB of memory.

The same occurs in Dutch:

- (26) #Er kwam tot middernacht iemand binnengelopen.  
 There came TOT midnight someone inside-walked  
 ‘#Someone entered the room until midnight’  
 (27) Er kwamen tot middernacht mensen binnengelopen.  
 There came TOT midnight people inside-walked  
 ‘People entered the room until midnight’  
 (28) #Mijn computer heeft tot 2GB aan geheugen.  
 My computer has TOT 2GB on memory.  
 (29) Computers van dit type hebben tot 2GB aan geheugen.  
 Computers of this type have TOT 2GB on memory.

Notice that English *until* is also sensitive to weak modals. In other words, English *until/up to* is exactly like Dutch *tot*.

- (30) a. #I arrived until 1AM.  
 b. I’m allowed to arrive until 1AM.

In sum, the use of *up to / tot* in numerals is constrained in exactly the same ways as *up to / tot* is in the spatial domain, and as *until / tot* is in the temporal domain. In the next section, I will propose that this can be accounted for if these prepositions express an homogeneity requirement.

## 5. Analysing ‘up to’/‘tot’

My proposal is the following:

*The spatial, temporal and numeral use of tot / up to share a homogeneity requirement.*

I will illustrate the existence of this homogeneity requirement first in the temporal domain, then in the spatial domain and finally for the numeral uses of *tot* and *up to*. For ease of exposition, I will limit myself mostly to English examples, but what is said about *until* and *up to* below applies equally to Dutch *tot* and I conjecture that similar prepositions in certain other languages are not any different.

### 5.1. Homogeneity in the temporal domain

‘Until’ and temporal ‘tot’ are inherently durative. They are compatible with stative and activity predicates only.

- (31) a. #I { ate an apple / arrived } until 3PM.  
 b. I { slept / was working on my handout } until 3PM.

It is common to analyse the contrast between (31-a) and (31-b) in terms of homogeneity: staves/activities are *temporally homogeneous* (Bennett and Partee 1972). The contrast is accounted for on the assumption that durative *until* lexically encodes homogeneity (e.g. de Swart (1996), Giannakidou (2002)). In (32), I illustrate the homogeneity that plays a role in (31-b). In (33), I give a simple semantics for *until*, which directly encodes homogeneity.

- (32) a. ‘I slept’ holds at interval  $I$   
 $\Rightarrow$  for each sub-interval  $I'$  of  $I$ , ‘I slept’ holds at  $I'$   
 b. ‘I was working on my handout’ holds at interval  $I$   
 $\Rightarrow$  for each sub-interval  $I'$  of  $I$ , ‘I was working on my handout’ holds at  $I'$

- (33)  $\llbracket p \text{ until } x \rrbracket = \forall I' \sqsubseteq I : p \text{ holds at } I'$ , where  $I$  is an interval ending in  $x$

The following illustrates how (33) applies to a felicitous example.

- (34) a. I slept until 3PM.  
 b. Let  $I$  be an interval [... 3PM]: I slept at each sub-interval of  $I$

I am omitting quite a few details here which could be important. First of all, one has to assume that the starting point of the interval is somehow contextually given. Second, what counts as a minimal sub-interval is probably very difficult to specify. Finally, note that the semantics in (33) does not actually place a temporal bound on the activity or state in question: that the state/activity does not continue after the time provided by the *until* adverb is not semantically encoded, but rather a

conversational implicature. In what follows, I will assume we have some suitable theory accounting for each of these three points.

### 5.2. Homogeneity in the spatial domain

Spatial *up to* and *tot* can be shown to be compatible with *spatially homogeneous* predicates only. This becomes clear from comparing ‘up to’ to ‘to’. The first of the examples below concerns the verb *relocate*. If I am first based in Utrecht and then in Amsterdam, then I have relocated to Amsterdam, but I cannot be said to have relocated to Breukelen (which is a place on the path between Utrecht and Amsterdam.) Similar considerations apply to *pointing* and *crossing*.

- (35) a. He relocated to Amsterdam.  
b. #He relocated up to Amsterdam.
- (36) a. The sign points to the auditorium.  
b. #The sign points up to the auditorium.
- (37) a. I crossed to the North side of the pass. (Piñón 1994)  
b. #I crossed up to the North side of the pass.

In contrast, *running* is compatible with spatial homogeneity:

- (38) Jasper ran (from here) up to the edge of the lake.
- (39) If ‘Jasper ran’ can be said to hold for a path [here... the edge of the lake]  
⇒ ‘Jasper ran’ holds for sub-paths of [here... the edge of the lake]

In parallel to durative *until*, I propose spatial *up to* to encode homogeneity:

- (40)  $\llbracket p \text{ up to } x \rrbracket = \forall P' \sqsubseteq P : p \text{ holds at } P'$ , where  $P$  is some path ending in  $x$
- (41) a. Jasper ran up to the edge of the lake.  
b. Let  $P$  be a path ending at the edge of the lake: ‘Jasper ran’ holds at every sub-path of  $P$

### 5.3. Homogeneity in the numeral domain

The question now is how homogeneity might be part of the numeral domain. I will make the following assumption about intervals in the numeral domain: in natural language semantics, predicates hold of cardinalities, not of intervals of cardinalities. So, given a scale of numbers, the relevant notion of sub-interval amounts to the discrete numbers themselves. A property of cardinalities is then homogeneous with respect to some interval if and only if the property holds of all values in the interval. This assumption explains why *up to*-numerals are incompatible with definite cardinalities, as in (42-a). If *up to* expresses homogeneity, then (42) will express that the total number of visitors last year is equal to several different numbers. This does not make sense: *the total number of visitors last year was  $n$*  can only hold for



a single value for  $n$ .

(42) #The total number of visitors last year was up to 1000.

Weak modals create a cardinality property that may apply to several cardinalities at the same time. In fact, the entailments in (43) show that this property can be homogeneous.

(43) Jasper is allowed to invite up to 10 friends  
 ⇒ Jasper is allowed to invite 10 friends  
 ⇒ Jasper is allowed to invite 9 friends  
 ⇒ Jasper is allowed to invite 8 friends  
 ⇒ Jasper is allowed to invite 7 friends  
 etc.

Notice that there is a contrast between the comparative numeral *less than 10* and *up to 9*.

(44) a. Jasper is allowed to invite less than 10 friends.  
 ≠ Jasper is allowed to invite 9 friends  
 b. Jasper is allowed to invite up to 9 friends.  
 ⇒ Jasper is allowed to invite 9 friends.

The contrast in (44) might not be immediately clear. The following context might be helpful. Say, Billy has permission to invite 10 friends to his birthday party and, say, that Jasper is not so lucky. In fact, Jasper isn't allowed to invite anyone. In such a situation, it makes sense to say (45-a), but not (45-b).

(45) a. Jasper is allowed to invite less than 10 friends. In fact, he isn't allowed to invite anyone!  
 b. Jasper is allowed to invite up to 10 friends. #In fact, he isn't allowed to invite anyone!

Similarly:

(46) a. The cars we sell are less than 3 years old. In fact, we don't sell cars older than a year.  
 b. The cars we sell are up to 3 years old. #In fact, we don't sell cars older than a year.

I showed that there is some sense in which *up to* displays a homogeneity requirement in the numeral domain. But how does this apply to the full range of data? In the next subsection, I will be a bit more precise about the underlying semantics of *up to*-numerals and investigate how this bears on their distribution.

#### 5.4. Semantics of ‘up to’ numerals

I will assume that modified numerals are quantifiers over cardinalities. That is, they combine with cardinality predicates. (Roughly as in Hackl (2000)). For instance, *less than 10* takes a cardinality predicate  $P$  and returns the proposition that the highest value for which  $P$  holds is 9 or fewer. Note that in an example like (47-a), the numeral has to move in order to be able to combine with something matching its type requirements, as illustrated by the logical form in (47-b).

- (47) a. The total number is less than 10.  
 b. less than 10 [  $\lambda_n$  [ the total number is  $n$  ] ]  
 c.  $\llbracket \text{less than 10} \rrbracket = \lambda P. \max_n(P) < 10$   
 d.  $\max_n(\lambda n. \text{the total number is } n) < 10$   
     = the total number < 10

For *up to*-numerals, I simply assume that they directly encode the notion of numeral homogeneity I discussed in the previous subsection. Again, I assume that homogeneity holds with respect to an interval which starts at some contextually salient value, indicated here as  $s$ . (In what follows, I will keep on indicating the starting value with the unbound variable  $s$ .)

- (48)  $\llbracket \text{up to } n \rrbracket = \lambda P. \forall m \in [s, \dots, n] : P(m)$

If we apply this semantics to the infelicitous (49-a), then we end up with contradictory truth-conditions.

- (49) a. #The total number is up to 10.  
 b. up to 10 [  $\lambda_n$  [ the total number is  $n$  ] ]  
 c.  $\forall m \in [s \dots 10] : (\lambda n. \text{the total number is } n)(m)$   
     =  $\forall m \in [s \dots 10] : \text{the total number is } m$

In (47) and (49), the modified numeral is in predicative position. For modified numerals in argument position, we need to borrow another assumption from Hackl (2000), namely that these quantifiers come with a counting quantifier *many*.

- (50)  $\llbracket m\text{-many} \rrbracket = \lambda A \lambda B. \exists x [|x| = m \ \& \ A(x) \ \& \ B(x)]$

Additionally, I will assume that this quantifier is ambiguous between the weak/existential form in (50) and the ‘exactly’ variant in (51), this to do justice to the general ambiguity of numerals between lower-bounded and double bounded readings:<sup>4</sup>

- (51)  $\llbracket m\text{-many} \rrbracket = \lambda A \lambda B. \exists !x [|x| = m \ \& \ A(x) \ \& \ B(x)]$

The following example will illustrate how these assumptions work out. For (52-a), we have two logical forms that yield identical truth-conditions.

<sup>4</sup>Thanks to 3(!) anonymous reviewers for urging me to be more explicit about the consequence of having these two options. The form of this ‘exactly’ variant is inspired by Geurts (2006).

- (52) a. Jasper is allowed to invite up to 10 friends.  
 b. [ up to 10 [  $\lambda_n$  [ allowed [ Jasper invite  $n$ -many friends ] ] ] ]  
 c.  $\forall m \in [s, \dots, 10] : \diamond \exists x [ |x| = m \ \& \ \text{friends}(x) \ \& \ \text{invite}(j, x) ]$   
*(Jasper is allowed to invite ... 8, 9 and 10 friends and maybe even more)*  
 d.  $\forall m \in [s, \dots, 10] : \diamond \exists! x [ |x| = m \ \& \ \text{friends}(x) \ \& \ \text{invite}(j, x) ]$   
*(Jasper is allowed to invite ... 8, 9 and 10 friends and maybe even more)*

Turning to (53-a), we get two possible readings:

- (53) a. ?I have up to 5 children.  
 b.  $\forall m \in [s, \dots, 5] : \exists x [ |x| = m \ \& \ \text{my\_child}(x) ]$   
*(I have 5 children)*  
 c.  $\forall m \in [s, \dots, 5] : \exists! x [ |x| = m \ \& \ \text{my\_child}(x) ]$   
*(I have exactly 5 children and I have exactly 4 children and I have exactly 3 children and ...)*

Neither (53-b) nor (53-c) is a good interpretation for (53-a): (53-c) is a contradiction and (53-b) makes ‘up to 5’ equivalent to bare ‘5’. This explains the general oddness of examples like (53-a). Turning to examples with strong modals, (54) shows that we get an account of their oddness in a way that is similar to (53).

- (54) a. ?Jasper is required to invite up to 10 people.  
 b.  $\forall m \in [s, \dots, 10] : \square \exists x [ |x| = m \ \& \ \text{people}(x) \ \& \ \text{invite}(j, x) ]$   
*(Jasper is required to invite 10 people)*  
 c.  $\forall m \in [s, \dots, 10] : \square \exists! x [ |x| = m \ \& \ \text{people}(x) \ \& \ \text{invite}(j, x) ]$   
*(Jasper has to invite exactly 10 people and he has to invite exactly 9 people etc.)*

Again, neither (54-b) nor (54-c) is a good interpretation for (54-a): (54-c) is a contradiction and (54-b) makes ‘up to 10’ equivalent to bare ‘10’.

The reading in (54-b) is a minimality reading. That is, it is about the number of people Jasper should *minimally* invite. Such readings can be observed with comparative numerals, as in (55). (The example is based on Hackl (2000)).

- (55) To get tenure at MIT, you have to have published 3 books. At Utrecht University, it is easier to get tenure. There, you are required to publish fewer than 3 books.  
 = the minimum amount of published books in order to get tenure  $< 3$

There is no similar reading available for *up to*-numerals, witness (56).<sup>5</sup>

<sup>5</sup>In fact, it seems that minimality readings do not occur with other spatial prepositions either:

- (i) Context: In Amsterdam you have to be 50 years or older to become a professor. In Utrecht, you have to be at least 40.  
 a. #At Utrecht University, you have to be under 50 years old to become a professor.  
 b. At Utrecht University, you have to be less than 50 years old to become a professor.

- (56) To get tenure at MIT, you have to have published 3 books. At Utrecht University, it is easier to get tenure. ??At Utrecht University, you are required to publish up to 2 books.  
 $\neq$  the minimum amount of published books = 2

So far, I have shown that the proposed semantics explains the contrast in acceptability of *up to*-numerals embedded under strong and weak modals. I propose that in simple sentences, one may accommodate a weak epistemic modal operator in order to make the sentence acceptable. For instance, according to the proposed semantics, the examples in (57) and (58) result in contradictory truth-conditions, unless a weak modal is inserted.

- (57) I have up to 5 children.  
 a.  $\forall m \in [s, \dots, 5] : \exists (!)x [ |x| = m \ \& \ \text{my\_child}(x) ]$   
 b.  $\forall m \in [s, \dots, 5] : \diamond \exists (!)x [ |x| = m \ \& \ \text{my\_child}(x) ]$
- (58) Up to 10 people died in the crash.  
 a.  $\forall m \in [s, \dots, 10] \exists (!)x [ |x| = m \ \& \ \text{people\_died\_in\_crash}(x) ]$   
 b.  $\forall m \in [s, \dots, 10] \diamond \exists (!)x [ |x| = m \ \& \ \text{people\_died\_in\_crash}(x) ]$

One might worry that the account sketched here overgenerates, for if the modified numeral quantifiers can scope over weak modals, then we would expect them to scope over any weak operator. The example in (59) shows that this is an unwanted prediction. The reading in (59-b), which corresponds to the logical form in (59-c), is simply unavailable.

- (59) a. Someone invited up to 10 children.  
 b. #For  $n$  up to 10: someone invited  $n$  children (unavailable)  
 c. #[ up to 10 [  $\lambda_n$  [ someone invited  $n$ -many children ] ] ]

However, this prediction is not a special feature of the proposed semantics. We would expect any modified numeral to be able to yield readings where it scopes over a non-modal operator. This is clearly not what we observe. Comparative modified numerals equally fail to scope over nominal quantifiers.

- (60) a. Someone invited less than 10 children.  
 b. #the maximal  $n$  such that someone invited  $n$  children  $< 10$

In other words, the lack of a wide scope reading for (59-a) can be assumed to be for the same reason (60-a) lacks the reading in (60-b). One way to account for this is by parallel with Kennedy's generalisation (Kennedy 1997, Heim 2000, Hackl 2000): if the scope of a quantificational DP contains the trace of a degree phrase, it also contains that degree phrase. If one takes a liberal notion of a degree, one which includes cardinalities, then this generalisation captures the data in (60). Prepositional numerals appear to behave exactly the same.

### 5.5. Summary

I have argued that *up to* and its kin encode a homogeneity requirement. What the usages of *up to / tot* share over all domains is that they express a property to hold over all parts of some interval. The lack of potential homogeneity is what characterises the examples where these prepositions are not acceptable. This non-homogeneity can be avoided by interpreting the predicate in question relative to a weak epistemic attitude.

## 6. Discussion

The main goal of this paper was to show that the semantics of *up to* in modified numerals is closely linked to its directional semantics in the spatial domain and its durative temporal semantics. This goal connects to the observation that modified numerals are a diverse class of relations, incorporating a wealth of semantic nuances. Given the results of this paper, one might wonder what are the properties of other directional prepositions that occur in modified numerals. It is beyond the scope of this paper to give a full account of such prepositions, but I will hint at some data suggesting that the kind of aspects of directionality that played a role in the distribution of *up to* are relevant to examples with other prepositions too. First of all, notice that start-point markers can occur in modified numerals, but are also very limited in their distribution. Again, modified numerals based on such prepositions are fine when embedded under a weak modal.

- (61) a. #Jasper invited from 10 friends up.  
 b. A tour can be organised for groups from 10 persons up

Similar data exists for complex prepositions expressing double bounds. For instance, (62) contains the arguably directional prepositional complex *from ... to ...*

- (62) The talks vary from 15 to 30 minutes.

Intuitively, the verb *vary* yields a set of potential lengths of my talk. When the length of my talk is given a definite value as in (63-a), the example becomes infelicitous. In contrast, the locative *between ... and ...* is perfectly acceptable in this case.

- (63) a. #My talk is from 15 to 30 minutes long.  
 b. My talk is between 15 and 30 minutes long.

Such data show that the kind of considerations we have been discussing to contrast directional *up to* to locative *under* also play a role for other directional-locative pairs. A detailed account of these data, however, is left for future research.

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