Laws for biscuits: Independence and dependence in conditionals

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Abstract Pragmatic theories of biscuit conditionals (BCs) claim that BCs have a standard conditional semantics and that the defining characteristic is a contextual assumption of independence. I argue that the standard formalization of independence is insufficient. This is shown with the phenomenon of factual uses of conditionals where the truth of the antecedent is mutually accepted by discourse participants. The standard account is amended in a framework which represents dependencies between facts and ‘grounds’ the standard formalization in the independence of antecedent and consequent facts.

Keywords: biscuit conditionals, independence, pragmatics, representation of context, information store, factual dependence

1 Introduction

Biscuit conditionals (BCs) are named after the famous example (1) brought up by Austin (1956). They are also referred to as relevance or speech-act conditionals. Other paradigmatic examples are given in (2) and (3).1

(1) There are biscuits on the sideboard, if you want them.
(2) If you are hungry, there are biscuits on the sideboard.
(3) If you need anything later, my name is Jill.

These if-constructions show a most prominent feature that sets them apart from so-called hypothetical conditionals (HCs) like (4).

(4) If Jill has done her groceries, there are biscuits on the sideboard.

(4) asserts the consequent, i.e. that there are biscuits on the sideboard, is true in case Jill has done groceries. We can say that the utterance of (4) is a restricted claim.

* I am indebted to María Biezma and Sven Lauer for discussion. Many thanks to Justin Bledin, Kyle Rawlins for helpful comments and to the audience at SALT 27, especially to Will Starr and Matthew Mandelkern. All errors are mine.

1 I ignore differences in the location of the antecedent and concentrate on preposed antecedents.
In contrast, with BCs like (1) to (3) the information conveyed is that the content of the main clause, the consequent, is true regardless of the truth of the antecedent. I adopt the term consequent entailment (CE) from Rawlins (2013). CE is surprising since if-constructions are standardly taken to convey conditional information. The if-form is strongly associated with restricting a claim of the truth of the consequent to antecedent possibilities. In the case of BCs, this does not hold, since the addressee can infer the unrestricted truth of the consequent. There are at least two main questions for a theory of BCs. First, why is the form of an if-construction used in a BC when the information conveyed is just the content of the consequent? The second question to be answered is why and how CE arises in BCs. Both of these questions point towards an answer for how hypothetical conditionals and biscuit conditionals are related.

There are roughly two major paths for a theory of BCs. Semantic theories start from the question of the function of the antecedent and claim that the antecedent has the same structure and function in HC and BCs, i.e. restricting an operator. Differences are claimed to be due to scope and the nature of the restricted operator. Regarding the latter, some theorists claim that in BCs, there is a special speech act operator that the antecedent restricts. CE is explained via the scope of the antecedent which, e.g., just applies to the speech act operator whereas the content of the consequent doesn’t get restricted. Whereas semantic accounts constitute the traditional camp dominant in the literature, the recent years brought up a specific pragmatic account of BCs. This kind of theory takes the question regarding CE as a starting point and gives a decisively pragmatic answer: CE is due to a contextual assumption of independence w.r.t. antecedent and consequent. The idea goes back to Franke (2007) with predecessors in Merin 2002 a.o. Independence is given when determining the truth-value of the antecedent doesn’t settle the truth-value of the consequent and vice versa. Importantly, it is claimed that HCs and BCs do not differ in their semantics. The question regarding the function of the if-form in BCs is rarely addressed by pragmatic accounts. However, Biezma & Goebel (2017) and Starr (2014b) provide theoretical explanations with reference to discourse structure.

The aim of this paper is to re-evaluate the notion of independence used by most pragmatic theories of BCs. Following Franke (2007), independence is modeled as a structural property of information states, i.e. as orthogonality of propositions with respect to a state. I will argue that orthogonality by itself doesn’t carve out the right notion of independence at play in BC interpretations. This is shown with the phenomenon of factual uses of if-constructions, where the truth of the antecedent

2 Note that Rawlins 2013 is a discussion of unconditionals where the interpretational effect of CE is similar to the one in BCs but due to semantic, not pragmatic factors.

3 Francez (2015) argues for a similar conclusion from a different perspective. Albeit this conclusion is not adopted in his formalizations.
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is already mutually accepted by the discourse participants. The pragmatic theory characterizes BCs in terms of an independence assumption. However, on the current accounts, the notion of independence is too entangled with the issue of deriving CE. In particular, on Franke’s (2007) account, it is not clear how independence in BCs and dependence in HCs are related. If we take assumptions of independence to trigger BC-readings, we should also be able to say something about HCs, where independence is not assumed.

My suggestion is to amend Franke’s (2007) notion of independence with a representation of factual dependencies in the framework of Veltman (2005). It was claimed for independent reasons that this kind of machinery is needed for the interpretation of counterfactual conditionals (Veltman 2005; Kratzer 1989) and deontic modals like should (Arregui 2011). This apparatus gives us the tools to derive orthogonality from discourse participants’ knowledge of dependencies between facts. Consequent entailment can then be viewed as a contextual inference due to pragmatic pressure to retrieve a coherent Common Ground (CG).

In Section 2, I review Franke’s (2007) account of independence and CE. In Section 2.1, I discuss factual conditionals and show why they pose a problem for the unamended account of independence as orthogonality by pointing out its conceptual shortcomings. I introduce the relevant elements of Veltman’s (2005) framework for representing factual dependencies in Section 3 and carve out the notion of independence following from it. In Section 3.1, this is transferred to a notion of law-like independence for a CG model that is to be claimed to be crucial for a BC-interpretation of an if-construction. As a stable assumption of interpreters, it is also responsible for CE in BCs (Section 3.2).

2 The Franke-Lauer account of independence and CE

In Franke 2007, 2009 independence of propositions φ and ψ for a BC φ > ψ is understood in the sense that the truth of the consequent does not depend on the truth of the antecedent and vice versa. The basic informal idea is that two propositions are independent with respect to an information state iff learning the truth-value of one of the propositions is not enough to determine the truth-value of the other proposition. Since independence is defined relative to an information state, it is a doxastic or epistemic notion and encodes a property of an information state.

Franke formalizes epistemic independence of propositions modifying Lewis’s 1988 notion of orthogonality of subject matters. The formalization is given in (5). Indeed, van Rooij (2007) shows that the two definitions are formally equivalent. Thus, I will use Lewis’s (1988) term ‘orthogonality’ for the formal property of an information state given in Franke 2007. For Franke, orthogonality is equivalent to the notion of independence at play in BC interpretations. However, in Section 2.1,
I argue that for a sufficient explication of the notion of independence, we have to make additional assumptions.

(5) Let $\phi$ and $\psi$ be propositions, i.e. sets of possible worlds, $X$ and $Y$ variables over propositions and $\sigma$ an information state, i.e. also a set of possible worlds. Propositions $\phi$ and $\psi$ are orthogonal iff
\[ \forall X \in \{\phi, \overline{\phi}\}, \forall Y \in \{\psi, \overline{\psi}\} : \text{if } \lozenge_{\sigma} X \text{ and } \lozenge_{\sigma} Y \text{ then } \lozenge_{\sigma} (X \cap Y) \]
where $\lozenge_{\sigma} P$ is shorthand for $P \cap \sigma \neq \emptyset$, i.e. compatibility of $P$ and the information state $\sigma$.

What (5) means is that all possible conjunctions of the propositions and their respective complements have to be compatible with the information state $\sigma$. More explicitly, the definition gives four conditions on an information state to satisfy orthogonality:

i. If $\lozenge_{\sigma} \phi$ and $\lozenge_{\sigma} \psi$, then $\lozenge_{\sigma} (\phi \cap \psi)$
ii. If $\lozenge_{\sigma} \phi$ and $\lozenge_{\sigma} \overline{\psi}$, then $\lozenge_{\sigma} (\phi \cap \overline{\psi})$
iii. If $\lozenge_{\sigma} \overline{\phi}$ and $\lozenge_{\sigma} \psi$, then $\lozenge_{\sigma} (\overline{\phi} \cap \psi)$
iv. If $\lozenge_{\sigma} \overline{\phi}$ and $\lozenge_{\sigma} \overline{\psi}$, then $\lozenge_{\sigma} (\overline{\phi} \cap \overline{\psi})$

Each condition presents a material implication. (5) is non-trivially satisfied by an information state, if each, protasis and apodosis of the conditions are true. However, the definition can also be satisfied if one of the propositions or its complement are true throughout the information state ($\Box_{\sigma} P$ for $\sigma \subseteq P$). That means that if we have a $\Box$-case for at least one of the propositions or its complements, this proposition is rendered orthogonal or respectively independent of any other proposition in $\sigma$. I will term this property the $\Box$-property of (5). It plays an important role in Franke’s (2007) and Lauer’s (2015) account for deriving consequent entailment in BCs.

For modeling the process of interpretation towards consequent entailment in BCs I follow the exposition in Lauer 2015. An interpreter $I$ reasons about what the information state $\sigma_S$ of the speaker $S$ looks like. Because of this setting Lauer 2015 is a reconstruction of the metareasoning about the speaker’s state an interpreter would engage in when faced with a BC.

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4 In the following I will dispense with indexing diamonds with $\sigma$ when I take it to be obvious w.r.t. which state compatibility is evaluated.
5 By case-by-case reasoning: If $\Box \phi$ holds, (5) is satisfied, because $\lozenge_{\sigma} \overline{\phi}$ is false with respect to $\sigma$, whereby the antecedents of (iii) and (iv) are false. So, conditions (iii) and (iv) as a whole are rendered true. For conditions (i) and (ii) we have to discriminate three cases: first, $\psi$ and $\overline{\psi}$ are both compatible with $\sigma$, i.e. there are $\psi$- and $\overline{\psi}$-worlds. Additionally we have $\Box \phi$ implying $\lozenge_{\sigma} \phi$. Thereby (i) and (ii) are true (protasis and apodosis are true). Second, if $\psi$ is true throughout $\sigma$, i.e. $\Box \psi$. The antecedent of (ii) is false and condition (ii) is true. $\Box \psi$ implies $\lozenge_{\sigma} \psi$, hence (i) is true. Third, if $\psi$ is false with respect to $\sigma$, i.e. $\Box \overline{\psi}$, the antecedent of (i) is false and condition (i) is true. Whereas, $\Box \overline{\psi}$ implies $\lozenge_{\sigma} \overline{\psi}$ and condition (ii) is true.
Laws for biscuits goes through. I’s hypotheses about \( \sigma_S \) are modeled as a set of information states \( \Sigma_{\sigma_S} \) whose elements are candidates for being the actual state \( \sigma_S \) of the speaker from the perspective of I.\(^6\) If the speaker utters a sentence this is regarded as evidence about the properties of her information state (under the assumption of truthfulness). Utterances bring up further constraints on the representation of the speaker’s state \( \Sigma_{\sigma_S} \), which is updated by eliminating states that do not satisfy these constraints.

I starts the interpretation of an if-construction as a BC with the assumption that \( \sigma_S \) satisfies orthogonality (in the sense of definition (5)) with respect to \( \phi \) and \( \psi \). This includes information states that satisfy (5) because of \( \Box \phi, \Box \psi, \Box \neg \phi \) or \( \Box \neg \psi \). Under the assumption that the speaker truthfully utters the conditional \( \phi \rightarrow \psi \) the interpreter I learns about \( \sigma_S \) that it also satisfies (6a) and (6b).\(^7\)

\[
\begin{align*}
(6) \quad &a. \quad \neg \Box (\phi \land \psi) \\
&b. \quad \Diamond \psi \text{ (presupposition of the antecedent)}\quad \text{8}
\end{align*}
\]

These constraints on the information state of the speaker are encoded in the semantics of the if-construction and its presuppositions. I now has to single out the information states that satisfy all constraints, i.e. pick from the states that satisfy (5) those that also satisfy (6a) and (6b). If (6a) has to be satisfied by \( \sigma \), the apodosis of (ii) must be false. To uphold (5), the protasis of (ii) must not be true, otherwise condition (ii) as a whole would be false. \( \Diamond \phi \) also has to be true because of (6b). Hence, \( \Diamond \psi \) has to be false, i.e. \( \Box \psi \) is true. The falsity of \( \Diamond \psi \) is taken over to condition (iv) and makes the protasis false, hence (iv) is true.

This means that the only information state that satisfies orthogonality as given by (5) and the two constraints from the semantics of the conditional \( \phi \rightarrow \psi \), i.e. (6a) and (6b), is one where the consequent \( \psi \) is true throughout the whole information state \( \sigma \), i.e. \( \Box \sigma \psi \). In order to satisfy all constraints the interpreter’s model of the speaker \( \Sigma_{\sigma_S} \) will only comprise states where the consequent \( \psi \) is taken to be true. All other states get eliminated. Thereby the interaction of the assumption of independence and the semantic constraints from the if-construction give CE as a contextual inference.

2.1 Factual conditionals and the shortcomings of orthogonality

Even though the Franke-Lauer account elegantly derives CE for BCs in a pragmatic way, in this section I point out some problems and shortcomings that motivate the

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\(^6\) My exposition abstracts from some complexities of Lauer’s (2015) account. For the comprehensive picture the reader is referred to the original paper.

\(^7\) Another way of framing this without the truthfulness assumption is that I reasons about the states that S publicly promotes and thereby publicly commits to. For further discussion see Lauer 2015, 2013.

\(^8\) This presupposition holds for the indicative case, but it carries over to the counterfactual case, where the if-construction is evaluated w.r.t. a state where the antecedent is true.
modified framework that I suggest in Section 3.

One problem for Franke’s (2007) notion of independence as orthogonality is that it is geared towards deriving CE. In contrast I claim that independence should be regarded as the main characteristic of BC readings. Furthermore, I will argue that the derivation of consequent entailment via the □-property is problematic.

An empirical problem comes from factual uses of conditionals in the sense of Constant 2014.9 These are if-constructions where the antecedent is already established and mutually presupposed. In Stalnakerian terms (Stalnaker 2014) the antecedent is Common Ground, i.e. it is among the propositions mutually taken for granted for the purpose of the conversation, and true throughout the context set cs (the intersection of all the proposition which are Common Ground). I take the latter to be the information state subject to the independence assumption. (7) is an example of a factual use of an if-construction taken from Akatsuka (1986).

(7) (A mother and her son are waiting for the bus on a wintry day. The son is trembling in the cold wind.)

Son: Mommy, I’m so cold.
Mother: Poor thing! If you’re so cold, put on my shawl.

With (7) it is obvious that the mother accepts the assertion of the son and all worlds in the context set are worlds where the son is cold. The informational effect of uttering the if-construction is the same like just uttering the consequent.10 Still, there is a difference with respect to discourse structure, i.e. with the if-construction it is made explicit what is discursively relevant for what.11

For a factual use of φ > ψ it was established beforehand that □φ holds with respect to the cs. The crucial point for current purposes about these factual uses of if-constructions is that they can come as BCs (8) or as HCs (9), (7).

(8) A: I am starving!
   B: If you are so hungry, there are sandwiches in the fridge.

(9) A: I am starving!
   B: If you are so hungry, I’ll make you some sandwiches.

9 Iatridou (1991) coined the term ‘factual conditionals’ for uses that carry the presupposition that somebody other than the speaker takes the antecedent to be true. I divert from this definition and assume following Constant (2014) that there are factual conditionals where the antecedent is actually taken to be Common Ground.

10 Note that how this is derived is different from consequent entailment in BCs. In factual conditionals the alleged consequent entailment is due to the fact that the restriction to antecedent worlds is already in place, i.e. the antecedent is CG. Consequent entailment then follows by something like modus ponens-reasoning.

11 For a comprehensive account see Biezma & Goebel 2017.
Indeed, Franke’s (2007) notion of independence applies to (8) and therefore predicts the \textit{if}-construction to be a BC, since orthogonality is satisfied w.r.t. antecedent and consequent. But it also wrongly predicts (9) to be a BC, where it clearly only has a hypothetical meaning conveying a dependence between antecedent and consequent. This is due to the fact that for both, (8) and (9), orthogonality holds because of $\Box \phi$ w.r.t. the context set. Admittedly, we have something like CE for (8) and (9), because in each case the addressee learns that the consequent is true. However, this is due to \textit{modus ponens} (MP) reasoning and doesn’t arise from reasoning about (in)dependence. MP reasoning is different since it is not inferred that the consequent is true \textit{regardless} of the truth of the antecedent. The latter is only inferred in (8). In contrast, (9) gives rise to the additional counterfactual implication that if the addressee was not hungry, the speaker wouldn’t prepare sandwiches. This implication is not even possible in (8) because antecedent and consequent are independent. However, Franke’s (2007) notion of independence as mere orthogonality cannot account for this difference.

What exactly are the problems about these examples? In each of these cases independence as defined in (5) is satisfied. But biscuithood thereby is predicted for a \textit{wrong reason}, i.e. that one of the propositions of the \textit{if}-construction (or its complement) already hold throughout the information state in question, i.e. the context set. As said, independence in the sense of (5) is satisfied by all accepted propositions in the information state. But especially from the factual uses in (8) and (9) we can take that the notion of independence being relevant here is more specific, i.e. independence between the specific antecedent and consequent proposition.

If we think about the reasoning process towards CE suggested by Lauer (2015), and in particular the information state after deriving CE, we arrive at a similar problem. Intuitively, we can describe the reasoning towards CE as reasoning to uphold or maintain independence between antecedent and consequent proposition in the representation of the speaker’s information state. But with arriving at the state where the consequent is true across the board, the information about the specific independence between antecedent proposition and consequent proposition is no longer recoverable. The consequent proposition is independent of \textit{any} other proposition in the state and not just from the antecedent proposition. It has the same relation to the antecedent proposition as any other accepted proposition.

A related shortcoming of the Franke-Lauer framework is that it is not clear how BCs and HCs or independence and dependence relate. A BC interpretation springs from the assumption of independence. But how is this related to the fact that from HCs we oftentimes or normally learn about a dependence holding between antecedent and consequent? Francez (2015) formulates a most plausible intuition about the connection of knowledge of dependence and independence: Independence in a BC reading springs from the fact that a relation of dependence between the
propositions at play is excluded by mutual knowledge of the participants about causal and epistemic dependencies. The connection between independence and dependence cannot be grasped by the metareasoning account. This gets evident when thinking about how to model how interpreters learn about dependencies from conditionals in this framework. There are three possible structures for $\Sigma_{\sigma S}$:

1. If orthogonality of $\phi$ and $\psi$ for all information states in $\Sigma_{\sigma S}$ is assumed by $I$, it is maintained
2. If non-orthogonality, i.e. dependence, of $\phi$ and $\psi$ for all information states in $\Sigma_{\sigma S}$ is assumed by $I$, it is maintained, too
3. If we find orthogonal and non-orthogonal information states in $\Sigma_{\sigma S}$, there is no update procedure that establishes either dependence or independence

With respect to (10c) it is a possibility just to rule out the state where orthogonality (5) holds. But on the current account there is no obvious way to get from a standard semantics for the conditional to the envisaged update that leads to learning about a dependence. How to get to a dependence reading should be something the pragmatic story should at least be able to explain, especially if it builds on a contextual assumption as its central element.

With (10c), another shortcoming arises. If we take (5) to exclusively model independence, the only way to model an information state which is undecided w.r.t. a dependence between propositions in question is by having a set of at least two information states, i.e. one where the propositions are orthogonal (independent) and one where the propositions are non-orthogonal (dependent). However, this model only encodes uncertainty about which state is the actual information state of the speaker on part of the interpreter. The idealized aim of the interpretational model in Lauer 2015 is to single out the actual state of the speaker. But what if he is genuinely undecided on a relation of dependence between two salient propositions? Considering such a state, it would satisfy (5) since for every possible combination of truth-values of the propositions in question there would be at least one world in the state. Because of this, Francez (2015) states that (5) is merely a structural property of an information state. This structural property is ambiguous between being interpreted as independence or ignorance about a dependence. The difference is that in the case of the independence assumption at play in a BC interpretation, (5) is a stable property of the information state, whereas for ignorance the state is ready to lose this structural property if a dependence is learned. In the next section I set

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12 I am indebted to Sven Lauer for pointing me to this fact.
13 In the case of a BC reading, independence is a ‘transcontextual’ constraint following the terminology of Merin (2007), i.e. independence holds for every future context set or information state.
out to model this difference to further carve out the notion of independence.

3 Modeling factual dependencies and independencies

Veltman (2005) sets up a framework for the interpretation of antecedents of counterfactual conditionals. A central element of his account is the representation of factual dependencies to account for the different role of dependent and independent facts in counterfactual revision.\footnote{The importance of factual dependencies is also claimed in Kratzer 1989 under the term law-like generalizations. However, the idea is implemented in a different way and build into what is called ‘natural propositions’.} I will here make use of the separate representation of facts, i.e. what is the case at a possible world, and factual dependencies, i.e. how facts are connected and which ‘stand and fall together’. I follow Veltman (2005) and a modification of his framework in Arregui 2011. Together these accounts provide the tools to define independence for facts at a world via the notion of factual dependence. This definition will be put to work in a Common Ground (CG) model of interpretation to carve out the notion of law-like independence that is responsible for a BC-reading of an if-construction.

A dependence between certain facts is a relation where one fact determines another, i.e. the dependent fact. Examples at hand are that whenever it rains Jones wears a hat, or, that if you go with someone for lunch you sit where they sit, or, if you are late and run out of the house you don’t put on a coat.\footnote{The first example is due to Veltman (2005), the latter two to Arregui (2011).} Note that these dependencies have default character, i.e. we might think of them to allow for exceptions. However, this dimension will not be encoded in the model where the idealized assumption is adopted that all the worlds in the model are ‘normal’, i.e. non-exceptional. Veltman dubs factual dependencies laws, very similar to Kratzer who uses the term law-like generalization. What counts as a law is deliberately vague and intuitive in Veltman’s (2005) exposition. It is any generalization that agents under normal circumstances take for granted, e.g. natural laws, habits, conventional rules (like the rules of chess or manners) and part-whole relations. Hence, the notion of law is very flexible and allows to account for a whole variety of types of dependencies between facts.

We can conceive of Veltman’s (2005) framework as giving possible worlds a finer-grained structure that not only models what is true at a world but also how some propositions are made true by this world via dependencies between facts.\footnote{This is how Arregui (2011) adopts Veltman (2005). A sympathetic view is presented in Starr 2014a.} Following Veltman (2005) classical possible worlds formally are valuation functions from sentences of a (finite) language $\mathcal{L}$ to the truth-values $\{0, 1\}$.\footnote{In different settings one would want to claim that possible worlds determine such functions, but are}
sentences are evaluated in the standard way. In the classical picture the values of the sentences are fixed one by one for each sentence. Veltman introduces special notation to indicate that an atomic sentence $p$ is true (or false) at a world $w$: $<p, 1> \in w$ for $w(p) = 1$ (or respectively $<p, 0> \in w$ for $w(p) = 0$). A pair like $<p, 1>$ is the representation of a fact in the model. Facts can be conceived of as minimal situational constituents of worlds. A situation generally defined is a subset of a world $s \subseteq w$, i.e. a partial function from $\mathcal{L}$ to the truth values, where all the facts of $s$ are also facts of $w$. Sentences have propositions, i.e. sets of possible worlds, as their denotations. The proposition expressed by $p$ is $\llbracket p \rrbracket$.

More structure comes in when we consider dependencies between facts. Many facts are not true on their own. Rather their truth depends on other facts being true (or false). This is to say that given that something is a fact of a world, i.e. that something has happened or is in a certain way, something other is a fact of this world, too. The facts of a world are connected by a web of factual dependencies. E.g. if there is a factual dependence between the weather being bad and Jones wearing a hat and it is a fact at world $w$ that the weather is bad, the relation of dependence determines that it is also a fact at $w$ that Jones wears a hat. The basic idea of Veltman (2005) is that we can identify a specific set of facts for a world that are independent from one another and which, together with the laws holding at a world determine all other dependent facts. This set of independent facts is called the base set of a world. So, we just need the base set and the laws to derive a complete world.

In presenting this, I will follow Arregui 2011, where, unlike in Veltman (2005), the starting point is a single world and the dependencies in that world. Factual dependencies themselves ‘belong’ to a world, because the laws holding for a world could have been different. Therefore, a world $w$ is associated with its law horizon $U_w$ which is the set of all possible worlds where the same set of laws and only this set of laws holds that pertains to $w$. $w$ itself is an element of $U_w$ (assuming that $w$ is normal). With the law horizon we have an indirect representation of all the laws that hold at $w$ and the system of factual dependencies they constitute. Base sets of a world, i.e. sets of independent facts that determine a complete valuation together with the laws, are defined w.r.t. this indirect representation of all the dependencies holding at the world.

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18 The values of the propositional denotations are indicated by small greek letters: $\phi, \psi$...
19 Literally facts are not true or false, rather it would be correct but arduous to talk about positive and negative facts.
20 Note that there might be more than one base set.
21 In this paper the question of how laws/factual dependencies are represented in the formal language itself is left open, because we can represent them indirectly. In Veltman 2005 laws are modalized propositions. Inspired by so-called causal models, Starr (2014a) and Snider & Bjorndahl (2015) adopt a different approach by representing laws/dependencies as functions with specific properties.
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Table 1: Strikethrough worlds do not obey the law $p \rightarrow q$

(11) a. A situation $s$ determines a world $w$ w.r.t. $U_w$ representing a body of factual dependencies iff for all $w'' \in U_w$: $s \subseteq w''$, $w'' = w$.

b. A situation $s$ is a base set for a world $w$ iff $s$ is a minimal situation that determines $w$ w.r.t. $U_w$, i.e. there is no $s' \subset s$ such that $s'$ determines $w$.

Hence, a base set is a set of atomic facts, i.e. a situation, that minimally determines a world, i.e. gives a complete valuation over the atomic sentences of $\mathcal{L}$ relative to a body of laws/factual dependencies represented by $U_w$. If it is a fact of the base set of a world $w$ that the weather is bad and we take the dependence between weather and Jones’ wearing a hat holding at $w$, this will give us the dependent fact that Jones is wearing a hat at $w$. How the base set is determined with the definitions given is illustrated with table 1. Assume that $p$ = ‘the weather is bad’, $q$ = ‘Jones is wearing a hat’, $r$ = ‘Diana is going out with Maggie’. Furthermore the factual dependence holds that if the weather is bad Jones is wearing his hat ($p \rightarrow q$). Worlds that do not belong to $U_{w_1}$, i.e. worlds where the law $p \rightarrow q$ does not hold, are struck through.

The base set identified on the basis of the factual dependence for $w_1$ is a situation $s$ including two facts: $s = \{< p, 1 >, < r, 1 >\}$. $< q, 1 >$ is derived by the law and there is no other world $w_x$ in $U_{w_1}$ for which $s \subseteq w_x$.

With the Veltman-Arregui framework we have a representation of possible worlds that, first, differentiates between the facts that hold at a world and the dependencies between the facts that hold at a world. This is facilitated by the notion of a base set which singles out independent atomic facts of a world. These base set facts together with the laws are fundamental for the distribution of facts among a world. Second, the framework gives a method to identify these independent facts of a world w.r.t. the laws that hold at this world. Hence, with base sets Veltman’s (2005) framework comes with a notion of what an independent fact is. The facts in the base set are

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The definitions given here are slight modifications of Arregui’s (2011) definitions.
those that are not dependent on any other fact. However, the notion of independence we are seeking has to be more general: we want to be able to arbitrarily pick two facts of a world and say whether they are independent from each other or not. With the definition of a base set we can say that two facts are independent iff no factual dependence/law holds between them. But for the more general case of independence we also have to take into account that dependent facts can be independent of each other. This means that there can be sequences of dependencies that are retracted to different base facts. For example, Jones wearing a hat is traced back to bad weather, whereas Diana’s going out with Maggie depends on the dinner arrangements. Jones wearing his hat and Diana’s going out with Maggie then are independent via the dependence on independent facts. Furthermore, it may be that two or more base facts together determine one dependent fact, e.g. if we have a law like \((r \land s) \rightarrow p\). Hence, two facts are independent w.r.t. a world \(w\) and \(U_w\) iff they can be retracted to two disjunct subsets in the base sets of \(w\).

In order to state a precise definition of factual dependence, I adopt Veltman’s (2005) forcing in the version of Arregui (2011) in (12). However, this requires to move from the level of facts to the level of propositions. A proposition \([p]\) describes a situation \(s\) (which can also just be a single fact) where \(p\) is true and collects all the worlds \(w\) where \(s \subseteq w\). Factual independence of propositions w.r.t. a world \(w\) and the laws holding at \(w\) represented by \(U_w\) is then defined w.r.t. those situations that together with the laws determine whether the proposition is true or false at \(w\).

\[
\begin{align*}
\text{(12) a. } & \text{ A situation } s \text{ forces } [p] \text{ within } U_{w'} \text{ iff for every world } w' \text{ in } U_{w'} \text{ such that } s \subseteq w', w \in [p] \\
\text{b. } & \text{ A situation } s \text{ minimally forces } [p] \text{ within } U_{w'} \text{ iff } s \text{ forces } [p], s \subseteq s', \text{ where } s' \text{ is the base set of } w' \text{ and there is no } s'' \subset s \text{ that forces } [p] \\
\text{(13) Two propositions } [p] \text{ and } [q] \text{ are factually independent with respect to } w \text{ and } U_w \text{ iff } s' \text{ minimally forces } [p], s'' \text{ minimally forces } [q], \text{ and } s' \cap s'' = \emptyset.
\end{align*}
\]

In words, propositions are factually independent for \(w\) iff their values, i.e. whether they are true or false at \(w\), can be traced to disjunct situations in the base set(s) of \(w\).

3.1 Towards law-like independence in a CG model

In the preceding section I have provided Arregui’s (2011) rendition of Veltman’s (2005) framework. It enabled a finer-grained view on possible worlds that takes into account dependencies between facts. Importantly, with this account at hand, we are able to define when facts of a world are independent and when propositions are independent w.r.t. a world. Since my aim is to give an account of the independence
assumption at play in BC interpretations, the next step is to add the fine-grained view
on possible worlds to a model of the interpretation of utterances.

To this end, I adopt the Stalnakerian model of communication and its dynamics
of information exchange (Stalnaker 1978, 2002, 2014). On this account conversation
happens w.r.t. a background of information shared by the discourse participants. This
is modeled by the Common Ground (CG), which is the set of propositions mutually
taken for granted for the purpose of the conversation. The most abstract goal of an
exchange is a communal inquiry regarding what the actual world looks like, i.e. which
propositions are true at the actual world. The CG has a context set $cs$ associated to it:
it is the set that comprises all the worlds that are live options or candidates for being
the actual world, i.e. all the worlds where the shared CG propositions hold. Modeling
propositions as sets of possible worlds, the $cs$ is the result of intersecting all the
propositions in the CG. Assertions, for example, if accepted by the interpreters, add
their content to the CG. If an assertion is accepted, the $cs$ shrinks by eliminating all
worlds that are not elements of the asserted proposition. Successful communication
in this framework is driven by the requirement that discourse participants coordinate
on their individual views of what is mutually taken for granted.\footnote{Cases of presupposition failure are examples where it surfaces that participants didn’t have the same view on what is CG.}

The standard Stalnakerian account is centered on shared information about facts.
It does not differentiate between two kinds of shared information: information about
which facts hold and information about factual dependencies. We now use the
Veltman-Arregui framework to enrich the context set. Differentiating between inform-
ation about facts and information about dependencies between facts is facilitated
by differentiating possible worlds where the shared facts hold and possible worlds
where the shared dependencies hold in the same manner as we differentiated between
a world $w$ and its law horizon $U_w$. However, now we have to deal with incomplete
information about facts and equivalently with incomplete information about laws. It
is the aim of the framework to be able to represent that and how agents can learn
information about laws.

The first element of our enriched model of the context set is the standard context
set which encodes the shared assumptions about which facts hold. To mark this we
add the subscript $F$ for ‘factual’: $cs_F$. Second, we have the law sphere $L$, which
represents the shared assumptions about which laws/factual dependencies hold. It is
the set that comprises all the structured worlds that are compatible with the mutually
assumed laws. Importantly, these two sets are related by subsethood: $cs_F \subseteq L$, since
every world where the shared facts hold also has to be a world where the assumed
laws hold. With this we arrive at an information store $IS$, which is a tuple of a
factual context set and a law sphere, $< cs_F, L >$.\footnote{The term ‘information store’ is due to Isaacs & Rawlins (2008).} Note that $L$ is not the universe
of the context set $cSF$. Rather, $L$ comprises the worlds where the shared factual dependencies hold. But since this set of dependencies is incomplete, for every world in $L$ there hold more laws, i.e. more laws that help to constitute the whole world. This makes it possible for agents to learn about these additional dependencies by adding them to their stock of assumptions or eliminate the possibility of a dependence by eliminating worlds from $L$.

Let us now put the structured possible worlds from Section 3 to work in building the information store $IS$. If it is shared that it is a fact that $<p,1>$, but the value of $q$ is undecided, in every world in $cSF <p,1>$ is a fact. But we will find worlds where $<q,1>$ is a fact and worlds where $<q,0>$ is a fact in $cSF$. We can adopt the same strategy for the shared information regarding laws. If it is a shared dependence that $p \to q$, all the worlds in $L$ will follow this law. However, if it is not decided whether $r$ and $s$ depend on each other, we will find worlds with laws like $r \to s$, $s \to r$ and worlds where $r$ and $s$ are factually independent amongst $L$. Note, that it is not possible to set up the $IS$ just on the basis of factual information, i.e. it does not suffice to take all the possible worlds $w$ where the shared facts hold and attend to their respective universes $U_w$. In contrast, $L$ (the shared laws) restricts what can be accepted factually by $cSF \subseteq L$. The worlds that are in the $IS$ are, first, the worlds compatible with the laws respectively the factual dependencies taken for granted in $L$ and, second, the worlds that are compatible with the shared facts and the laws in $cSF$.

This perspective gives us the means to model the context set in a way that encodes accepted facts and dependencies and even indecision about dependencies. In the latter case we find worlds in $L$ where there is dependence between the facts in question and also worlds where these facts are independent. Learning about a dependence then amounts to ruling out worlds incompatible with this dependence. For an assumption of direct dependence of $s$ on $r$ for example, we will only find worlds in $L$ where this dependence holds. We are now able to model an assumption of law-like independence in the very same way: Two propositions (or the respective facts) are law-like independent w.r.t. a body of assumed laws/dependencies represented by $L$ iff the propositions in question are factually independent (in the sense of (13)) for every world in $L$. Or, conversely, there is no world in $L$ where any kind of dependence between the facts/propositions in question holds.

Importantly, the assumption of law-like independence is linked to orthogonality (5). If orthogonality is satisfied in a non-trivial way, i.e. in non-$\square$ cases, the information state in question has a specific structure. Assume that propositions

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25 This is the same as saying that $cSF \subseteq \Box p$.

26 “The general laws set a limit to the factual information one can have.” (Veltman 2005: 166)

27 Ruling out $\square$-cases (by definition) is much closer to the original suggestions in Lewis 1988, where he defines orthogonality w.r.t. subject matters (see Bledin & Rawlins 2016). However, he rules out...
$[p]$ and $[q]$ are orthogonal non-trivially to one another w.r.t. information state $\sigma$. This means that we will find $p \land q$-worlds, $p \land \neg q$-worlds, $\neg p \land q$-worlds and $\neg p \land \neg q$-worlds distributed over $\sigma$. An assumption of law-like independence induces the very same structure on $L$, where we have all possible combinations of truth-values for the propositions in question represented by the worlds in $L$. This is because in all the worlds the facts in question are independent and thereby all combinations have to be present in $L$. As noted in 2.1, orthogonality as a structural property of $L$ can also encode indecision w.r.t. a factual dependence. This is due to the fact that orthogonality is not sensitive to the factual dependencies that hold at a world but only to the mere facts of worlds in an information state. If we look at the finer structure of worlds with the factual dependencies holding for them, we can differentiate between the different grounds of orthogonality. If there is an assumption of law-like independence established in the $IS$ this is a stable property of $L$. And so is orthogonality. Ruling out worlds from $L$ will maintain orthogonality since there are only worlds where independence holds. In contrast, orthogonality from indecision w.r.t. a dependence is not a stable property of the $IS$. Since we have both, worlds where the facts in question stand in a dependence relation and worlds where they are independent, future $IS$ after updating can be ones where it is decided on dependence or independence, and orthogonality of the propositions in question does not hold any more. Establishing dependence or independence then just becomes a question of eliminating worlds with certain structures.\textsuperscript{28}

3.2 Deriving CE from IS coherence

To explain consequent entailment (CE) within the factual dependencies framework, I adopt a dynamic semantics for if-constructions following Rawlins (2010) (building on Heim 1983). The update potential of an if-construction $\phi > \psi$ acts on an $IS$.\textsuperscript{29} I assume the restriction that an if-construction in the indicative mood targets $cs_F$. Informally, the antecedent $\phi$ restricts the context set to the worlds in $cs_F$ where $\phi$ is true. This restricted set gets updated with the consequent $\psi$. If it is a declarative, all the worlds where $\psi$ does not hold get eliminated from the restricted context set. After this update, the restriction of the $cs_F$ to the $\phi$-worlds is lifted. From the perspective of the unrestricted $cs_F$, all worlds where $\phi$ is true but $\psi$ is not, get eliminated.

Simplifying Rawlins’s (2010) framework, the information store $IS$ gets extended

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\textsuperscript{28} Starr (2014a) presents a similar account of dependencies which retains the notion of informativity of possible worlds accounts.

\textsuperscript{29} We will from here on directly talk about the propositional denotations of $[p]$ and $[q]$: $\phi$ is the antecedent proposition, $\psi$ is the consequent proposition.
with a slot $a$, the view on the context set, that tracks restrictions on $csF$. So, the IS now is a triple: $<a, csF, L>$. The ASSUME operator introduces restrictions on the view, the POP operator eliminates all restrictions.

(a) $IS + \text{ASSUME } \phi = <a \cap \phi, csF, L>$
(b) $IS + \text{POP } \phi = <\emptyset, csF, L>$

Updates under a (possibly empty ($a = \emptyset$)) restriction are modeled by domain-limited update (cf. Kaufmann 2000):

$$x \vdash_a \phi = (x \cap a) \cup (x \cap a \cap \phi)$$

Assertive updates (ASSERT) then are defined in the following way:

(16) $IS + \text{ASSERT } \phi = <a, csF \vdash_a \phi, L>$

With this, the definition of the semantic update potential of an (indicative) if-construction comes down to the following:

(17) $IS + \text{"if } \phi, \psi \text{"} = IS + \text{ASSUME } \phi + \text{ASSERT } \psi + \text{POP } \phi = <a \cap \phi, csF \vdash_a \psi, L> + \text{POP } \phi$

Consequent entailment only arises if the $csF$ has a certain default structure before the utterance of an if-construction. This structure is given if neither the antecedent- nor the consequent-proposition or their complements are established w.r.t. $csF$. Hence, there will be at least one world for all truth-value distributions over the conjunction of $\phi$ and $\psi$. This default context structure satisfies (5) in the non-trivial sense. If we also take $L$ to satisfy law-like independence, the structure of the information store $IS$ can be depicted like in fig. (1a). Applying the update potential (17) leads to the exclusion of $\emptyset \cap \Psi$-worlds from $csF$. The resulting structure is represented in fig. (1b). Domain-limited update by the if-construction induces non-orthogonality on $csF$, i.e. $csF$ does not satisfy (5) any more. But non-orthogonality between propositions also means dependence, or at least non-independence. Remember that the informal idea about independence was that in learning the truth-value of one of the propositions, nothing about the other proposition is learned and vice versa. With the structure of $csF$ in fig. (1b), if it is learned that the antecedent is true, it is also immediately learned that the consequent is true. Thereby, the two propositions are dependent with respect to $csF$.

In brief, we have non-orthogonality w.r.t. $csF$ but orthogonality induced by law-like independence w.r.t. $L$. In particular, law-like independence dictates orthogonality

30 Where $\emptyset$ is the underlying domain of possible worlds.
31 The reader is referred to Rawlins 2010 for a comprehensive explanation of these definitions.
as defined in (5) for $L$ since it is determined by factual independence holding for all the worlds in $L$. Since $L$ restricts what can be known factually, $cs_F$ has to be compatible with the assumptions in $L$. But then, the structure of the information store depicted in (1b) is **incoherent**. There are two possible ways to react if an interpreter $I$ arrives at an incoherent perspective on the shared information store: (i) $I$ might come to the conclusion that her assumption that law-like independence is a feature of the $IS$, i.e. that it is a **shared** assumption, is wrong. The speaker doesn’t assume independence. So $I$ would have to revise her take on the $IS$. (ii) $I$ searches for a minimal modification to the update proposed by the speaker which satisfies simultaneously the constraints by the semantics of the $if$-construction and law-like independence. Furthermore, $s/he$ takes this minimal modification to be intended by the speaker.

(i) is excluded for normal contexts and the paradigmatic examples (1) to (3), since law-like independence between the propositions in question is assumed to be fundamental and easily accessible so that it can easily be taken to be shared. If $I$ takes law-like independence to be shared, easily accessible to all discourse participants and well entrenched with respect to the system of laws they share, (i) is excluded. Giving up assumptions about factual dependencies and independencies is not done without good reasons, which $I$ lacks in paradigmatic biscuit scenarios. It is a general rule that "we are not prepared to give up propositions that we consider to be general laws" (Veltman 2005: 166), where we might want to add an ‘easily’.

Hence, if law-like independence is sufficiently entrenched in this sense, $\phi > \psi$ receives a BC reading. If the prior context is additionally a default context where neither the truth nor the falsity of antecedent or consequent are established, consequent entailment will arise as a consequence of (ii). $I$ takes the speaker to likewise presuppose law-like independence and to propose to update $cs_F$ such that all antecedent worlds are consequent worlds. All constraints on $cs_F$ from (6a), (6b) and from $L$ have to be satisfied by an update. $\phi$-worlds cannot be ruled out because of the presupposition that the antecedent has to be a possibility with respect to $cs_F$. The only minimal modification to the update of $cs_F$ that satisfies the constraints of the semantics of the $if$-construction and aligns with orthogonality by law-like independence in $L$ is the exclusion of all $\overline{\psi}$-worlds from $cs_F$ (depicted in fig. (1c)). With this, $\psi$ is true throughout the whole context set $cs_F$ and we have derived consequent entailment from a requirement of **law-like coherence** on the information store $IS$.\textsuperscript{32}

For both varieties of factual conditionals, BCs and HCs, the update to $cs_F$ will be the same, i.e. excluding the antecedent and non-consequent worlds. However, the difference lies within $L$: if there is an assumption of law-like independence, the

\textsuperscript{32} The formalization of the condition of law-like coherence is left for another occasion.
antecedent and non-consequent worlds in $L$ will be retained (depicted in fig. (1d)). If, however, $L$ is compatible with a dependence between antecedent and consequent, an HC reading will arise by also eliminating antecedent and non-consequent worlds from $L$ (depicted in fig. (1e)). The counterfactual implication of (9), that if the addressee was not hungry, the speaker wouldn’t make biscuits, is a consequence of the fact that independence is not presupposed in this case. How exactly dependence readings are to be analyzed in the given framework is an issue for further research.\footnote{Snider & Bjorndahl (2015) provide a framework where counterfactual conditionals put constraints on explanations in terms of factual dependencies. This may be generalized to indicative HCs as well. An interesting question arising from this is how indicative and counterfactual conditionals relate w.r.t. BC and HC readings.}

4 Conclusion

The Franke-Lauer account of the role of independence in the interpretation of if-constructions as BCs cannot deal with factual uses of conditionals. Mere orthogonality is not sufficient to carve out the notion of independence since we have to take into account the level of dependencies between facts. I have argued for adopting the model of Veltman (2005) and Arregui (2011) to give possible worlds more structure. This enables to formulate a notion of independence of facts. Furthermore, if certain facts are taken to be independent, the corresponding propositions are orthogonal with respect to an information state, but not \textit{vice versa}. Consequent entailment (CE) for a BC is a contextual inference from pragmatic pressure to maintain a coherent
information store (IS). This aspect arises from an interplay of a standard restrictional semantics of the if-construction and specific shared world knowledge about dependencies between facts.

References


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