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Reasoning with Partial Orders: Restrictions on Ignorance Inferences of Superlative Modifiers*

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Abstract The present study is concerned with Ignorance Inferences associated with Superlative Modifiers (SMs) like *at least* and *at most*. Experimental evidence is presented showing that the Ignorance Inferences associated with SMs depend on their associate: when the associate of an SM is totally ordered (e.g. a numeral), the exhaustive interpretation of the prejacent must necessarily constitute an epistemic possibility for the speaker. However, when the associate of the SM is partially ordered, the exhaustive interpretation of the prejacent can but need not constitute an epistemic possibility for the speaker.

Keywords: ignorance inferences, implicatures, superlative modifiers, experimental semantics & pragmatics

1 Introduction

There are two well-known and uncontroversial facts about Superlative Modifiers (SMs henceforth) like *at least* and *at most*: (i) that they can take a variety of expressions as their complements (Krifka 1999), and (ii) that they give rise to certain Ignorance Inferences (Nouwen 2010). For instance, both (1a) and (1b) below may convey that Bill, the speaker, is ignorant about something. More specifically, (1a) suggests that while Bill knows the lower/upper bound for the number of dogs Ed has, he is ignorant about the exact number; (1b) suggests that while he knows the maximal/minimal number of party attendees, he is ignorant as to who exactly came.

(1) a. Bill said: Ed has {at least / at most} <u>four</u> dogs.
$\rightsquigarrow$ *Bill is ignorant about the exact number of dogs.*

b. Bill said: {At least / At most} <u>Liz and Sue</u> came.
$\rightsquigarrow$ *Bill is ignorant about exactly who came.*

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These type of Ignorance Inferences are pervasive, as they appear with a variety of complements–as long as SMs are used by a cooperative speaker within a discourse that meets certain pragmatic conditions (Westera & Brasoveanu 2014).

The division of labor between semantics and pragmatics in deriving the various implications conveyed by sentences like (1) is debated. On one commonly held view, the lower and upper bound of SMs are part of their semantic content, whereas Ignorance Inferences are pragmatic. A number of different proposals, each introducing its own machinery, has been put forward to derive the lower and upper bound restrictions: they have been analyzed as modals (Geurts & Nouwen 2007), as *minima* and *maxima* operators (Nouwen 2010), as inquisitive expressions (Coppock & Brochhagen 2013b), as operators of meta-speech acts (Cohen & Krifka 2014), and as epistemic indefinites (Nouwen 2015). With respect to Ignorance Inferences, at least two main lines of research have been pursued: a neo-Gricean analysis (Büring 2007, Schwarz 2016) and those relying on grammatical approaches to implicatures (Mayr 2013, Mayr & Meyer 2014).

Despite an abundance of literature on certain aspects of SMs (e.g., lack of Scalar Implicatures, behavior in embedded contexts, etc.), the discussion falls short in two respects. First, a formal description of what exactly these Ignorance Inferences look like is lacking. Second, investigation into Ignorance Inferences has asymmetrically focused on the "numeral" case, where SMs modify numeral or measure phrases, leaving the cases involving associates of other categories (DPs, VPs, etc.) largely unexplored. In spite of this, there seems to be a tacit assumption in the literature that SMs behave alike in their Ignorance Inferences with numeral and non-numeral associates.

This paper seeks to fill the gap in the literature by experimentally investigating the formal properties of SMs across associate types, and uses the findings to adjudicate between theories of SMs. I begin by asking whether it is justified to assume that there are no formal differences between the inferences that come with the numeral case and other cases. Call this question UNIFORMITY.

(2) UNIFORMITY
Are the inferences that come with SMs the same across the board?

The experimental results seem to answer the question in the negative: participants treat Ignorance Inferences associated with sentences like (1a) and (1b) differently. I argue that this difference is related to the particular scale structure of the different types of associates. In particular, (*i*) when SMs modify a scale which constitutes a total order, the exhaustive interpretation of the prejacent *must* be an epistemic possibility for the speaker. However, (*ii*) when SMs modify a scale which constitutes a partial order, the exhaustive interpretation of the prejacent *need not* be an epistemic possibility for the speaker. These results constitute a novel finding and introduce yet another property that formal theories of SMs must explain.

The paper takes off by presenting three different theories of Ignorance Inferences with SMs, which crucially lead to three different sets of predictions. The experimental design and results are then discussed against the backdrop of these predictions.

2 Calculating Ignorance Inferences of SMs

This section reviews three theories of Ignorance Inferences with SMs, with a particular focus on the aforementioned UNIFORMITY question.

2.1 A neo-Gricean account

Schwarz (2016) uses a double Horn-Set strategy to calculate the Ignorance Inferences of SMs with numerals. Mendia (2016) modifies and extends the proposal to account for the distributional flexibility of SMs. This amendment rests on the following two assumption: (i) SMs are focusing elements (Krifka 1999) whose association with focus is Conventional (*à la* Beaver & Clark 2008), and (ii) SMs form a Horn-Set with *only*: $\langle$*at least, only*$\rangle$ and $\langle$*at most, only*$\rangle$. In addition, the set of alternative propositions that are relevant for the Gricean calculus is provided by two independent mechanisms: (i) Association with Focus, whereby a set of alternatives may be obtained by replacing the focus-bearing constituent with contextually relevant alternatives (Rooth 1992), and (ii) the neo-Gricean substitution method for implicature calculation, whereby elements participating in a Horn-Set can be replaced with one or more of their scalemates (Sauerland 2004). Below we present Mendia's (2016) proposal in more detail.

In the neo-Gricean framework Ignorance Inferences of SMs are derived as a kind of Quantity Implicature. For instance, take an assertion like *at least 2 people came*, expressed as $[\geq 2]$.[1] Assuming Hintikka's (1962) K_S and P_S operators of epistemic necessity and possibility (relativized to a speaker S), such assertion is assumed to convey that $K_S[\geq 2]$ (cf. Maxim of Quality). Assume as well that some version of the Maxim of Quantity is at work, here defined in terms of asymmetric entailment: for any two relevant and true propositions $[\phi]$ and $[\psi]$, if the denotation of $[\phi]$ asymmetrically entails $[\psi]$, the speaker should choose $[\phi]$ over $[\psi]$. Those

1 For simplicity, propositions are enclosed in square brackets, such that $[\phi]$ stands for some proposition containing a relevant expression ϕ. The idea is to informally represent the associate of the SM within the square brackets as a mnemonic for the relevant expression for the purposes of implicature calculation. For instance, a sentence like *4 students came* is represented as $[4]$ and *Al and Mary came* as $[A \oplus M]$. With modifiers, $[\geq\ \phi]$ stands for [*at least* ϕ], $[\leq\ \phi]$ for [*at most* ϕ], and $[O\ \phi]$ for [*only* ϕ].

alternatives $[\phi]$ are often called Stronger Alternatives. Upon hearing $[\psi]$, a hearer may reason that, since the speaker did not choose the Stronger Alternative, it must be because it is not the case that $K_S[\phi]$, i.e., $\neg K_S[\phi]$. This is a Primary Implicature. In the case of $[\geq 2]$, the number of Primary Implicatures is bigger, since Stronger Alternatives can be obtained by replacing both *at least* with *only* and the numeral 2 by its scalemates.

(3) a. Primary Implicatures of $[\geq 2]$:
$\neg K_S[O\,2] \wedge \neg K_S[\geq 3] \wedge \neg K_S[O\,3] \wedge \neg K_S[\geq 4] \wedge \neg K_S[O\,4]\ldots$

b. Implicature Base of $[\geq 2]$:
$K_S[\geq 2] \wedge \neg K_S[O\,2] \wedge \neg K_S[\geq 3] \wedge \neg K_S[O\,3] \wedge \neg K_S[\geq 4] \wedge \neg K_S[O\,4]\ldots$

We do not need to do anything else to derive the Ignorance Inference of $[\geq 2]$. The Implicature Base in (3b) entails that two and only two of the Stronger Alternatives, namely $[O\ 2]$ and $[\geq\ 3]$, must constitute epistemic possibilities for the speaker: $\neg K_S\neg[O\ 2]$ and $\neg K_S\neg[\geq 3]$.[2] This is so because negating any one of the two entailed propositions would entail the truth of one other Stronger Alternative, thus contradicting the corresponding Primary Implicature. As an illustration, consider that given $K_S[\geq\ 2]$, if $\neg P_S[O\ 2]$ (or alternatively $K_S\neg[O\ 2]$), it follows that $K_S[\geq 3]$, which contradicts the Primary Implicature that $\neg K_S[\geq 3]$. Therefore, it must the case that $\neg K_S\neg[O\ 2]$ (or $P_S[O\ 2]$). Together, the entailment that $\neg K_S\neg[O\ 2]$ and the Primary Implicature that $\neg K_S[O\ 2]$ express that the speaker is ignorant about whether $[O\,2]$. The same derivation can account *mutatis mutandis* for the ignorance of the speaker about whether $[\geq 3]$.

The first key observation is that what facilitates these entailments is a configuration where there are two Stronger Alternatives that jointly exhaust the space of possibilities denoted by the assertion. Schematically:

(4) $$\underbrace{[\geq 2]}_{\text{assertion}} \quad \leftrightsquigarrow \quad \underbrace{[O\ 2] \vee [\geq 3]}_{\text{exhaust all possibilities}}$$

As a consequence, one or the other Stronger Alternative, $[O\ 2]$ or $[\geq 3]$, must be true, and so negating any one of them entails the truth of the other.

The second key observation is that this configuration always arises when SMs modify complements that constitute total orders. With the current assumptions this means that the exhaustive interpretation of the prejacent must constitute an epistemic possibility when SMs modify a complement that is totally ordered. However,

2 In Hintikka's (1962) epistemic logic K and the possibility epistemic operator P are interdefinable, since $K\phi \leftrightarrow \neg P\neg[\phi]$ and $P\phi \leftrightarrow \neg K\neg\phi$. Thus, the entailments of the Implicature Base above, $\neg K_S\neg[O\,2]$ and $\neg K_S\neg[\geq\ 3]$, could also be expressed in terms of their more transparent equivalents $P_S[O\,2]$ and $P_S[\geq 3]$.

things are different when SMs modify complements that are partially ordered. Take now instead the sentence *at least Bill came*, expressed as $[\geq\ B]$. As before, upon hearing $[\geq\ B]$, the hearer infers that $K_S[\geq\ B]$ and–for a reduced domain consisting of $\{Bill,\ Mia,\ Sue\}$–derives the corresponding Primary Implicatures and Implicature Base.

(5) a. Primary Implicatures of $[\geq\ B]$:
$\neg K_S[O\,B] \wedge \neg K_S[\geq\ B\oplus S] \wedge \neg K_S[\geq\ B\oplus M] \wedge \neg K_S[O\,B\oplus S] \wedge \neg K_S[O\,B\oplus M] \wedge \neg K_S[\geq\ B\oplus S\oplus M] \wedge \neg K_S[O\,B\oplus S\oplus M]$

b. Implicature Base of $[\geq\ B]$:
$K_S[\geq\ B] \wedge (5a)$

Unlike in the case where SMs modifies a numeral, the Implicature Base does not entail that any one of the Stronger Alternatives is an epistemic possibility for the speaker. Unlike in (3b), (5b) fails to generate a pair of Stronger Alternatives that jointly exhaust the possibility space of the prejacent (it requires at least three Stronger Alternatives to do so).

(6) $$\underbrace{[\geq B]}_{\text{assertion}} \leftrightsquigarrow \underbrace{[O\,B] \vee [\geq\ B\oplus S] \vee [\geq\ B\oplus M]}_{\text{exhaust all possibilities}}$$

As a consequence, it is possible to negate any one of the Stronger Alternatives because its conjunction with the Implicature Base results in a contingent set.

The prediction is that, for instance, knowing that at least Bill came is not at odds with the certainty that he did not come alone, expressed as $K_S\neg[O\,B]$, albeit it requires ignorance about the identity of the companions (in our simplified scenario).

(7) $$K_S[\geq B] \wedge K_S\neg[O\,B] \wedge \neg K_S[O\,B] \wedge \neg K_S[\geq B\oplus S] \wedge \neg K_S[\geq B\oplus M]$$

Following the same logic, negating two such Stronger Alternatives, for instance $K_S\neg[\geq B\oplus S]$ and $K_S\neg[\geq B\oplus M]$, would entail that the speaker is knowledgeable about the third corresponding Stronger Alternative in (7), $K_S[O\ B]$, contradicting the Primary Implicature that $\neg K_S[O\,B]$ and resulting in oddness once again.

In sum, the neo-Gricean rendition of the implicatures of SMs defended in Mendia (2016) predicts that when an SM takes a complement that denotes a total order, it pragmatically entails that the exhaustive interpretation of the prejacent is an epistemic possibility for the speaker. In turn, if the SMs' complement denotes a partial order, the exhaustive interpretation of the prejacent is a contingent epistemic possibility for the speaker.

2.2 Inquisitive Semantics for Superlative Modifiers

Coppock & Brochhagen (2013b) develop a theory of SMs couched within Inquisitive Semantics. In this proposal it is assumed that SMs share with other epistemic operators (like disjunction) the property of being *interactive*: SMs are taken to require that there be minimally two relevant epistemic possibilities that are compatible with the speaker's knowledge.

In Inquisitive Semantics, denotations are represented as sets of possible worlds (possibilities) corresponding to the set of possible answers to the QUD. The set of available possibilities is then further constrained by the information state of the speaker, that is, by the set of possible worlds epistemically accessible to the speaker. For instance, assuming that k_s represents the information set of the speaker, a proposition p restricted to k_s is expressed as the set $\{p' \mid \exists q \in p : p' = k_s \cap q\}$. Notice that nothing in the way that k_s restricts p requires the restricted set to contain more than one possibility. To fix this, the authors propose an additional pragmatic principle to derive Ignorance Inferences (elaborating on Groenendijk & Roelofsen 2009).

(8) MAXIM OF INTERACTIVE SINCERITY
If p is interactive, then p is interactive in the speaker's information set.

Since by assumption SMs are interactive, the proposition p must be interactive in the information state k_s, and so the denotation of p restricted to k_s must contain more than one possibility. The principle in (8) enforces this requirement. For cases where SMs modify a phrasal (non–numeral) complement, the calculation of the Ignorance Inference proceeds as follows. Consider the following proposition:

(9) At least Ann snores. [Coppock & Brochhagen 2013b: 28]

Assume a small domain like $\{Ann,\ Bill\}$ and suppose that the speaker knows that only Ann snored and Bill did not. In this situation (9) is odd or misleading at best. The denotation of (9) amounts to the set of sets of worlds that verify (9), that is, $\{\{[A \oplus B], [O\ A]\}, \{[O\ A]\}\}$. The speaker information set is smaller in this case, it contains the singleton $\{[O\ A]\}$, the set of worlds where only Ann snores. If we restrict the set of propositions in p with those in k_s, only $\{[O\ A]\}$ survives:

(10) $\{p' \mid \exists q \in \{\{[A \oplus B], [O\ A]\}, \{[O\ A]\}\} : p' = \{[O\ A]\} \cap q\} \qquad \leftrightarrow \{[O\ A]\}$

Since this proposition is not interactive, the maxim of Interactive Sincerity is violated, and (9) is predicted to be odd in these situations. It follows that in the case of *at least* sentences there must be some Stronger Alternative to the prejacent that is consistent with the speaker's information set. Although the authors do not explicitly discuss how to derive these Ignorance Inferences, this could be done by virtue of SMs' signaling that the speaker is unable to restrict her epistemic state to a singleton (cf. Alonso-Ovalle & Menéndez-Benito 2010 for epistemic indefinites). In the case of SMs taking numeral complements, Ignorance Inferences are derived the same way. Consider:

(11) # A hexagon has at least three sides. [Nouwen 2010: 4]

Presumably (11) is odd because the Ignorance Inference that it conveys conflicts with the common assumption that most people know the number of sides of a hexagon. Under this approach, (11) denotes the set $\{\bigcup\{[O\ n] \mid n \geq 3\}\}$. The information set of anyone who knows that hexagons have exactly six sides is just the singleton $\{[O\ 6]\}$.[3] Restricting the denotation of (11) with the information state of the speaker delivers again a singleton and so Interactive Sincerity is violated, resulting in the oddness of (11).

Thus, in this system, if the speaker knows that *only p*, it is predicted that she cannot use the sentence *at least p*, because her epistemic state already includes which one of the possibilities expressed by *at least p* holds. In the current framework, this means that a speaker who knows *only p* has all the information required to settle the relevant issue with respect to some QUD, and so the use of an interactive expression is not warranted. Conversely, it is predicted that a speaker can always utter a proposition of the form *at least p* when she knows that *only p* is false, provided that there are at least two other higher ranked alternatives that she considers possible. Moreover, this is true of SMs modifying both numerals and other phrases.

2.3 Inquisitive Semantics with a twist

In the system of Coppock & Brochhagen (2013b) presented above, all that it is required of SMs is that they denote any two possibilities, but nothing is said about which possibilities. As a consequence, no particular proposition is required to be in the information state of the speaker when she utters an SM-proposition. This can be problematic: among other things, it means that the speaker is not required to consider the exhaustive interpretation of the prejacent as an epistemic possibility. As mentioned above, the account predicts that there should be nothing odd about (1a) with *at least*, even if the speaker knows for certain that Ed has either exactly five or exactly six dogs. Since there are two possibilities alive for the speaker, Interactive Sincerity is observed and no oddness is predicted, contrary to intuitions. For the same reasons, (1b) above does not require that Bill mandatorily considers the possibility that only Liz and Sue came.

3 This is assuming a two-sided semantics for numerals, but the authors show that the same results are obtained assuming a one-sided semantics of numerals.

Experiments conducted by the same authors in Coppock & Brochhagen (2013a) confirmed that these predictions are indeed problematic. In a truth value judgment task speakers had to judge whether a picture with n objects could be verified by sentences of the form *at most* n and *at most* $n+1$. The results showed that acceptance rates dropped for *at most* $n+1$ (44.3%) compared to *at most* n (97.8%). This is unexpected if all that SMs require is for the speaker to consider any two epistemic possibilities; as long as the true possibility is included in the information set of the speaker, n in this case, it should be inconsequential to choose a proposition like $n+1$, and yet this is not the case.

In order to fix the problem, the authors provided in Coppock & Brochhagen (2013a) a new pragmatic principle.

(12) MAXIM OF DEPICTIVE SINCERITY
If p highlights a possibility q, then the speaker considers q possible.

Depictive Sincerity rests on the notion of *highlighting*, borrowed from work in Inquisitive Semantics on polar questions: a possibility q is highlighted if it is overtly expressed. Assuming that propositions of the form *SM n* overtly express the prejacent, speakers uttering (1a) must consider the possibility that Ed has four dogs. Extending Depictive Sincerity to the conjunctive case in (1b), that Liz and Sue came must be considered an epistemic possibility.

As a consequence of Depictive Sincerity, the prejacent of an SM-statement must always constitute an epistemic possibility for the speaker. Unlike Interactive Sincerity alone, Depictive Sincerity enforces this requirement and hard-wires it in the form of a pragmatic constraint regulating the use of SMs.[4]

2.4 Summary of predictions

All three theories presented above make different predictions about the kind of knowledge that is compatible with uttering SMs. Recall that our question concerns whether or not the prejacent of an SM statement constitutes an epistemic possibility *uniformly*, that is, irrespective of the nature of the SMs associate. Only the neo-Gricean approach predicts a difference based on the ordering properties of the complement: SMs modifying a totally ordered complement must entail that the exhaustive interpretation of the prejacent is a possibility for the speaker, whereas with partially ordered complements this is not a requirement. In contrast, both Inquisitive approaches predict uniform behaviors (albeit different in each version). The

4 Notice that what is required of SMs is that the prejacent is true, leaving open the question as to whether it has to be exhaustively true. For the purposes of this paper, I will assume that it is indeed the exhaustive interpretation of the prejacent what is required to be an epistemic possibility; otherwise its application would be vacuous in some cases (e.g., *at least Ann snored*).

	Structure of complement					
	Total Order			Partial Order		
NG	$K_S[SM\ \phi]$	$\models$	$P_S[O\ \phi]$	$K_S[SM\ \phi]$	$\not\models$	$P_S[O\ \phi]$
IS-	$K_S[SM\ \phi]$	$\not\models$	$P_S[O\ \phi]$	$K_S[SM\ \phi]$	$\not\models$	$P_S[O\ \phi]$
IS+	$K_S[SM\ \phi]$	$\models$	$P_S[O\ \phi]$	$K_S[SM\ \phi]$	$\models$	$P_S[O\ \phi]$

Table 1 Summary of predictions wrt. the interpretation of the prejacent

Inquisitive Semantics approach of Coppock & Brochhagen (2013b) predicts that the exhaustive interpretation of the prejacent does not have to be in the epistemic state of the speaker, regardless of the type of complement. Finally, the amendment presented by Coppock & Brochhagen (2013a) does require the exhaustive interpretation of the prejacent to be an epistemic possibility, but this happens for both totally and partially ordered complements alike. These predictions are summarized in Table 1 above.[5]

3 Experiment

The goal of the experiment is two-fold: (i) to ascertain the facts about UNIFORMITY and (ii) to assess which of the three theories reported above fares better in terms of explaining the behavioral data.

3.1 Participants

Thirty-six native speakers of English participated in the experiment, recruited via the online platform Amazon Mechanical Turk. Prior to analysis, four participants were removed due to low accuracy (below 75%) in filler questions.

3.2 Design & Materials

Two factors, "Type" (TOTAL ORDER *vs.* PARTIAL ORDER) and "Acceptability" (BAD, GOOD and TARGET) were crossed in an Acceptability Judgment Task to create 30 critical items in a 2×3 factorial design. All TOTAL ORDER items used numerals, whereas all PARTIAL ORDER items involved plurals formed by conjunction. In the Acceptability factor, BAD items were semantically ill-formed, GOOD items involved semantically and pragmatically felicitous sentences and TARGET items, the

5 IS- stands for the Inquisitive Semantic approach by Coppock & Brochhagen (2013b), IS+ for Coppock & Brochhagen (2013a) and NG for the neo-Gricean approach in Mendia (2016).

critical condition, involved sentences where the exhaustive interpretation of the prejacent was explicitly negated. Finally, "Quantifier" (AT LEAST *vs.* AT MOST) was counterbalanced. Examples (13) through (15) present a sample of the full paradigm used in the experiment.

(13) *Context*: Sue is teaching a class to four students: Mary, Liz, Al and Bill. A colleague asks her:

(14) Type TOTAL ORDER:

a. *Question*: How many students completed the quiz?

b. *Answer*–AT LEAST: I don't remember, at least two...

i. ...maybe one. BAD

ii. ...maybe more. GOOD

iii. ...but not only two. TARGET

c. *Answer*–AT MOST: I don't remember, at most two...

i. ...maybe four. BAD

ii. ...maybe less. GOOD

iii. ...but not exactly two. TARGET

(15) Type PARTIAL ORDER:

a. *Question*: Who completed the quiz?

b. *Answer*–AT LEAST: I don't remember, at least Mary and Liz...

i. ...maybe only Liz. BAD

ii. ...maybe somebody else too. GOOD

iii. ...but not only them. TARGET

c. *Answer*–AT MOST: I don't remember, at most Mary and Liz...

i. ...maybe Al and Bill too. BAD

ii. ...maybe only Mary. GOOD

iii. ...but not both. TARGET

Items consisted of a short Q&A dialog where subjects had to judge the naturalness of the answer to the question. The first screen presented a short sentence that served to set the context of the Q&A dialog. Subjects advanced by pressing a key to a second screen, where one character asked to another character a relevant question relative to the context. After reading the question, subjects advanced to the third and last screen, which contained the response of the second character to the previous question. In the same screen, subjects were asked the comprehension question

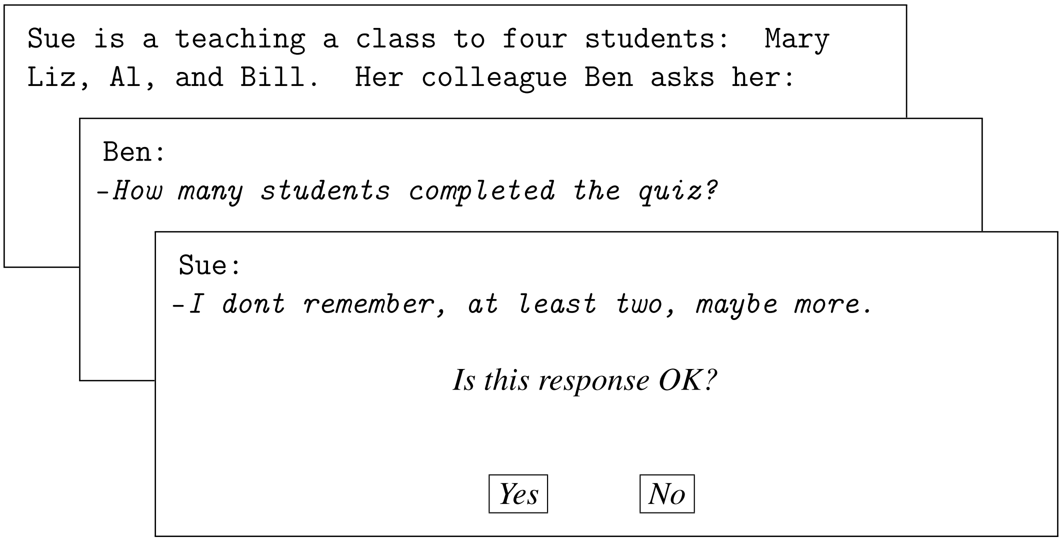

Figure 1 The components of each item, depicted in order from back to front.

Is this response OK?, which they had to answer by pressing a key for either "Yes" or "No". All items, fillers included, proceeded alike. The three components of each item (context, question, answer and assessment of acceptability) were presented in that order in three subsequent screens, as illustrated in Figure 1 above. All experimental items were presented using the Ibex Farm experiment presentation platform (Drummond 2015).

3.3 Analysis

Trials with Reading Times below 500ms or 2.5 *SD*s above the mean were discarded from the analysis. In addition, one *at most* item was removed due to a coding error. After the cleaning, 32 subjects and 29 critical items were analyzed for a total of 905 observations. Acceptance Rates ("Yes" answers) were analyzed using a logistic mixed-effects model, computed with the `lme4` package in R.

The model was estimated using the `glmer` function with Type (numeral for TOTAL ORDER *vs.* conjunction for PARTIAL ORDER) and Quantifier (*at least vs. at most*) as fixed effects and participant and item as random intercepts. In addition, two critical experimental contrasts were included in the analysis as variables with deviation coding. The first contrast measured the difference between GOOD and TARGET trials, GOOD$\sim$TARGET. This contrast is interpreted as the penalty associated with accepting a TARGET item as compared to a GOOD item. In theoretical terms, the contrast reflects the penalty associated with holding an epistemic state

	TOTAL ORDER	PARTIAL ORDER
BAD	18.87(3.11)	23.78(3.57)
GOOD	91.61(2.23)	86.30(2.85)
TARGET	**32.00(4.68)**	**67.77(3.80)**

Table 2 Mean Acceptance Rates and and *SE*s.

	Estimate	*SE*	*t*	
Type	−0.707	0.217	−3.24	**
Quantifier	9.715	0.289	3.350	***
BAD∼GOOD	4.038	0.336	12.015	***
GOOD∼TARGET	−07.18	0.298	−2.406	*
Type * BAD∼GOOD	−0.335	0.535	−0.629	
Type * GOOD∼TARGET	−2.758	0.561	−4.916	***

Table 3 Fixed effects estimates from LME modelling. Significance at $|t| > 2$.

that includes both $K_S[SM\ \phi]$ and $P_S[O\ \phi]$, as opposed to an epistemic state containing, for instance, $K_S[\geq\ \ \phi]$ and $P_S[>\ \ \phi]$. The second contrast measured the difference between BAD trials and GOOD trials, BAD∼GOOD. This was included as a control, since we know that the differences between BAD and GOOD are independent from any other factor: BAD trials are uniformly semantically deviant, whereas GOOD trials are always both semantically and pragmatically felicitous. Lastly, to test for differences in our experimental contrasts, terms for the interaction of Type with each of the contrasts were included.

3.4 Results

The general results are summarized in Table 2 and Table 3. With TOTAL ORDER acceptability rates for TARGET are comparable to those for BAD. With PARTIAL ORDER, however, acceptability rates double, bringing TARGET items closer to GOOD. The statistical analysis confirmed these trends: the interaction between Type and GOOD∼TARGET was significant. Type and BAD∼GOOD were both significant predictors of acceptability as well. Crucially, however, the two factors do not interact, suggesting that the effect of Type on the TARGET items was not modulated by generalized differences between the two types of sentences.

The analysis also revealed a main effect of Quantifier. To explore this effect, a post-hoc comparison was conducted between the quantifiers and found pronounced differences in behavior with *at least* and *at most*. Figure 2 represents the acceptance

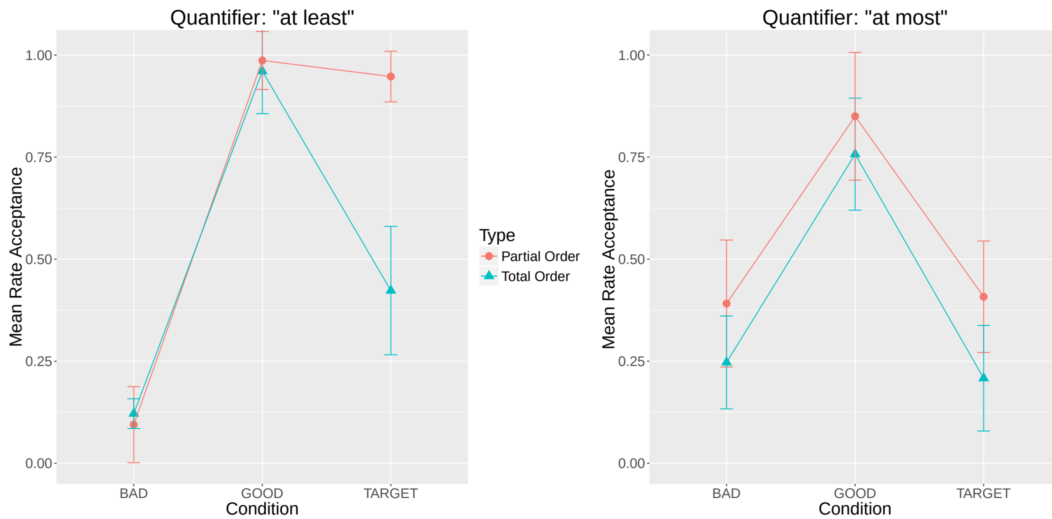

Figure 2 By Quantifier Type*Acceptability interaction plots

rates by Type and Acceptability separately for each quantifier.

In the case of the SM *at least*, there is a clear difference in acceptability on TARGET depending on whether we have a total or partial order associate: TARGET is accepted at the same rate as GOOD with PARTIAL ORDER (conjunctions), but rejected at much higher rates with TOTAL ORDER (numerals). Thus, the Type*Acceptability interaction we found with our earlier model is evident with *at least* ($p < 0.001$). However, we do not see the same pattern with *at most*: TARGET items are rejected across-the-board, behaving very much like BAD. Numerically, TARGET seems to show greater rates of acceptance, but the difference is not significant. In sum, it appears as though the interaction observed above is largely driven by the *at least* items.

Finally, *at most* differs from *at least* in a number of other respects. First, BAD trials are rated higher than the corresponding BAD trials for *at least* in both Types. Second, GOOD trials are rated lower for *at most* in comparison to *at least*, also in both Types. Finally, with *at most* we find that there is a marked preference for PARTIAL ORDER across all three Acceptability conditions, an asymmetry that is absent from the *at least* cases.

4 Discussion

We started by posing a question about the nature of the Ignorance Inferences that come with SMs. We dubbed this question UNIFORMITY, repeated below.

(16) UNIFORMITY
Are the inferences that come with SMs the same across the board?

Our results reveal a complex picture suggesting that a simple "Yes" or "No" answer cannot suffice. For the SM *at least* the answer appears to be "No", supporting the neo-Gricean account defended in Mendia (2016). On the other hand, for *at most*, the answer to the question seems to be "Yes", with the results being more consistent with Coppock & Brochhagen's (2013a) account. Let us therefore consider each quantifier in turn.

With *at least*, we observe generally that even with TOTAL ORDER, TARGET items are rated somewhere in between fully ungrammatical sentences and fully grammatical ones. One way to interpret these results is to take TARGET's penalization as a sign of pragmatic oddness. On this view, GOOD's high acceptance rates reflect a semantically and pragmatically felicitous utterance, and BAD's lower ratings are reflective of its semantic ill-formedness. Correspondingly, the lower acceptance rates of TARGET in TOTAL ORDER might reflect that, although not semantically deviant, these propositions are pragmatically odd in their context. This is a hypothesis that requires further investigation. What is critical, however, is the clear difference on TARGET between TOTAL ORDER and PARTIAL ORDER. Altogether, these results point to a negative answer to the UNIFORMITY question for the SM *at least* and support Mendia's (2016) account.

These results are not reproduced with the SM *at most*. Although BAD and GOOD follow the same trends as *at least* for both Types, neither TOTAL ORDER nor PARTIAL ORDER behaves like *at least* on TARGET. First, TOTAL ORDER items in TARGET show acceptance rates lower than BAD. Second, PARTIAL ORDER items in the TARGET condition do not show the same sharp improvement in acceptability rates as their *at least* counterparts. Thus, in general, we observe that both Types in TARGET are accepted at similar rates as BAD items. Therefore, in the case of *at most*, the results are more consistent with theories where the prejacent of an SM statement must always be amongst those alternatives that the speaker considers in her epistemic state, as defended by Coppock & Brochhagen's (2013a).

Nevertheless, we should be prudent when interpreting the results obtained for the SM *at most*. In contrast to the results obtained for *at least*, which closely match what is expected by the neo-Gricean approach, it seems difficult to extract definitive conclusions about *at most* from the results of this experiment alone due to a potential methodological issue. Notice that in the experimental paradigm there is no perfect counterpart of the *at least* stimuli for *at most* in the PARTIAL ORDER Type, the reason being that there is no *only* equivalent for *at most*. That is, a speaker uttering a sentence of the form [*at least* ϕ] could consider [*only* ϕ] as both a grammatical and plausible alternative to her utterance. This is not so for *at most*: in a run-of-the-mill *at most* sentence like *at most Bill and Al came*, the sentence *only Bill and Al came* would never figure as a plausible alternative to the former. This might be due to the presuppositional content of *only* itself, or due to the entailment properties of *at least* versus *at most*. Informally speaking, [*only* ϕ] requires there to be some higher ranked alternative that does not hold. Thus, in many theories, *only* brings in an existential commitment about such denied higher ranked alternatives. In turn, [*at most* ϕ], which is often described as contributing universal quantification over alternative propositions, does not (see discussion in Coppock & Beaver 2011). This property of *only* makes it problematic to find a context where both [*at most* ϕ] and [*only* ϕ] are felicitous, which is precisely one of the key data-points required to assess the reported theories.

5 Conclusion

This paper presented a study that addresses the question of whether or not the Ignorance Inferences that come with SMs are the same across the board. The results show that, in the case of the SM *at least*, this is clearly not the case, and the critical factor that drives this difference is the structural properties of the SMs' associate, i.e., whether it is totally or a partially ordered. This is a novel observation about the behavior of *at least* that any semantic/pragmatic theory of SMs must account for, as, for instance, is done in Mendia (2016). On the other hand, the SM *at most* does behave uniformly across the different kinds of complements tested, and so the results lend support to Coppock & Brochhagen's (2013a) suggestion that the prejacent must always constitute a possibility in the epistemic state of the speaker. The resulting state of affairs is one where two pragmatic theories make divergent predictions, and each of them can account only for the behavior of one of the SMs, but not the other. There are, in addition, a number of questions raised by these experimental findings that will have to wait for a future occasion. Below I comment on some of them.

One such question concerns the behavior of *at least* in the TARGET condition. As it was already discussed, the fact that TOTAL ORDER items in TARGET where only accepted half as often as PARTIAL ORDER was interpreted as reflecting pragmatic oddness, but not semantic infelicity. While this strikes us as a plausible working hypothesis, further confirmation is required (e.g., by investigating whether the alleged pragmatic oddity is reflected in Reading Time latencies).

A second issue for further research concerns the overall differences between the two SMs. For instance, consider the differences between *at most* and *at least* on the TARGET items. The SM *at most* shows acceptance rates similar to BAD in TARGET. Since the BAD condition was meant to be a control of semantic ill-formedness and no theory of SMs predicts the semantic ill-formedness of the TARGET items, the question arises as to what exactly leads to these unexpected low acceptance rates.

A final issue that I leave open for future consideration is the less crisp judgments adults have on *at most* sentences. The experiment showed that the acceptance rates for both BAD and GOOD items gravitated towards 50%: (*i*) semantically infelicitous sentences in the BAD conditions were rated much higher than with *at least* (10.81% versus 31.16% for *at most*), and (*ii*) semantically and pragmatically felicitous sentences in the GOOD conditions were rated lower (80.66% for *at most* versus 97.35% for *at least*). This behavior is, however, not expected by any theory of SMs, and raises the possibility that the semantics of the two SMs are not perfectly parallel.

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