

## Contrastive negation and the theory of alternatives\*

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**Abstract** This paper is a contribution to the theory of focus; in particular, it is concerned with what counts as a valid focus alternative. Empirical data from contrastive negation associating with presupposition triggers is presented, and it is observed that association with the definite article and *both* lead to an existence inference. It is shown that the existence inference is unexplained by the current theory of focus, but that it can be explained by placing constraints on focus alternatives. The constraints I propose are that focus alternatives are limited to the meanings of linguistic expressions that satisfy one of two novel notions of equivalence: Strawson-equivalence or P-equivalence. Strawson-equivalence is the bidirectional variant of von Fintel’s (1999) Strawson-entailment, and P-entailment holds between two expressions which have equivalent presuppositions.

**Keywords:** negation, presupposition, focus, alternatives, quantifiers, Strawson-entailment, Strawson-equivalence, P-equivalence

### 1 Introduction

This paper presents novel data on contrastive negation to motivate a more constrained theory of focus alternatives. The main empirical contribution is this: it is observed that when the definite article associates with negation, its uniqueness presupposition can be denied, but its existence presupposition cannot. Similarly, when *both* associates with negation, an analogous existential inference is perceived. This behavior can be explained if focus alternatives are limited to the meanings of lexical expressions that satisfy one of two (somewhat) novel semantic conditions: Strawson-equivalence or P-equivalence. Strawson-equivalence (inspired by von Fintel’s (1999) Strawson-entailment) holds between two semantic objects that recursively Strawson-entail each other. P-equivalence holds between two semantic objects which recursively have equivalent presuppositions. By using these two notions to

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characterize focus alternatives, one can explain the behavior of the definite article and *both* under association with negation.

This paper is organized as follows. Our crucial observations regarding association with the definite article are reported in §2, which includes observations regarding other presupposition triggers. §3 presents the basic theoretical treatment of contrastive negation that is assumed here. §4 further develops the base theory to handle association with presupposition triggers in general, and §5 shows that this theory makes wrong predictions when applied to the definite article. §6 develops the proposal. §6.1 defines Strawson-equivalence and §6.2 shows how Strawson-alternatives can be fruitfully applied to association with the definite article. §6.3 presents observations on *both* and finally §6.4 defines P-equivalence and shows how P-alternatives can be applied to association with *both*.

## 2 Observations

Contrastive negation (or association with negation) describes a negated sentence containing a focus-marked constituent. In terms of its meaning, a contrastively negated sentence contrasts the negated sentence with some other sentence formed by replacing the focus marked item with something else. A simple example, taken from Beaver & Clark 2009, is shown in (1).<sup>1</sup> Compare (1) with (2).

- (1) a. Kim doesn't study LINGUISTICS at Northwestern.  
 b. Kim doesn't study linguistics at NORTHWESTERN.
- (2) Kim doesn't study linguistics at Northwestern.

Focus marking affects the sentence's felicity conditions. (2) is true and felicitous in any world where it is not the case that Kim studies linguistics in Northwestern. But (1a) is only true and felicitous in a world where both (2) holds and Kim studies something else at Northwestern.

Contrastive negation also affects the felicity conditions of presupposed content. When a presupposition trigger associates with negation, as in (3), it is felicitous to deny the presupposed content.

- (3) a. Jimi isn't bringing BOTH his guitars, he has three guitars.  
 b. Chris didn't KNOW that Hillary was going to win the election; in fact, she didn't.

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<sup>1</sup> The examples in (1), as well as all other examples in this paper, should be read with the *rise fall rise* (RFR) contour (Pierrehumbert & Steele 1989), also known as the B-accent (Jackendoff 1972). According to Beaver & Clark 2009, this is the intonation that is most commonly associated with contrastive negation. Other intonations may have other interpretations, and are not discussed here; see Beaver & Clark 2009 for an overview of the different intonations and their interpretations.

c. Emilia isn't coming to Montréal AGAIN, this is the first time she's coming.

The continuations in each of these three sentences make it clear that the presupposition triggered by the focus marked item is false, where those presuppositions are listed informally in (4).

- (4) a.  $\llbracket \text{both} \rrbracket(P)(Q)$  presupposes  $|P| = 2$ .  
b.  $\llbracket \text{know} \rrbracket(p)(x)$  presupposes  $p$  is true.  
c.  $\llbracket \text{again} \rrbracket(p)$  presupposes  $p$  occurred in the past.

Now, consider a case where definite article associates with negation. In the classic analysis (Frege 1892; Strawson 1950), the definite article presupposes existence and uniqueness. One way to state this is in (5).<sup>2</sup>

- (5)  $\llbracket \text{the} \rrbracket(P)(Q)$  presupposes  
a. *Existence*.  $P \neq \emptyset$ , and  
b. *Uniqueness*.  $|P| \leq 1$ .

Together this amounts to the presupposition that  $|P| = 1$ .

(6-8) shows that when the definite article associates with negation, the uniqueness presupposition can be denied, but not the existence presupposition.

- (6) a. Jake isn't THE student who failed the midterm, many students failed.  
b. # Jake isn't THE student who failed the midterm, no one failed.
- (7) a. Sam didn't invite THE syntax professor at McGill, he invited A syntax professor.  
b. # Sam didn't invite THE syntax professor at McGill, he didn't invite any syntax professors.
- (8) a. Sam didn't invite THE syntax professor at McGill, there are three syntax professors at McGill.  
b. # Sam didn't invite THE syntax professor at McGill, there no syntax professors at McGill.

In (6), the relevant presupposed content is that there exist students (existence) and that there is at most one student (uniqueness). (6a) shows that if many students failed the exam, then contrastively negating the definite is felicitous; this is the situation in which the uniqueness presupposition is false. (6b) shows that if no one failed the

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<sup>2</sup> Here, the definite article is analyzed as a quantificational determiner (Barwise & Cooper 1981) instead of a function from predicates to entities in order to capture a similarity between the definite article and *both*, which is presented in §6.3.

exam, then contrastively negating the definite is not felicitous; this is the situation in which the existence presupposition is false. (7-8) show the same phenomenon in argument position. (7) shows that contrastively negating the definite is compatible with a continuation that replaces the definite with the indefinite, but not compatible with a continuation that replaces it with *not . . . any*. Furthermore, uttering (7a) gives rise to the inference that Sam invited multiple syntax professors, and hence that there are multiple syntax professors, denying the uniqueness presupposition. (8) shows the same facts as (7) with continuations that directly deny either the existence or uniqueness presupposition.

The contrasts in (6-8) present a puzzle for the semantics of contrastive negation. If contrastive negation can deny presuppositions, how do we explain the contrast between the uniqueness-denying examples (the (a) examples) and the infelicitous existence-denying examples (the (b) examples)? In more general terms, why does association with the definite article always lead to an existence inference?

- (9) **Puzzle.** Negation associating with the definite article invariably leads to an existence inference, in which the existence of the restrictor of  $\llbracket \text{the} \rrbracket$  is inferred.

The remainder of this paper is dedicated to solving this puzzle.

### 3 Theoretical background

Let us adopt the focus-based analysis of contrastive negation in Beaver & Clark 2009 (see also Jackendoff 1972) using the notation for focus contributed by Rooth 1992 (I will refer to this as the Beaver-Clark-Rooth theory of contrastive negation). Rooth 1992 defines the focus alternative set ( $\llbracket \cdot \rrbracket^f$ ) and the focus operator ( $\sim$ ) which is defined in terms of focus alternative sets. To differentiate the focus alternative set  $\llbracket S \rrbracket^f$  of an utterance  $S$  from the ordinary meaning of  $S$ , Rooth uses  $\llbracket S \rrbracket^o$  for the ordinary meaning of  $S$ . Definitions of these two notions are given in (10).

- (10) a. *Focus alternative set.*  $\llbracket S \rrbracket^f$  is the set of all semantic objects  $\phi$  such that  $\phi$  is obtained from  $\llbracket S \rrbracket^o$  by replacing the interpretation of every focus marked item  $X_F$  in  $S$  with any semantic object of the same type as  $\llbracket X_F \rrbracket^o$ .
- b. *Focus operator.*  $\llbracket [\sim S] \rrbracket^o$  asserts  $\llbracket S \rrbracket^o$  and presupposes  $\exists \psi \in \llbracket S \rrbracket^f [\psi \neq \llbracket S \rrbracket^o \wedge \psi]$ .

The focus operator is a purely presuppositional operator: it contributes nothing to the asserted content, but contributes the *focus presupposition* which states that there is some true alternative in the focus alternative set of its argument.<sup>3</sup>

<sup>3</sup> Note that our formulation of the focus operator includes the disputed *existential presupposition*: the

A contrastively negated sentence has an LF where  $\sim$  scopes below negation.<sup>4</sup> For example, (1a) has the LF (11a). Its focus alternative set is (11b). Given the definition of the focus operator, its interpretation is (11c). In (11c), the asserted content is just the ordinary interpretation of (11a), and the presupposed content is the focus presupposition, which states that there is some true alternative in the focus set. In this example, the focus presupposition is equivalent to stating that there is some  $x$  other than linguistics that Kim studies at Northwestern; this is what is written in (11c-ii)

- (11) a. [not [ $\sim$  [Kim studies [linguistics]<sub>F</sub> at Northwestern]]]  
 b. {**study-at**( $k, x, \text{northwestern}$ ):  $x \in D_e$ }  
 c. [(11a)]<sup>o</sup> =  
 i. *Assertion.*  $\neg$ **study-at**( $k, \text{ling}, \text{northwestern}$ )  
 ii. *Presupposition.*  $\exists x \in D_e$ [**study-at**( $k, x, \text{northwestern}$ )  $\wedge x \neq \text{ling}$ ]

#### 4 Association with presupposition triggers

We saw in §2 that contrastive negation can deny presuppositions. The analysis of contrastive negation laid out in §3 cannot capture this property. If  $\sim$  scopes over a sentence which has a presupposition, that presupposition will project through  $\sim$  and proceed to project through negation, since negation is a hole (Karttunen 1973).

To allow contrastively negated sentences to deny presuppositions, Beaver & Krahmer's (2001) *assertion* operator is used.<sup>5</sup> The assertion operator is defined by the truth table in (12).<sup>6</sup>

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content which states that the alternative  $\psi$  is true. It is disputed whether the existential presupposition should be part of the semantics of focus (see e.g. Rooth 1999 for arguments against the existential presupposition). However, it is clear that contrastively negated sentences have the inference that there is a true alternative. The existential presupposition is included as an easy way to capture this inference, though the content of this paper should be compatible with other ways of capturing this inference.

<sup>4</sup> This is the preferred scope ordering given in Beaver & Clark 2009 for association with negation under the RFR intonation. Other intonations prefer the opposite scope ordering, but those are outside the scope of this paper.

<sup>5</sup> The notion of an operator with the truth conditions of the assertion operator goes back to Bochvar's (1938) *truth* operator.

<sup>6</sup> Following the formalization in Beaver & Krahmer 2001, a trivalent logic is assumed; the set of truth values is  $\{0, 1, \#\}$ , where presupposition failure is treated as the # truth value. Negation is a hole for presuppositions (Karttunen 1973), so it has the truth table in (i).

(i) *Negation in a trivalent logic.*

(12) *Assertion operator.*

	A
1	1
0	0
#	0

When  $A$  scopes below negation, it can be used as a “presupposition wipe-out device”. The general effect of  $A$  is that it shifts presupposed content into asserted content, so that if  $p_q$  presupposes  $q$  and asserts  $p$ , then  $A(p_q) \equiv (p \wedge q)$ .<sup>7</sup> Then, if negation scopes over  $A$ , the presupposed content can be denied. In other words, we have that  $\neg A(p_q) \equiv \neg(p \wedge q)$ , so  $\neg A(p_q)$  can be made true if  $q$  is false.

An example is shown in (13), in which the sentence *Jacques didn’t invite the king of France* is interpreted with the LF (13a).<sup>8</sup> In the actual world, *Jacques invited the king of France* is a presupposition failure, since France is a republic. But  $A$  converts # into the truth value 0, which negation flips to 1, so the whole proposition is true. In other words,  $A(p)$  is true if  $p = 1$ , and false if  $p \neq 1$ . In our example,  $p = \llbracket \text{Jacques invited the king of France} \rrbracket^o = \#$ , so  $p \neq 1$ ; hence,  $\neg A(p)$  is true. This is what is stated in (13b).

- (13) a. [not [A [Jacques invited the king of France]]]  
 b.  $\neg(A(\llbracket \text{Jacques invited the king of France} \rrbracket^o))$   
 $\equiv \neg(\llbracket \text{Jacques invited the king of France} \rrbracket^o = 1)$   
 $\equiv \llbracket \text{Jacques invited the king of France} \rrbracket^o \neq 1$

The  $A$ -operator can be combined with the focus operator to capture the behavior of negation associating with presupposition triggers. This is illustrated with example (3b), repeated in (14).

- (14) Chris didn’t KNOW that Hillary was going to win the election (, because she didn’t).

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	¬
1	0
0	1
#	#

<sup>7</sup> Technically, in the logic defined by Beaver & Krahmer 2001,  $A(p_q) \equiv Ap \wedge Aq$ . This is due to the fact that their formalism allows  $p$  and  $q$  to have presuppositions themselves. In the informal presentation here, assume that  $p$  and  $q$  are not presuppositional themselves, in which case the equivalence in the body of this paper holds.

<sup>8</sup> In the interest of a cleaner presentation, the symbol  $A$  is used in both our meta-language and object language (i.e., in both the LF and its meaning).

(14) should be true in a world where Hillary didn't win the election. Furthermore, with its continuation (14) intuitively has the implication that Chris merely believed that Hillary was going to win. This reading can be generated by the LF in (15a), where  $A$  scopes below both negation and the focus operator. Its focus alternative set is (15b) and its interpretation is (15c).<sup>9</sup>

- (15) a. [not [ $\sim$  [ $A$  [Chris [knew]<sub>F</sub> that Hillary was going to win]]]]  
 b.  $\{\mathcal{P}(\lambda w'.\mathbf{win}_{w'}(\mathbf{h}))(\mathbf{c}): \mathcal{P} \in D_{\langle\langle e, \langle s, t \rangle \rangle, \langle s, t \rangle \rangle}\}$   
 c. i. *Assertion.*  $\llbracket \text{know} \rrbracket^o(\lambda w'.\mathbf{win}_{w'}(\mathbf{h}))(\mathbf{c})(w_0) \neq 1$   
 ii. *Presupposition.*  $\exists \mathcal{P}[\mathcal{P}(\lambda w'.\mathbf{win}_{w'}(\mathbf{h}))(\mathbf{c})(w_0) \wedge \mathcal{P} \neq \llbracket \text{know} \rrbracket^o]$

If Hillary didn't win, then the assertion in (15c) is true, thanks to the contribution of the  $A$  operator. Furthermore, the focus presupposition states that there is some alternative propositional functor  $\mathcal{P}$  that relates Chris and the proposition that Hillary was going to win. Clearly, in this context, the relevant alternative propositional functor is  $\llbracket \text{believe} \rrbracket^o$ , which lacks the factive presupposition of  $\llbracket \text{know} \rrbracket^o$ .

## 5 Association with the definite article

This section revisits the crucial case of negation associating with the definite article. The analysis developed in the last section is applied to this case, and it is shown that the analysis makes the incorrect prediction; it does not predict any of the contrasts observed in §2.

Let us take (7) as our running example, repeated in (16).

- (16) Sam didn't invite THE syntax professor,  
 a. ... he invited A syntax professor.  
 b. # ... he didn't invite any syntax professors.

Assume that the definite article has an entry like in (17).<sup>10</sup> If (16) has the LF in (18a), its interpretation is (18b).

<sup>9</sup> Note that in (15b), the focus alternatives are each within the scope of  $A$ . The presence of  $A$  makes no difference here, since  $A$  does not embed negation, nor any other holes or filters for presuppositions. If  $p$  contains no holes or filters, then  $Ap$  has the same truth conditions as  $p$ , in the sense that  $Ap = 1$  iff  $p = 1$ .

Note also that in (15c) the world parameter is included since it is relevant for the meaning of *know*. In the rest of this paper, world parameters are generally not included so as to not clutter the notation.

<sup>10</sup> Set-notation is used as a shorthand for formulas in predicate logic: (i)  $P \neq \emptyset$  is short for  $\exists x[P(x)]$ , (ii)  $|P| \leq 1$  is short for  $\forall x[P(x) \rightarrow \forall y[P(y) \rightarrow x = y]]$ , (iii)  $P \subseteq Q$  is short for  $\forall x[P(x) \rightarrow Q(x)]$ .

$$(17) \quad \llbracket \text{the} \rrbracket^o(P)(Q) := \begin{cases} P \subseteq Q & \text{if } P \neq \emptyset \wedge |P| \leq 1 \\ \# & \text{otherwise} \end{cases}$$

- (18) a. [not [~ [A [Sam invited [the]<sub>F</sub> syntax professor]]]]  
 b. i. *Assertion*.  $\llbracket \text{the} \rrbracket^o(\lambda x. \mathbf{prof}(x))(\lambda x. \mathbf{invite}(s, x)) \neq 1$   
 ii. *Presupposition*.  $\exists \delta [\delta(\lambda x. \mathbf{prof}(x))(\lambda x. \mathbf{invite}(s, x)) \wedge \delta \neq \llbracket \text{the} \rrbracket^o]$

(18b) asserts that the ordinary interpretation of *Sam invited the syntax professor* is not true, so it is either false or a presupposition failure. (18b) presupposes that there is some other determiner meaning that can replace  $\llbracket \text{the} \rrbracket^o$  that leads to a true proposition.

One determiner meaning that could replace  $\llbracket \text{the} \rrbracket^o$  is  $\llbracket \text{a} \rrbracket^o$ , giving the alternative  $\llbracket \text{Sam invited a syntax professor} \rrbracket^o$ . This correctly predicts that (16a) is felicitous, since the continuation in (16a) is identical to an existing focus alternative. This also predicts that (8a) is felicitous, since the continuation is compatible with an existing focus alternative. Another determiner meaning that could replace  $\llbracket \text{the} \rrbracket^o$  is  $\llbracket \text{no} \rrbracket^o$ , giving the alternative  $\llbracket \text{Sam invited no syntax professor} \rrbracket^o$ . Here, we run into a problem: this incorrectly predicts that (16b) is felicitous, since this focus alternative is compatible with the continuation in (16b). This also incorrectly predicts that (8b) is felicitous, for the same reason.

This problem is the one we set out to solve. The base theory does not give the right predictions for association with the definite article because the definite's focus alternative set contains items like  $\llbracket \text{no} \rrbracket^o$  which lead to focus alternatives that correspond with infelicitous interpretations.

In more general terms, here is a summary of the analysis thus far. We observed that when contrastive negation associates with a presupposition trigger, the presupposition can be denied. This was modelled by combining the Beaver-Clark-Rooth theory with Beaver & Krahmer's assertion operator. We crucially observed that when contrastive negation associates with the definite article, the definite presupposition can still be denied, but an existence inference remains. To capture this, we seek an analysis which generates an existence inference in those cases. The Beaver-Clark-Rooth theory fails at this task, because there are alternatives in the focus alternative set associated with the definite article which do not entail existence of their restrictor (like  $\llbracket \text{no} \rrbracket^o$ ).

Our solution, presented in §6, is to constrain the set of focus alternatives, so that alternatives like  $\llbracket \text{no} \rrbracket^o$  are ruled out, but alternatives like  $\llbracket \text{a} \rrbracket^o$  remain. §6 shows how to define a few general constraints which are capable of deriving a suitable focus alternative set for the definite article. These constraints are formulated in terms of two notions – Strawson-equivalence and P-equivalence – to be defined below. Then, it is shown how the proposed theory extends to other examples of contrastive negation.

## 6 Constraints on alternatives

### 6.1 Strawson-equivalence

A constrained theory of alternatives must at the very least exclude  $\llbracket \text{no} \rrbracket^o$  from  $\llbracket \text{the} \rrbracket^f$ . On the other hand, it would be safe to include  $\llbracket \text{a} \rrbracket^o$  in  $\llbracket \text{the} \rrbracket^f$ . Notice that  $\llbracket \text{the} \rrbracket^o$  and  $\llbracket \text{a} \rrbracket^o$  share a curious property that  $\llbracket \text{the} \rrbracket^o$  and  $\llbracket \text{no} \rrbracket^o$  do not. If the presupposition triggered by  $\llbracket \text{the} \rrbracket^o$  is taken for granted, then any sentence containing  $\llbracket \text{the} \rrbracket^o$  is semantically equivalent to that same sentence containing  $\llbracket \text{a} \rrbracket^o$ . For example, if there is a unique queen of England, then  $\llbracket \text{Nigel invited the queen of England} \rrbracket^o$  entails that  $\llbracket \text{Nigel invited a queen of England} \rrbracket^o$ ; furthermore, since there is at most one queen of England,  $\llbracket \text{Nigel invited a queen of England} \rrbracket^o$  entails that  $\llbracket \text{Nigel invited the queen of England} \rrbracket^o$ . This property does not hold between  $\llbracket \text{the} \rrbracket^o$  and  $\llbracket \text{no} \rrbracket^o$ , since these two entries are actually contradictory. For example,  $\llbracket \text{Nigel invited the queen of England} \rrbracket^o$  contradicts  $\llbracket \text{Nigel invited no queen of England} \rrbracket^o$ .

This property is called Strawson-equivalence. To define it, first define the notion of Strawson-entailment, originally contributed by von Fintel 1999, given in (19) (where  $\rightarrow$  is material implication).

(19) *Strawson-entailment.*

Let  $p, q$  be expressions of type  $\langle s, t \rangle$ . Then  $p \xrightarrow{S} q$  iff for every  $w \in D_s$  such that  $q(w) \neq \#$ ,  $p(w) \rightarrow q(w)$ .

It will be useful to talk about Strawson-entailment holding between lexical items, not just propositions. To that end, (21) extends the notion of Strawson-entailment to all types which end in  $\langle s, t \rangle$ .<sup>11</sup>

(20) For any type  $\tau$ , define the notion *ends in  $\tau$*  recursively:

- a.  $\tau$  ends in  $\tau$ .
- b.  $\langle \sigma, \sigma' \rangle$  ends in  $\tau$  if  $\sigma'$  ends in  $\tau$ .

(21) *Cross-categorical Strawson-entailment.*

Let  $f, g$  be expressions of type  $\langle \sigma, \tau \rangle$  where  $\tau$  ends in  $\langle s, t \rangle$ . Then  $f \xrightarrow{S} g$  iff for every  $x \in D_\sigma$  such that  $g(x) \neq \#$ ,  $f(x) \xrightarrow{S} g(x)$ .

Finally, define Strawson-equivalence as bidirectional Strawson-entailment.

(22) *Strawson-equivalence.* Let  $f, g$  be expressions of type  $\tau$  which ends in  $\langle s, t \rangle$ . Then  $f \xleftrightarrow{S} g$  iff  $f \xrightarrow{S} g$  and  $g \xrightarrow{S} f$ .

<sup>11</sup> Von Fintel does not define Strawson-entailment recursively in this way, so there are some cases in which the two definitions diverge. The notion of Strawson-entailment defined here resembles the notion of  $\xrightarrow{ST}$ -entailment defined in Sharvit 2017, though is not exactly the same.

It can be easily proven that  $\llbracket \text{the} \rrbracket^o$  is Strawson-equivalent to  $\llbracket \text{a} \rrbracket^o$ , but that  $\llbracket \text{the} \rrbracket^o$  is not Strawson-equivalent to  $\llbracket \text{no} \rrbracket^o$ , assuming the lexical entries in (23a,23b).

- (23) a.  $\llbracket \text{a} \rrbracket^o(P)(Q) := P \cap Q \neq \emptyset$   
 b.  $\llbracket \text{no} \rrbracket^o(P)(Q) := P \cap Q = \emptyset$

(24) **Proposition.**

- a.  $\llbracket \text{the} \rrbracket^o \stackrel{S}{\Leftrightarrow} \llbracket \text{a} \rrbracket^o$   
 b.  $\llbracket \text{the} \rrbracket^o \not\stackrel{S}{\Leftrightarrow} \llbracket \text{no} \rrbracket^o$

## 6.2 Strawson-alternatives

Define the Strawson-alternatives of  $\alpha$  as the items Strawson-equivalent to  $\alpha$ .

- (25) *Strawson-alternatives.*  $A_S(\alpha) := \{\gamma: \alpha \stackrel{S}{\Leftrightarrow} \gamma\}$

As a first step, let us hypothesize that focus alternatives are Strawson-alternatives.

- (26) **Hypothesis 1.** For any F-marked constituent  $[X]_F$ ,  $\llbracket X \rrbracket^f := A_S(\llbracket X \rrbracket^o)$ .

Given proposition (24),  $\llbracket \text{no} \rrbracket^o$  is no longer a member of  $\llbracket \text{the} \rrbracket^f$ , but  $\llbracket \text{a} \rrbracket^o$  is. Assuming that the focus alternative set for a whole proposition is constructed recursively from the alternative sets of lexical items via piece-wise composition (as in Rooth 1996), the focus alternatives for any sentence with an F-marked *the* cannot be constructed by replacing *the* with *no*, as desired.

In general,  $\llbracket \text{the} \rrbracket^f$  should include only those semantic objects which guarantee existence. More precisely,  $\llbracket \text{the} \rrbracket^f$  should only include expressions  $\delta$  such that  $\delta(P)(Q)$  guarantees  $P \neq \emptyset$ . At this point, our hypothesized focus alternative set is still not constrained enough to guarantee this; it still admits alternatives which do not guarantee existence in this way.

**Problem: Arbitrary non-linguistic alternatives.** It is easy to formulate arbitrary semantic objects that are Strawson-equivalent to  $\llbracket \text{the} \rrbracket$  but do not guarantee existence. Consider the expression in (27).

- (27)  $\mathbf{the}'(P)(Q) := \begin{cases} P \subseteq Q & \text{if } |P| \leq 1 \\ \# & \text{otherwise} \end{cases}$

$\mathbf{the}'$  is just like  $\llbracket \text{the} \rrbracket^o$ , except it lacks the existence presupposition. There is no linguistic expression in English which has the meaning of  $\mathbf{the}'$ , but it is easily formulable using predicate logic (abbreviated as set theory) as a lambda-expression of the same type as  $\llbracket \text{the} \rrbracket^o$ . Since focus alternatives are stated upon semantic objects,

**the'** can be considered as an alternative. It is easy to see that **the'** is Strawson-equivalent to **the**, but **the'**( $P$ )( $Q$ ) does not guarantee that  $P \neq \emptyset$ .

In fact, there are an infinite amount of expressions which are Strawson-equivalent to  $\llbracket \text{the} \rrbracket^o$  but do not guarantee existence. Any function  $\delta_n$ , for any  $n \in \mathbb{N}$ , of the form

$$\delta_n(P)(Q) := \begin{cases} P \subseteq Q & \text{if } |P| \leq n \\ \# & \text{otherwise} \end{cases}$$

is Strawson-equivalent to  $\llbracket \text{the} \rrbracket^o$  but does not guarantee existence for  $P$ .

To solve this problem, notice that neither **the'** nor any of the  $\delta_n$ 's correspond to linguistic expressions. This suggests that our alternatives should be restricted to linguistic expressions, a conclusion which was also reached by Fox & Katzir 2011 for entirely different reasons. There are many ways to implement this idea; one solution, which minimally alters the first hypothesis, is formulated in (28). Here, it is assumed that  $\llbracket \cdot \rrbracket^o$  has already been defined as function from linguistic expressions (actually LFs generated by some grammar) to type-theoretic objects. By restricting alternatives to items of the form  $\llbracket y \rrbracket^o$  where  $y \in \text{dom}(\llbracket \cdot \rrbracket^o)$ ,  $\llbracket y \rrbracket^o$  is implicitly restricted to meanings of linguistic expressions.

(28) **Hypothesis 2.** For any F-marked constituent  $[X]_F$ ,  $\llbracket X \rrbracket^f := \{ \llbracket y \rrbracket^o : \llbracket y \rrbracket^o \in A_S(\llbracket X \rrbracket^o), y \in \text{dom}(\llbracket \cdot \rrbracket^o) \}$ .

**Result.** After imposing these constraints, let us see what the resulting set of alternatives for the definite article looks like. Since our alternatives are limited to the meanings of linguistic expressions, let us delineate the set of English quantificational determiners that our alternatives draw from. Let us be generous about what is considered a quantificational determiner and assume that they include all and only these: *some/a*, *every/all*, *no/none*, *both*, *neither/either*, *at least n*, *exactly n*, *the (n)*, *not all*, *at most n*, *fewer than n*, and *most*. Assume also that these determiners have the interpretations assigned to them by Barwise & Cooper 1981, except that I follow De Jong & Verkuyl 1985 in assuming that *every* and *all* presuppose existence, as in (29) (originally suggested by Strawson 1950).

$$(29) \quad \llbracket \text{every/all} \rrbracket^o(P)(Q) := \begin{cases} P \subseteq Q & \text{if } P \neq \emptyset \\ \# & \text{otherwise} \end{cases}$$

With these assumptions, the set of alternatives to *the* are the expressions listed in (30).<sup>12</sup>

<sup>12</sup>  $\llbracket \text{all} \rrbracket^o$  and  $\llbracket \text{some} \rrbracket^o$  are not shown since they are assumed to be respectively semantically equivalent to  $\llbracket \text{every} \rrbracket^o$  and  $\llbracket \text{a} \rrbracket^o$ . Since this is a set, duplicates are not allowed.

$$(30) \quad \llbracket \text{the} \rrbracket^f = \{ \llbracket \text{every} \rrbracket^o, \llbracket \text{a} \rrbracket^o, \llbracket \text{most} \rrbracket^o, \llbracket \text{the} \rrbracket^o, \llbracket \text{both} \rrbracket^o, \llbracket \text{exactly } 1 \rrbracket^o, \llbracket \text{neither} \rrbracket^o, \llbracket \text{the } 2 \rrbracket^o, \llbracket \text{the } 3 \rrbracket^o, \dots \}$$

The set in (30) has two notable properties, one is desirable and the other is potentially problematic. First, the desired property that (30) has is that all of these alternatives guarantee existence.  $\llbracket \text{every} \rrbracket^o$ ,  $\llbracket \text{most} \rrbracket^o$ ,  $\llbracket \text{both} \rrbracket^o$ ,  $\llbracket \text{neither} \rrbracket^o$ , and  $\llbracket \text{the } (n) \rrbracket^o$  presuppose that their first argument is non-empty, while  $\llbracket \text{a} \rrbracket^o$  and  $\llbracket \text{exactly } 1 \rrbracket^o$  entail that their first argument is non-empty.

Second, the problematic property is that (30) contains the alternative  $\llbracket \text{neither} \rrbracket^o$ . This predicts that (31) should be felicitous, though I find it quite strange.

(31) ?? Sam didn't invite THE syntax professor, he invited neither of them.

If (31) is indeed infelicitous, then  $\llbracket \text{neither} \rrbracket^o$  should be excluded from  $\llbracket \text{the} \rrbracket^f$ . Let us examine why  $\llbracket \text{neither} \rrbracket^o$  is a valid focus alternative, i.e., why it is Strawson-equivalent to  $\llbracket \text{the} \rrbracket^o$ .  $\llbracket \text{neither} \rrbracket^o(P)(Q)$  presupposes that  $|P| = 2$ , which contradicts the presupposition of  $\llbracket \text{the} \rrbracket^o(P)(Q)$ , which presupposes that  $|P| = 1$ . These two lexical items end up Strawson-entailing each other because the base case of Strawson-entailment is material implication. If we apply the definition of Strawson-equivalence to  $\llbracket \text{the} \rrbracket^o$  and  $\llbracket \text{neither} \rrbracket^o$ , shown in (32), we can see that the contradictory presuppositions cause the antecedent to be false, making the material conditional true. One can easily check that the same occurs in the opposite direction, making  $\llbracket \text{the} \rrbracket^o$  and  $\llbracket \text{neither} \rrbracket^o$  Strawson-equivalent.

$$(32) \quad \llbracket \text{the} \rrbracket^o(P)(Q) \stackrel{S}{\Rightarrow} \llbracket \text{neither} \rrbracket^o(P)(Q) \text{ iff } P \subseteq Q \wedge |P| = 1 \wedge |P| = 2 \rightarrow P \cap Q = \emptyset.$$

In fact, if two expressions have contradictory presuppositions, they will always Strawson-entail each other. This is also the reason why  $\llbracket \text{both} \rrbracket^o$  and  $\llbracket \text{the } n \rrbracket^o$  for  $n > 1$  are listed as valid focus alternatives.

Given that (31) is infelicitous, I take the view that this case is an undesirable edge case of Strawson-entailment that should be excluded from the focus alternatives. To do so, hypothesis 2 is altered as follows:

$$(33) \quad \textbf{Hypothesis 3.} \text{ For any F-marked constituent } [X]_F, \llbracket X \rrbracket^f := \{ \llbracket y \rrbracket^o : \llbracket y \rrbracket^o \in A_S(\llbracket X \rrbracket^o), y \in \text{dom}(\llbracket \cdot \rrbracket^o), \text{presupp}(\llbracket y \rrbracket^o) \text{ does not contradict } \text{presupp}(\llbracket X \rrbracket^o) \}.$$

To summarize, it was observed that association with presupposition triggers could deny their presuppositions, but that association with definites still preserved the existence presupposition. It was shown that this behavior can be explained if the focus alternatives for the definite article are constrained to Strawson-equivalent linguistic items with non-contradictory presuppositions. Given such a constraint, the set of focus alternatives is restricted just to those determiner meanings which

preserve existence on their first argument. As a result, the definite's existence presupposition cannot be false. This derives the contrasts observed in §2.

### 6.3 Association with *both*

This section presents evidence that hypothesis 3 is in fact too tightly constrained. In §6.4, I propose to weaken it by allowing alternatives to be either Strawson-alternatives or P-alternatives, a notion to be defined shortly.

Consider association with *both*. (34) shows that association with *both* shares a property with association with the definite article: when negated, existence cannot be denied.

- (34) a. Jimi isn't bringing BOTH guitars, he has three  
 b. # Jimi isn't bringing BOTH guitars, he has no guitars

The explanation given for the definite extends to this case. The set of linguistic items Strawson-equivalent to  $\llbracket \text{both} \rrbracket^o$  (its focus alternatives under hypothesis 3) are given in (35).

- (35)  $\llbracket \text{both} \rrbracket^o$ ,  $\llbracket \text{every} \rrbracket^o$ ,  $\llbracket \text{at least two} \rrbracket^o$ ,  $\llbracket \text{exactly two} \rrbracket^o$

All of the expressions in (35) preserve existence:  $\llbracket \text{both} \rrbracket^o$  and  $\llbracket \text{every} \rrbracket^o$  presuppose existence for their first argument, and  $\llbracket \text{at least two} \rrbracket^o$  and  $\llbracket \text{exactly two} \rrbracket^o$  entail existence for both arguments. If hypothesis 3 is right and (35) is the set of focus alternatives for *both*, then the contrast in (34) is explained analogously to the definite case.

However, the alternatives in (35) cannot account for (36), which is perfectly felicitous.

- (36) Jimi isn't bringing BOTH guitars  
 a. ... he isn't bringing EITHER of them.  
 b. ... he's bringing NEITHER of them.

It is clear that both continuations in (36) contrast  $\llbracket \text{both} \rrbracket^o$  with  $\llbracket \text{neither} \rrbracket^o$ .<sup>13</sup> But  $\llbracket \text{neither} \rrbracket^o$  does not show up in (35) and none of the alternatives in (35) are compatible with  $\llbracket \text{neither} \rrbracket^o$ . As a result, hypothesis 3 incorrectly predicts that (36) should be a presupposition failure.

<sup>13</sup> Assume, for simplicity, that  $\llbracket \text{neither} \rrbracket = \llbracket \text{not} \dots \text{either} \rrbracket$ .

#### 6.4 P-equivalence and P-alternatives

We wish to find a way to include  $\llbracket \text{neither} \rrbracket^o$  into the set of focus alternatives for *both*. Notice that  $\llbracket \text{both} \rrbracket^o$  and  $\llbracket \text{neither} \rrbracket^o$  have a property in common. Assuming the entries in (37), their presuppositions are equivalent.

$$(37) \quad \text{a. } \llbracket \text{both} \rrbracket^o(P)(Q) := \begin{cases} P \subseteq Q & \text{if } |P| = 2 \\ \# & \text{otherwise} \end{cases}$$

$$\text{b. } \llbracket \text{neither} \rrbracket^o(P)(Q) := \begin{cases} P \cap Q = \emptyset & \text{if } |P| = 2 \\ \# & \text{otherwise} \end{cases}$$

Call this property *P-equivalence* and define it recursively in (38). Say that  $\llbracket \text{both} \rrbracket^o$  is P-equivalent to  $\llbracket \text{neither} \rrbracket^o$  (and vice-versa), written in (39).

(38) *P-equivalence.*

a. (Base case) Let  $p, q$  be expressions of type  $\langle s, t \rangle$ . Then  $p \stackrel{P}{\Leftrightarrow} q$  iff for all  $w \in D_s$ ,  $p(w) = \#$  iff  $q(w) = \#$ .

b. (Inductive case) Let  $f, g$  be expressions of type  $\langle \sigma, \tau \rangle$  where  $\tau$  ends in  $t$ . Then  $f \stackrel{P}{\Leftrightarrow} g$  iff for all  $x \in D_\sigma$ , (i)  $f(x) = \#$  iff  $g(x) = \#$ , and (ii)  $f(x) \stackrel{P}{\Leftrightarrow} g(x)$ .

(39) **Proposition.**  $\llbracket \text{both} \rrbracket^o \stackrel{P}{\Leftrightarrow} \llbracket \text{neither} \rrbracket^o$

Then, alter the previous hypothesis by allowing alternatives to be either Strawson-equivalent or P-equivalent. This is the final formulation.

(40) *P-alternatives.*  $A_P(\alpha) := \{\gamma : \alpha \stackrel{P}{\Leftrightarrow} \gamma\}$

(41) **Final Hypothesis.** For any F-marked constituent  $[X]_F$ ,  
 $\llbracket X \rrbracket^f := \{\llbracket y \rrbracket^o : \llbracket y \rrbracket^o \in A_S(\llbracket X \rrbracket^o) \cup A_P(\llbracket X \rrbracket^o), y \in \text{dom}(\llbracket \cdot \rrbracket^o), \text{presupp}(\llbracket y \rrbracket^o)$   
 does not contradict  $\text{presupp}(\llbracket X \rrbracket^o)\}$

As a result,  $\llbracket \text{neither} \rrbracket^o$  is now included as a possible focus alternative for  $\llbracket \text{both} \rrbracket^o$ . Now, (36) corresponds to a valid focus alternative, so it is no longer a presupposition failure, as desired.

To address one final worry: one might ask whether the final hypothesis alters our explanation for association with the definite article based on hypothesis 3. To answer this question, we must first find out whether adding the P-alternatives of  $\llbracket \text{the} \rrbracket^o$  to its focus alternatives really adds any new alternatives to  $\llbracket \text{the} \rrbracket^f$ . In fact, the addition of the P-alternatives of  $\llbracket \text{the} \rrbracket^o$  adds nothing to its focus alternative set, because there are no linguistic expressions in English whose meanings are P-equivalent to  $\llbracket \text{the} \rrbracket^o$ . Then,  $\llbracket \text{the} \rrbracket^f$  remains the same, and the final hypothesis can still be suitably applied to association with the definite article.

## 7 Conclusion

In this paper, new evidence was provided for focus alternatives being more tightly constrained than previously thought. Evidence came from contrastive negation, specifically negation associating with presupposition triggers. It was observed that when presupposition triggers associate with negation, their presuppositions can be denied. When considering the presuppositional quantificational determiners *the* and *both*, we saw that for both of these cases, there is always an existential inference present even under negation (i.e., their first argument must be non-empty). We found that we could predict this behavior by constraining focus alternatives to lexical items which are either Strawson-equivalent or P-equivalent.

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