

Scope-related cumulativity asymmetries and cumulative composition*

Nina Haslinger
University of Vienna

Viola Schmitt
University of Vienna

Abstract Some elements in German and English, e.g. *every* DPs, give rise to cumulativity asymmetries: They allow for cumulative readings only if they occur in the scope of another semantically plural expression. We present a surface-compositional and event-less analysis of this pattern, expanding Schmitt’s (2017) ‘plural projection’ framework. In this system, any constituent containing a semantically plural subexpression denotes a set of (possibly higher-type) pluralities. Cumulativity is built into the rules implementing this ‘projection’ of semantic plurality.

Keywords: cumulativity, pluralities, universal quantifiers, distributive conjunction

1 The problem: Asymmetrically distributive universals

Some elements in English and German, such as DPs headed by *every*, give rise to a particular semantic asymmetry when they co-occur with plural expressions like (*the*) *two dogs*. We call these elements **asymmetrically distributive universals (ADUs)**.

1.1 Basic asymmetries

In some configurations, *every* DPs are restricted to a **distributive reading**: Thus, (2) is true in the ‘distributive’ scenario (1a) where the predicate *fed (the) two dogs* applies to each girl individually, but false in scenario (1b) where it does not.

- (1) CONTEXT: There are two girls, Ada and Bea, and two dogs, Carl and Dean.
- a. SCENARIO: Ada fed Carl and Dean. Bea fed Carl and Dean.
- b. SCENARIO: Ada fed Carl. Bea fed Dean.

* We thank Kübra Atasoy, Daniel Buring, Lucas Champollion, Mojmír Dočekal, Enrico Flor, Izabela Jordanoska, Clemens Mayr, Max Prüller, Eva Rosina, Magdalena Roszkowski, Dóra Kata Takács, Marcin Wągiel, Susi Wurmbrand, Henk Zeevat, Eytan Zweig, the audiences at the Universities of Göttingen, Tübingen and Brno, the 19th Szklarska Poręba Workshop on the Roots of Pragmasemantics and SALT 28, and the SALT 28 reviewers, for comments. Our research was funded by the Austrian Science Fund (FWF), project P-29240 ‘Conjunction and disjunction from a typological perspective’.

- (2) *Every girl in this town fed (the) two dogs.* **true in (1a), false in (1b)**

The fact that (2) is false in ‘cumulative’ scenarios like (1b) shows that it lacks a **cumulative reading**, where the feeding relation holds cumulatively of the girls and the dogs – which is to say that each of the girls fed at least one dog, and each of the dogs was fed by at least one girl. In this respect, *every* DPs differ from plural definites or indefinites like *(the) two girls*, which exhibit cumulative readings in parallel syntactic contexts. For instance, (3) is true in scenario (1b).

- (3) *The two girls in this town fed (the) two dogs.* **true in (1a), true in (1b)**

However, it cannot be part of the semantic contribution of *every* to block cumulativity across the board. As discussed by Schein (1993), Kratzer (2003), Ferreira (2005), Zweig (2008) and Champollion (2010), *every* DPs allow for cumulative readings if another semantically plural expression occurs in a higher syntactic position. For instance, (4) is true in the ‘cumulative’ scenario (1b).

- (4) *(The) two girls fed every dog in this town.* **true in (1a), true in (1b)**

The contrast between (2) and (4) illustrates the basic property of ADUs: They are limited to distributive readings relative to syntactically ‘lower’ plural expressions, but allow for cumulative readings relative to syntactically ‘higher’ plural expressions.

In German, the determiner *jed-* (‘every’) also shows the hallmarks of ADUs. But for some speakers, they are also found with another class of expressions, namely conjunctions of the form *sowohl A als auch B* ‘A as well as B’: While (7) can be true in the cumulative scenario in (5), (6) is false, which mirrors the behavior of *every*.¹

- (5) SCENARIO: Two skiing World Cup races took place today. Ada and Bea were the only Austrian participants. Ada competed in the downhill and won it. Bea competed in the slalom and won it.

- (6) *Heute haben sowohl die Ada als auch die Bea die zwei Rennen gewonnen!*
today have PRT the Ada PRT also the Bea the two races won
‘Today, both Ada and Bea won the two races.’ **false in (5)**

- (7) *Heute haben die zwei Österreicherinnen sowohl die Abfahrt als auch den Slalom gewonnen!*
today have the two Austrians PRT the downhill PRT also the
slalom won
‘Today, the two Austrians won both the downhill and the slalom.’ **true in (5)**

¹ Preliminary data suggests that conjunctions of the forms *i A i B* in Polish and *A is és B is* in Hungarian, which are usually said to be ‘distributive’ in the literature, might exhibit similar asymmetries.

We follow [Champollion 2010](#) in assuming that the restrictions on ADUs should be described in terms of scope (or c-command relations at LF) rather than thematic roles, contra [Kratzer 2003](#) a.o. More research is clearly needed, but one argument for this comes from German sentences like (9): The ADU in (9) differs from that in (2) above regarding its syntactic position (it occurs below another plural, as the subject of the infinitival clause), but not wrt. its thematic role (in both cases, the ADUs are agents). Nevertheless, (9), as opposed to (2), has a cumulative reading, which shows that a restriction in terms of thematic roles is insufficient.

- (8) SCENARIO: Detectives Ada and Bea were observing three suspects. Ada saw suspect 1 smoke a cigar. Bea saw suspects 2 and 3 smoke one cigar each.
- (9) *Ada und Bea haben jeden Verdächtigen eine Zigarre rauchen gesehen.*
 Ada and Bea have every suspect a cigar smoke seen
 ‘Ada and Bea saw every suspect smoke a cigar.’ **true in (8)**

1.2 Schein sentences

This ‘simple’ structural asymmetry is not the only property of ADUs that any theory of cumulativity has to account for. We also observe a particular interaction between cumulativity and distributivity when ADUs are ‘sandwiched’ between two plural expressions, as in (11), which is true in the scenario in (10). These cases were first discussed in detail by [Schein \(1993\)](#), so we call them **Schein sentences**.

- (10) SCENARIO: There are two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.
- (11) *Ada and Bea taught every dog two new tricks.* **true in (10)**
 (adapted from [Schein 1993](#))

On the relevant reading of (11), *every dog* seems to cumulate with *Ada and Bea*, as it is not the case that *each* of the girls taught every dog two tricks. Yet, *every dog* is distributive wrt. *two tricks*, since each dog is taught two (potentially different) tricks.

This reading cannot be straightforwardly captured in terms of a single cumulative relation holding between individuals. It is not the case that the relation $[\lambda x_e. \lambda y_e. y \text{ taught } x \text{ two new tricks}]$ holds cumulatively of the two girls and the two dogs: This would predict that for each girl x , some dog was taught two tricks by x , which is false in scenario (10). It is also not the case that the relation $[\lambda x_e. \lambda y_e. y \text{ taught } x \text{ to every dog}]$ holds cumulatively of the two girls and some plurality of two tricks: This would falsely predict that there must be two tricks that each of the dogs was taught. For the same reason, the truth conditions of (11) cannot be captured by assuming that the three-place relation $[\lambda x_e. \lambda y_e. \lambda z_e. z \text{ taught } x \text{ to } y]$ holds cumulatively.

In other words, all three semantically plural expressions seem to ‘participate’ in the cumulative interpretation, but this cannot be accounted for in terms of a single cumulative relation between individuals since *every dog* has scope over *two tricks*.

1.3 Interim summary

We saw that English and German have a class of expressions – asymmetrically distributive universals (ADUs) – that exhibit the following behavior:

1. They allow for cumulative readings wrt. syntactically higher plural expressions.
2. They prohibit cumulative readings wrt. syntactically lower plural expressions.
3. When they occur in Schein sentences, the resulting mixed cumulative/distributive reading cannot be analyzed via a single cumulative relation between individuals.

In the next sections, we present a new account of ADUs, which we illustrate using English *every* DPs. It is based on a novel view of cumulativity, adapted from a proposal by Schmitt (2017), which derives cumulativity in a step-by-step process by means of a special composition mechanism that is sensitive to syntactic structure.

2 Motivating plural projection and higher-order pluralities

Before we spell out our proposal in detail in Sections 3 and 4, we informally outline its core properties and show where it deviates from existing accounts of ADUs.

2.1 The basic intuition behind our proposal

The basic idea is that model-theoretic objects of higher types like predicates or propositions can also form pluralities that participate in cumulative relations.² If so, the truth conditions of (11) can be paraphrased as follows: We consider all unary predicates of the form **taught**(x)(y), where x is a trick and y is a dog.³ We then form a set containing all pluralities of such predicates that (i) ‘cover’ both of the dogs and (ii) relate each dog to two tricks. This set is sketched in (12), where + symbolizes a cross-categorical sum operation, to be defined in Section 3 below.

$$(12) \quad \{ \mathbf{taught}(\mathbf{T}_1)(\mathbf{C}) + \mathbf{taught}(\mathbf{T}_2)(\mathbf{C}) + \mathbf{taught}(\mathbf{T}_1)(\mathbf{D}) + \mathbf{taught}(\mathbf{T}_2)(\mathbf{D}), \\ \mathbf{taught}(\mathbf{T}_1)(\mathbf{C}) + \mathbf{taught}(\mathbf{T}_2)(\mathbf{C}) + \mathbf{taught}(\mathbf{T}_2)(\mathbf{D}) + \mathbf{taught}(\mathbf{T}_3)(\mathbf{D}), \\ \mathbf{taught}(\mathbf{T}_1)(\mathbf{C}) + \mathbf{taught}(\mathbf{T}_3)(\mathbf{C}) + \mathbf{taught}(\mathbf{T}_1)(\mathbf{D}) + \mathbf{taught}(\mathbf{T}_2)(\mathbf{D}), \dots \}$$

² See Schmitt 2013, 2017 for a defense of this point, and von Stechow 1980, Krifka 1990, Beck & Sharvit 2002 for earlier arguments that some higher-type conjunctions behave like plural individuals.

³ In the remainder of the paper, we will represent the denotations of lexical predicates and proper names by boldfaced versions of (abbreviations of) the respective object-language expressions.

If the VP *taught every dog two new tricks* denotes the set in (12), we can characterize the cumulative reading of (11) as follows: (11) is true iff Ada and Bea cumulatively satisfy at least one predicate sum in this set. In other words, there must be some predicate sum in (12) such that each of the two girls satisfies at least one predicate in this sum, and each predicate in this sum is satisfied by at least one of the two girls.

This intuition does not amount to a convincing account of Schein sentences, of course, unless we have a principled way of deriving denotations like (12). In this paper, we will adapt a proposal developed by Schmitt (2017): Whenever a constituent that would ‘normally’ be assigned semantic type a contains a plural, it will actually denote a set of pluralities of denotations of type a – a **plural set**. The property of denoting a plural set ‘projects’ from a constituent to its mother via special composition rules. Any node dominating a semantically plural expression will itself be semantically plural unless an intervening operator blocks this process. Cumulativity falls out from the rules implementing this ‘projection’ mechanism.

2.2 Informal outline of plural projection

The basic system roughly looks like this: We start with the assumption that plural definites and conjunctions both denote singleton sets containing a plurality. For instance, we have $\llbracket \textit{Carl and Dean} \rrbracket = \{\mathbf{Carl} + \mathbf{Dean}\}$ and $\llbracket \textit{smoke and drink} \rrbracket = \{(\lambda x.\mathbf{smoke}(x)) + (\lambda x.\mathbf{drink}(x))\}$. Further, any non-plural meaning can be shifted to a singleton set containing it, i.e. $\llbracket \textit{smoke} \rrbracket$ is shiftable to the set $\{\lambda x.\mathbf{smoke}(x)\}$.

Unlike ordinary sets, these ‘plural sets’ combine with one another via a special composition rule. In the simplest case, when a non-plural functor combines with a plural argument as in (13), the output will be a singleton set containing the sum of those values which result from applying the functor to an atomic part of the argument plurality. In (13), this sum has two atomic parts – the property of feeding Carl and the property of feeding Dean. Similarly, when a plural functor combines with a non-plural argument (14), the resulting set will contain the plurality obtained by applying each atomic part of the functor to the argument. The part structure of the embedded plural expression ‘projects up’ in the syntactic tree in a way that resembles Hamblin/Rooth-style Alternative Semantics (Hamblin 1973; Rooth 1985).

$$\begin{array}{ccc}
 (13) \quad \textit{feed Carl and Dean} & & (14) \quad \textit{feed and brush Dean} \\
 \{\mathbf{feed}(\mathbf{C})_{\langle e,t \rangle} + \mathbf{feed}(\mathbf{D})_{\langle e,t \rangle}\} & & \{\mathbf{feed}(\mathbf{D})_{\langle e,t \rangle} + \mathbf{brush}(\mathbf{D})_{\langle e,t \rangle}\} \\
 \underbrace{\hspace{10em}} & & \underbrace{\hspace{10em}} \\
 \{\mathbf{feed}_{\langle e, \langle e,t \rangle \rangle}\} \quad \{\mathbf{C}_e + \mathbf{D}_e\} & & \{\mathbf{feed}_{\langle e, \langle e,t \rangle \rangle} + \mathbf{brush}_{\langle e, \langle e,t \rangle \rangle}\} \quad \{\mathbf{D}_e\}
 \end{array}$$

If no other plural expressions occur in the structure, this projection process continues to the sentence level, where we end up with a plurality of propositions.

If both the functor set and the argument set contain pluralities, the output of the composition rule must be more complex. A single predicate sum is an inadequate denotation for a predicate involving cumulativity like (15), which is true of individuals who feed Carl and brush Dean, but also of individuals who feed Dean and brush Carl, etc. Generally, when a plural functor encounters a plural argument, there are many ways of matching parts of the functor plurality with parts of the argument plurality. This is why we need *sets* of pluralities rather than simply pluralities: The resulting set will contain all those value pluralities that ‘cover’ all atomic parts of the functor as well as all atomic parts of the argument, as sketched in (16) for (15).

(15) *feed and brush Carl and Dean*

(16) $\{\mathbf{feed}(\mathbf{C})+\mathbf{brush}(\mathbf{D}), \mathbf{feed}(\mathbf{D})+\mathbf{brush}(\mathbf{C}), \mathbf{feed}(\mathbf{C})+\mathbf{feed}(\mathbf{D})+\mathbf{brush}(\mathbf{D}) \dots\}$

$$\underbrace{\{\mathbf{feed}_{\langle e, \langle e, t \rangle} + \mathbf{brush}_{\langle e, \langle e, t \rangle} \}}_{\{\mathbf{C}_e + \mathbf{D}_e\}}$$

At the root clause level, we obtain a truth value: A sentence denoting a plural set is true iff the set contains at least one plurality all atomic parts of which are true.

This should give an idea of the system that forms the backbone of our analysis of ADUs. Of course, several components are still missing, most importantly, a treatment of ADUs themselves that will derive us something along the lines of (12). We will introduce them once we have spelled out the system in detail. But first, we present independent motivation for the basic ideas just sketched. The motivating examples will show an interesting analogy to the problem posed by Schein sentences.

2.3 Plural projection vs. syntactically derived cumulative relations

Our approach challenges the widespread idea that cumulative readings are due to **cumulation operators** that attach to a relation-denoting expression in the syntax. Beck & Sauerland (2000), for instance, derive the cumulative reading of a sentence like (17) from an LF like (18). (Indices are interpreted as in Heim & Kratzer 1998.)

(17) *Ada and Bea wanted to buy the two dogs.*

(18) $[[[Ada \text{ and } Bea] [[the \text{ two } dogs] [** [2 [1 [t_1 \text{ wanted to buy } t_2]]]]]]]]$

(19) For a binary relation $R_{\langle e, \langle e, t \rangle} \}, [**](R)$ is the smallest relation $R'_{\langle e, \langle e, t \rangle} \}$ such that (i) for all individuals x, y , if $R(x)(y)$, then $R'(x)(y)$ and (ii) for every set $S \subseteq \{(x, y) \mid R(x)(y)\}$, $R'(\mathbf{+}\{x \mid \exists y. (x, y) \in S\})(\mathbf{+}\{y \mid \exists x. (x, y) \in S\})$.

Here, the crucial work is done by the cumulation operator $**$, defined in (19). This operator essentially takes a relation between individuals and enriches its extension by adding ‘pointwise sums’ of the pairs in the original relation. For each set of pairs

in the original relation, the pair consisting of the sum of all the first components and the sum of all the second components will appear in the derived relation. For example, if $R(\mathbf{A}, \mathbf{C})$ and $R(\mathbf{B}, \mathbf{D})$ holds, so does $\llbracket ** \rrbracket (R)(\mathbf{A} + \mathbf{B}, \mathbf{C} + \mathbf{D})$.

The operator $**$ attaches to relation-denoting constituents in the syntax. If the cumulative relation does not correspond to a surface constituent, as in (17), covert movement of the two plurals is invoked to derive a suitable LF constituent. Hence cumulativity is always tied to relation-denoting LF constituents – a property of the cumulation-operator account that Schmitt (2017) argues is problematic.⁴ Her point is based on cumulative readings of sentences in which a predicate conjunction contains another plural expression. For instance, (21) is true in scenario (20).

- (20) SCENARIO: Ada owns a dog, Carl. Bea owns another dog, Dean, and a cat, Eric. Ada and Bea went on a trip and made Gene take care of their pets: Ada made Gene feed Carl, and Bea made Gene feed Dean and brush Eric.
- (21) *The two girls made Gene* $\llbracket \text{feed the two dogs} \rrbracket_P$ and $\llbracket \text{brush Eric} \rrbracket_Q$ when all he wanted to do was take care of his hamster. **true** in (20) (Schmitt 2017)

In (21), the predicate conjunction P and Q has a cumulative reading relative to *the two girls*: In scenario (20), it is not the case that each girl made Gene brush Eric, as a distributive interpretation of predicate conjunction (e.g. von Stechow 1974; Gazdar 1980; Partee & Rooth 1983 a.o.) would require. Rather, the relation $\llbracket \lambda P. \lambda x. x \text{ made Gene do } P \rrbracket$ intuitively applies cumulatively to the two girls and the two predicates P and Q . As there is no surface constituent denoting this relation, the obvious solution within the cumulation-operator account would be to assume an LF like (22). (This requires an extension of $**$ to higher types, cf. Schmitt 2013.)

- (22) $\llbracket \llbracket \text{the two girls} \rrbracket \llbracket \llbracket \text{feed the two dogs and brush } E \rrbracket \llbracket ** \rrbracket [2 [1 [t_1 \text{ made } G t_2]]] \rrbracket \rrbracket$

Yet, in scenario (20) *the two dogs* also has a cumulative reading relative to *the two girls*, since neither of the girls made Gene feed both of the dogs. There is no obvious way of interpreting *feed the two dogs* in (22) that accounts for this fact. Even if we interpret *feed the two dogs* as being true of plural individuals that cumulatively feed both of the dogs (for instance by inserting a second instance of $**$ that modifies *feed*), the problem persists, since the semantic argument of *feed the two dogs* in (22) is the singular individual Gene rather than any plurality of girls.⁵

The problem is that three plural expressions participate in cumulation – *the two girls*, *the two dogs* and the predicate conjunction – but there is no way of deriving a

⁴ This point does not extend to accounts that cumulate thematic role relations, see Section 2.4.

⁵ Analyses of cumulative predicate conjunction like Link 1984, Krifka 1990, Heycock & Zamparelli 2005 do not extend $**$ to predicates. They won't help us with (22) since their scope is restricted to cases where the predicate conjunction directly combines with a semantically plural argument.

relation that might form the input for **, since one plural expression syntactically contains another. If we move only two plural expressions in the syntax, as in (22), the resulting LF won't give us the right semantics. But if we move *the two dogs* out of the predicate conjunction, it will be unclear how to interpret the resulting structure since the predicate conjunction would contain an unbound trace.

This phenomenon – which we call the **flattening effect** – resembles the problem with Schein sentences discussed in Section 1.2: We find a cumulative reading that cannot be accounted for by a single cumulative relation. Our solution for this problem will resemble the solution sketched for Schein sentences in (12): The truth conditions of (22) can easily be stated if we appeal to cumulative relations involving higher-order pluralities. The two girls must cumulatively satisfy the predicate plurality in (23) – i.e. each girl must satisfy at least one predicate in the sum (23), and each predicate must be satisfied by at least one girl.

$$(23) \quad \llbracket \text{feed the two dogs and brush Eric} \rrbracket = \mathbf{feed}(\mathbf{C}) + \mathbf{feed}(\mathbf{D}) + \mathbf{brush}(\mathbf{E})$$

So we need a system that derives (23) as the denotation of the VP conjunction. This system must guarantee that if one plural expression is contained in another, the resulting expression denotes a single 'flat' plurality that preserves the part structure of the embedded plural expression (*the two dogs* in (23)). The projection mechanism sketched above will do just that (with minor variations), as shown in Section 3.

2.4 Comparison to previous analyses of ADUs

Given this sketch of our approach to ADUs and the general system underlying it, how does it compare to previous analyses? There are essentially two existing treatments of the puzzle posed by Schein sentences like (11), both of which assume that such sentences involve multiple cumulative relations. They both derive the right truth conditions for examples with *every*⁶, so our proposal won't improve on the *status quo* in this respect. Rather, the main advantage of our analysis is its broader scope.

Under the first approach, the relevant cumulative relations are thematic role relations that relate individuals to events (Schein 1993; Kratzer 2003; Ferreira 2005; Zweig 2008). These theories resemble ours in that cumulativity is no longer restricted to relations between individuals. But our proposal, unlike theirs, allows us to remain agnostic about the role of events in the semantics of predicates. First, we do not have to claim that every predicate that allows for cumulativity has an event or state argument. Second, event-based accounts need additional assumptions to account for the fact, illustrated in (24), that cumulative relations can 'reach inside' arguments

⁶ Some of the event-based analyses derive truth conditions that are too weak. For instance, given Kratzer's (2003) lexical entry for *every*, (11) should be true if Ada taught every dog two tricks and Bea didn't contribute. However, Ferreira's (2005) compositional implementation avoids this problem.

that denote neither individuals nor events, like complements of attitude verbs. Our system allows us to express the cumulative reading of (24) in terms of a plurality of propositions that the ambassadors cumulatively believe (cf. Schmitt 2017).⁷

- (24) The Georgian ambassador called this morning, the Russian one at noon.
They think that Trump should talk to Putin and build a hotel in Tbilisi, but
 neither addressed the Caucasus conflict! **true in (25)**
- (25) SCENARIO: The Georgian ambassador thinks Trump should build a hotel in
 Tbilisi. The Russian ambassador thinks Trump should talk to Putin.

The second approach (Champollion 2010) only requires cumulative relations between individuals, but posits a complex LF for Schein sentences that contains cumulation operators in two different syntactic positions. The distributive interpretation of *every* DPs wrt. lower plurals is accounted for by means of a restriction on the syntax-semantics interface: Traces of *every* DPs must range only over atomic individuals. For Schein sentences with *every* DPs, the predictions of our proposal coincide with Champollion's. But our theory generalizes more easily to other cases of cumulativity. First, it derives the flattening effect from Section 2.3, which poses a problem for any theory based on syntactically derived cumulative relations. Second, since Champollion (2010) requires the traces of ADUs to range over atoms, his account does not extend to distributive conjunctions like German *sowohl . . . als auch*, which also occur in Schein sentences, but seem to quantify over plural individuals: (27) has a reading in which the girls, as well as the boys, *cumulatively* fed two dogs.

- (26) SCENARIO: The girls fed two dogs between them and the boys fed two dogs
 between them.
- (27) *Sowohl die Mädchen als auch die Buben haben zwei Hunde gefüttert.*
 PRT the girls PRT also the boys have two dogs fed
 'The girls as well as the boys fed two dogs.' **true in (26)**

While we cannot give a full analysis of (27) here for reasons of space (but see Haslinger & Schmitt 2018), the plural projection approach does not predict any link between distributivity in Schein sentences and quantification over atoms.

2.5 Interim summary

This section outlined our basic idea for Schein sentences and sketched the essentials of the system underlying it: Pluralities can 'project' up the tree in the sense that

⁷ In some analyses, attitude verbs combine with eventualities, such as belief states, rather than propositions (Kratzer 2006; Moulton 2015; Elliott 2017). However, such analyses still involve an operator within the complement clause that maps propositions to eventualities. Since a purely event-based system for cumulativity cannot 'reach' below this operator, the problem discussed in the text remains.

denotations of embedding nodes reflect the part structure of embedded pluralities. Cumulativity is encoded in this ‘plural projection’ mechanism. The **flattening effect** motivates this approach, as it shows that a theory based on cumulation operators which attach to syntactically derived relations is not powerful enough. While this argument is independent from the data presented in Section 1, there is a structural analogy between the flattening effect and our description of Schein sentences.

3 Plural projection: The formal analysis

Our next step is to introduce the details of the plural projection mechanism and show how it derives the flattening effect. (What we present here overlaps with the account by Schmitt (2017), but introduces some novel concepts and generalizations.) This will form the backbone of our treatment of ADUs in Section 4.

3.1 Ontology

Above, we motivated a semantic category of so-called ‘plural sets’ that do not interact with the composition rules in the same way as the ‘ordinary’ sets unary predicates denote. We implement this distinction in the type system: For any semantic type a , there is also a type a^* for plural sets with elements of type a .

(28) The set T of **semantic types** is the smallest set such that $e \in T, t \in T$, for any $a, b \in T$, $\langle a, b \rangle \in T$, and for any $a \in T$, $a^* \in T$.

The elements of plural sets are pluralities, which we assume to be available for any semantic type, following Schmitt 2013, 2017. This claim requires a cross-categorical notion of sum. We posit that sums of any semantic type stand in a one-to-one correspondence to nonempty sets of atomic meanings of that type. In other words, for any type, the domain is enriched by a ‘flat’ plural semantics.

In (29), we formalize this idea by defining a cross-categorical operation \oplus that maps any nonempty set of denotations (of the same type) to its sum. For any type a , the set A_a of atomic domain elements is extended to a set D_a that also includes sums. Clause (29b) says that the sum operation on D_a is isomorphic to the union of nonempty sets of atomic meanings from A_a . Clause (29c) bans pluralities from being identified with such sets, as they are distinguished by the composition rules.

(29) For each type a , there is an **atomic domain** A_a and a **full domain** D_a with the following properties:

- a. D_a is a set s.th. $A_a \subseteq D_a$ and there is an operation $\oplus_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \rightarrow D_a$.
- b. There is a function $pl_a : \mathcal{P}(A_a) \setminus \{\emptyset\} \rightarrow D_a$ s.th.:
 - i. $pl_a(\{x\}) = x$ for each $x \in A_a$

- ii. and pl_a is an isomorphism from $(\mathcal{P}(A_a) \setminus \{\emptyset\}, \cup)$ to (D_a, \oplus) .
- c. For any type $b \neq a$, D_a and D_b are disjoint.⁸

The atomic domains are defined recursively in the usual way in (30).⁹ For types of the form a^* , we assume that the domain is isomorphic to, but disjoint from the power set of D_a . The disjointness condition allows us to define operations that are sensitive to whether their arguments are plural sets (type a^*) or ‘regular’ sets (type $\langle a, t \rangle$).

- (30) a. $A_e = A$, the set of individuals; $A_t = \{0, 1\}^W$, where W is the set of possible worlds
- b. For any types a, b : $A_{\langle a, b \rangle} = D_b^{D_a}$, the set of partial functions from D_a to D_b .
- c. For any type a , A_{a^*} is a set that is disjoint from $\mathcal{P}(D_a)$ and on which the operations \cup , \cap and \setminus are defined. Further, there is a function $pl_a^* : \mathcal{P}(D_a) \rightarrow A_{a^*}$ that is an isomorphism wrt. \cup , \cap and \setminus .

Finally, we introduce some notational conventions:

- (31) a. We use ‘starred’ variables like x^* , P^* etc. for types of the form a^* .
- b. We sometimes omit type subscripts on cross-categorial operations.
- c. For variables x, x_1, \dots, x_n of any type, we write $[x_1, \dots, x_n]$ for the plural set $pl^*(\{x_1, \dots, x_n\})$, and $[x \mid \phi]$ for $pl^*(\lambda x. \phi)$.
- d. For any type b and $x, y \in D_b$:
- i. $x +_b y =_{\text{def}} \oplus_b(\{x, y\})$
 - ii. $x \leq y \Leftrightarrow_{\text{def}} x +_b y = y$
 - iii. $x \leq_a y \Leftrightarrow_{\text{def}} x \leq y \wedge y \in A_b$

Let’s look at a few examples for illustration. (32) shows that, while D_e contains both atoms and pluralities of type e , the domain A_{e^*} of plural sets of individuals is isomorphic to the power set of D_e . (D_{e^*} , in turn, would contain sums of such plural sets, e.g. $[\mathbf{A}] + [\mathbf{A} + \mathbf{B}]$, plus the elements of A_{e^*} .) (33) illustrates our assumption that $D_{\langle e, t \rangle}$ contains sums of predicates in addition to the familiar ‘atomic’ predicates.

- (32) a. $A_e = \{\mathbf{A}, \mathbf{B}\}$, $D_e = \{\mathbf{A}, \mathbf{B}, \mathbf{A} + \mathbf{B}\}$
- b. $A_{e^*} = \{[\], [\mathbf{A}], [\mathbf{B}], [\mathbf{A} + \mathbf{B}], [\mathbf{A}, \mathbf{B}], [\mathbf{A}, \mathbf{A} + \mathbf{B}], [\mathbf{B}, \mathbf{A} + \mathbf{B}], [\mathbf{A}, \mathbf{B}, \mathbf{A} + \mathbf{B}]\}$
- (33) a. $A_{\langle e, t \rangle} = \{\lambda x. \mathbf{smoke}(x), \lambda x. \mathbf{dance}(x), (\lambda x. \mathbf{smoke}(x) \vee \mathbf{dance}(x)), \dots\}$
- b. $D_{\langle e, t \rangle} = \{\lambda x. \mathbf{smoke}(x), \lambda x. \mathbf{dance}(x), (\lambda x. \mathbf{smoke}(x) \vee \mathbf{dance}(x)), \lambda x. \mathbf{smoke}(x) + \lambda x. \mathbf{dance}(x), \lambda x. \mathbf{smoke}(x) + (\lambda x. \mathbf{smoke}(x) \vee \mathbf{dance}(x)), \lambda x. \mathbf{dance}(x) + (\lambda x. \mathbf{smoke}(x) \vee \mathbf{dance}(x)), \dots\}$

⁸ The empty partial function should be exempt from the disjointness conditions.

⁹ Atomic functional meanings can have pluralities in their domain, or return plural values. This is not needed here, but might help with collective predicates, which, however, we still lack an analysis of.

3.2 Semantics of plurals and conjunction

Both plural definites and indefinites denote plural sets of type e^* , as in (34). The denotations of indefinites can have more than one element, e.g. *two pets* denotes the set of all sums of two pets.¹⁰

$$(34) \quad \begin{aligned} \text{a. } \llbracket \textit{the girls} \rrbracket &= \llbracket \textit{the} \llbracket \text{PL girl} \rrbracket \rrbracket = [\mathbf{A} + \mathbf{B}] \\ \text{b. } \llbracket \textit{two pets} \rrbracket &= \llbracket \textit{two} \llbracket \text{PL pet} \rrbracket \rrbracket = [\mathbf{C} + \mathbf{D}, \mathbf{C} + \mathbf{E}, \mathbf{D} + \mathbf{E}] \end{aligned}$$

This is achieved by the definitions in (35), which allow the determiner meanings to manipulate plural sets directly. We assume that plural DPs involve a pluralization operator PL, defined in (35b), that forms all sums of atomic individuals satisfying the restrictor predicate.¹¹ (35c) implements the familiar idea that the definite determiner selects the maximal element from the set of pluralities formed by PL. Numerals also apply to the output of PL and filter out the elements of a certain cardinality, as defined in (35d).

$$(35) \quad \begin{aligned} &\text{Plural definites and upward-monotonic indefinites} \\ \text{a. } \mathcal{A}(P_{\langle e,t \rangle}^*) &= \lambda x_e. (\exists P_{\langle e,t \rangle}. P \in pl^{*-1}(P^*) \wedge \exists P'_{\langle e,t \rangle}. P' \leq_a P \wedge P'(x)) \\ &\approx \text{the set of individuals that satisfy an atomic part of a predicate sum in } P^* \\ \text{b. } \llbracket \text{PL}_{\langle \langle e,t \rangle^*, e^* \rangle} \rrbracket &= \lambda P_{\langle e,t \rangle}^*. [x_e \mid \forall y_e (y \leq_a x \rightarrow \mathcal{A}(P^*)(y))] \\ &\approx \text{the function mapping a plural set } P^* \text{ to the closure of } \mathcal{A}(P^*) \text{ under sum} \\ \text{c. } \llbracket \textit{the}_{\langle e^*, e^* \rangle} \rrbracket &= \lambda x_{e^*}. [\exists x \in pl^{*-1}(x^*) (\forall y \in pl^{*-1}(x^*) (y \leq x)). [tx \in pl^{*-1}(x^*). \forall y \in \\ &pl^{*-1}(x^*). y \leq x] \\ &\approx \text{the function mapping a plural set to the singleton with its maximal} \\ &\text{element} \\ \text{d. } \llbracket \textit{two}_{\langle e^*, e^* \rangle} \rrbracket &= \lambda x_{e^*}. [x_e \mid pl^{*-1}(x^*)(x) \wedge |x| = 2], \text{ where } |x| \text{ is the number of} \\ &\text{atomic parts of } x \\ &\approx \text{the function extracting all elements of cardinality 2 from a plural set} \end{aligned}$$

Next, we turn to conjunction. Apart from the plural projection rule specified below, conjunction is the only binary operation in our fragment that directly combines two plural sets. These two operations turn out to have a common core: a cross-categorial operation we call \oplus . For arguments that are not plural sets, \oplus coincides with the sum operation $+$, as shown in (36a). When applied to plural sets, however, it has a ‘distributive’ effect: It produces the set of all pluralities that can be obtained by

¹⁰ This approach does not extend to non-upward-monotonic indefinites like *exactly/less than ten pets*.

¹¹ The restrictor predicate may itself be a plural set of type $\langle e, t \rangle^*$, which allows us to give a plausible semantics for examples with cumulativity in the restrictor, such as *the dogs and cats of the linguists*.

selecting one element from each argument set and summing up all the selected elements. Thus, $\llbracket a \text{ girl and two pets} \rrbracket$ in (36b) ends up denoting a plural set containing all sums of a girl and two pets.

$$(36) \quad \begin{aligned} \text{a. } \llbracket \text{smoke and drink} \rrbracket &= \lambda x. \mathbf{s}(x) \oplus \lambda x. \mathbf{d}(x) = \lambda x. \mathbf{s}(x) + \lambda x. \mathbf{d}(x) \\ \text{b. } \llbracket a \text{ girl and two pets} \rrbracket &= \llbracket a \text{ girl} \rrbracket \oplus \llbracket \text{two pets} \rrbracket \\ &= [\mathbf{A}, \mathbf{B}] \oplus [\mathbf{C} + \mathbf{D}, \mathbf{C} + \mathbf{E}, \mathbf{D} + \mathbf{E}] \\ &= [\mathbf{A} + \mathbf{C} + \mathbf{D}, \mathbf{A} + \mathbf{C} + \mathbf{E}, \mathbf{A} + \mathbf{D} + \mathbf{E}, \mathbf{B} + \mathbf{C} + \mathbf{D}, \mathbf{B} + \mathbf{C} + \mathbf{E}, \mathbf{B} + \mathbf{D} + \mathbf{E}] \end{aligned}$$

The formal definition of this operation for an arbitrary number of arguments is given in (37). Informally, (37b) says that we consider all the different ways of choosing an element from each of the argument sets (represented by the function variable f) and, for each such choice, we sum up the selected elements.¹²

$$(37) \quad \begin{aligned} \text{The operation } \oplus_a : \mathcal{P}(D_a) \setminus \{\emptyset\} &\rightarrow D_a \text{ is defined for any type } a \text{ as follows:} \\ \text{a. For any type } a \text{ that is not of the form } b^*, &\text{ and any nonempty } S \subseteq D_a, \\ \oplus S &= \mathbf{+}S. \\ \text{b. For any type } b^* \text{ and any nonempty } S \subseteq D_{b^*}, &\oplus_{b^*} S = [X \mid \exists f : f \text{ is a} \\ &\text{function from } S \text{ to } D_b \wedge \forall X^* \in S : f(X^*) \in pl^{*-1}(X^*) \\ \wedge X &= \oplus_b(\{f(X^*) \mid X^* \in S\})] \end{aligned}$$

Armed with this definition, we can now give a semantics for conjunction that integrates well with our meanings for plural definites and indefinites:

$$(38) \quad \text{Notational convention: For any type } a \text{ and any } x, y \in D_a: x \oplus_a y = \oplus_a(\{x, y\}).$$

$$(39) \quad \llbracket \text{and}_{\langle a, \langle a, a \rangle} \rrbracket = \lambda x_a. \lambda y_a. x \oplus_a y \text{ for any type } a$$

(39) yields the results in (36). Only one piece is missing now: The ‘projection’ rule combining plural sets, which will also make use of the operation \oplus .

3.3 Adding plural projection to the compositional system

As sketched in Section 2.2, the projection rule considers all the different ways of matching parts of the functor plurality with parts of the argument plurality such that each atomic part is covered, and constructs a plurality of values for each such matching. We formalize this by defining the notion of a **cover** of (P, x) , defined in (40). This is a relation between atomic parts of P and atomic parts of x in which each atomic part of P and each atomic part of x occurs at least once. Some examples of covers for a simple functor-argument pair are given in (41).

¹² The recursion involved in applying \oplus , rather than $\mathbf{+}$, in (37b) is needed for the analysis of German *sowohl . . . als auch* ‘as well as’. For the examples analyzed here, it would be sufficient to use $\mathbf{+}$.

(40) Let $P \in D_a, x \in D_b$. A relation $R \subseteq A_a \times A_b$ is a **cover** of (P, x) iff $\mathbf{+}(\{P' \mid \exists x' : (P', x') \in R\}) = P$ and $\mathbf{+}(\{x' \mid \exists P' : (P', x') \in R\}) = x$.

(41) $P = \mathbf{smoke} + \mathbf{dance}, x = \mathbf{Ada} + \mathbf{Bea}$
 Covers: $\{\langle \mathbf{smoke}, \mathbf{A} \rangle, \langle \mathbf{dance}, \mathbf{B} \rangle\}, \{\langle \mathbf{smoke}, \mathbf{B} \rangle, \langle \mathbf{dance}, \mathbf{A} \rangle\},$
 $\{\langle \mathbf{smoke}, \mathbf{A} \rangle, \langle \mathbf{dance}, \mathbf{A} \rangle, \langle \mathbf{dance}, \mathbf{B} \rangle\}, \dots$

(42) defines the composition rule for plural sets. For each cover of some plurality in the functor set and some plurality in the argument set, we use regular functional application for all functor-argument pairs related by the cover. The values are ‘summed up’ via \oplus from (37). (In this paper, \oplus usually coincides with $\mathbf{+}$.)

(42) Cumulative Composition

- a. For any $P^* \in D_{\langle a, b \rangle^*}$ and $x^* \in D_{a^*}$:
 $\mathcal{C}(P^*, x^*) = [\oplus(\{P'(x') \mid (P', x') \in R\}) \mid \exists P \in pl^{*-1}(P^*), x \in pl^{*-1}(x^*) : R \text{ is a cover of } (P, x)]$
- b. For any meaningful expressions ϕ of type $\langle a, b \rangle^*$ and ψ of type a^* , $[\phi \ \psi]$ is a meaningful expression of type b^* , and $\llbracket \phi \ \psi \rrbracket = \mathcal{C}(\llbracket \phi \rrbracket, \llbracket \psi \rrbracket)$.

(43) illustrates the effect of this rule. For each plurality of two people, and each way of matching these people with the two predicates such that both predicates and both people are covered, we obtain a plurality of propositions in the output set.

- (43) a. *Two people are smoking and dancing.*
 b. $\mathcal{C}([\mathbf{smoke} + \mathbf{dance}])([\mathbf{Ada} + \mathbf{Bea}, \mathbf{Ada} + \mathbf{Gene}, \mathbf{Bea} + \mathbf{Gene}])$
 $= [\mathbf{smoke}(\mathbf{A}) + \mathbf{dance}(\mathbf{B}), \mathbf{smoke}(\mathbf{A}) + \mathbf{dance}(\mathbf{G}),$
 $\mathbf{smoke}(\mathbf{B}) + \mathbf{dance}(\mathbf{G}), \mathbf{dance}(\mathbf{A}) + \mathbf{smoke}(\mathbf{B}) \dots]$

Finally, we relate these plural sets of propositions to truth conditions via (44):¹³

(44) A plural set $p^* \in A_{t^*}$ of propositions is **true** in a world w iff there is a plurality $p \in pl^{*-1}(p^*)$ such that for all $q \leq_a p$, $q(w) = 1$, and **false** in a world w iff for all pluralities $p \in pl^{*-1}(p^*)$, there is a $q \leq_a p$ such that $q(w) = 0$.

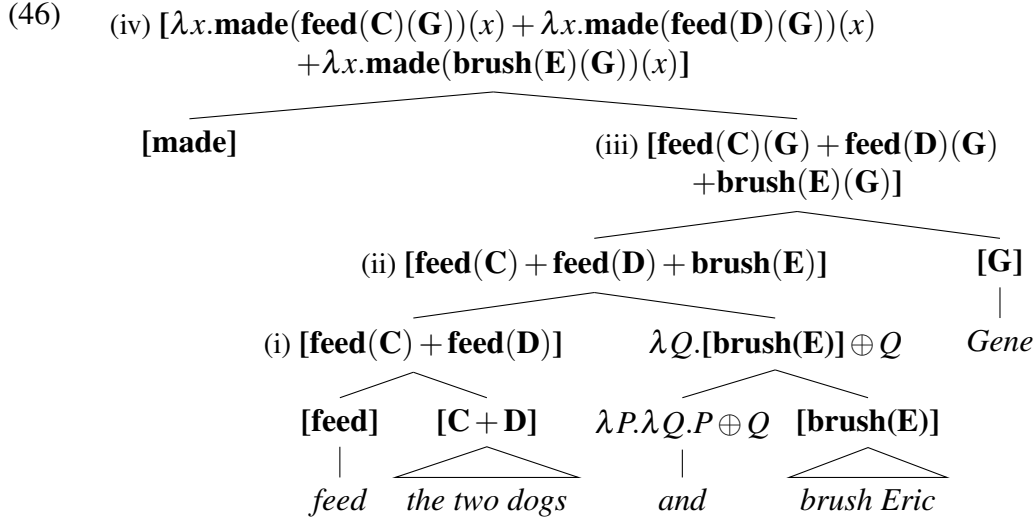
3.4 Deriving the flattening effect

As claimed in Section 2, the present system, as opposed to analyses with individual-level cumulation operators, derives the fact that (45) (= (21)) is true in scenarios like (20) above. The relevant part of the semantic composition is sketched in (46).¹⁴

¹³ We take it to be an open question how cumulativity interacts with presupposition projection and other cases of partiality, and how best to account for homogeneity in cumulative sentences.

¹⁴ We assume that the ordinary meaning of lexical expressions may be shifted to singleton plural sets via a freely available operation.

(45) *The two girls made Gene* $[[\text{feed the two dogs}]_P \text{ and } [\text{brush Eric}]_Q]$.



The meaning of the first VP conjunct, labeled (i), follows from the Cumulative Composition rule in (42). In this case, there is a unique cover and the functor **feed** is applied to each atomic part of the argument plurality. The resulting predicate sum is combined with the second conjunct by means of \oplus . Since there is only one way of choosing an element from each set, \oplus yields a ‘trivial’ singleton plural set, labeled (ii). In (iii), this set combines with the singleton plural set [**G**] via another application of Cumulative Composition, yielding the by now familiar projection behaviour. Next, we apply the meaning of the matrix predicate *made*. Strictly speaking, this is not covered by our current version of Cumulative Composition, but the rule can easily be extended to allow for intensional functional application (cf. Schmitt 2017). If so, and if the atomic parts of our pluralities are intensions, the resulting set (iv) contains a single predicate sum that can be cumulated with the sum of the two girls.

3.5 Interim summary

The main properties of the system that let us analyze the cumulative readings of examples like (45) were as follows: We posited pluralities and a sum operation for every semantic category. We then defined a mechanism that lets semantic plurality ‘project up’ in the syntactic tree: Any expression containing a semantically plural subexpression denotes a set of pluralities. This assumption let us reduce apparent cases of non-lexical cumulative relations to a series of local steps, rendering cumulation operators in the syntax and the corresponding LF movement obsolete.

We now show how all this can help us capture the behaviour of ADUs.

4 Analysis of ADUs and cumulativity asymmetries

Like conjunction and plural determiners, ADUs are analyzed as operators that directly manipulate plural sets of predicates, thus blocking application of the Cumulative Composition rule. Here we focus on *every/jeder*, but the approach extends to ADU conjunctions like *sowohl ... als auch* (Haslinger & Schmitt 2018).

The lexical entry for *every* is given in (47). Since *every* DPs can take any argument type of the form $\langle e, a \rangle^*$, they can combine with predicates of any arity.

- (47) a. For any $P_{\langle a, b \rangle, x_a}$: $\mathcal{D}(P, x) = \bigoplus(\{Q(x) \mid Q \leq_a P\})$
 b. $\llbracket \text{every}_{\langle \langle e, t \rangle^*, \langle \langle e, a \rangle^*, a^* \rangle} \rrbracket = \lambda P_{\langle e, t \rangle^*}^* . \lambda R_{\langle e, a \rangle^*}^* . [\bigoplus(\{\mathcal{D}(f(x), x) \mid x \in \mathcal{A}(P^*)\}) \mid f \text{ is a function from } \mathcal{A}(P^*) \text{ to } pl^{*-1}(R^*)]$

Put informally: When an *every* DP combines with a plural set R^* of predicates, we consider different functions that map each individual in the NP extension to an element of R^* . For each such function, we take every NP individual, apply all the predicates in its respective predicate plurality and sum up the results over all the individuals. Finally, all the sums obtained in this way are collected into a plural set.

4.1 Cumulativity asymmetries

Let us now use this lexical entry to derive the cumulativity asymmetry from Section 1. The crucial examples are repeated in (48) (with the context from (1) above).

- (48) a. *Every girl in this town fed two pets.* **distributive reading only**
 b. *The two girls in this town fed every pet.* **cumulative reading available**

We consider the derivation for (48a) first. Given our assumptions from Section 3, the VP denotes the plural set in (49a): The structure of the plural set $\llbracket \text{two pets} \rrbracket$ projects due to an application of Cumulative Composition. Definition (47) then requires that we consider all possible different functions from $\mathcal{A}(\llbracket \text{girl} \rrbracket)$ – the set of all atomic girls – to the set (49a). Two examples of such assignments are given in (49b).

- (49) a. $\llbracket \text{fed two pets} \rrbracket = [\mathbf{feed(C)} + \mathbf{feed(D)}, \mathbf{feed(C)} + \mathbf{feed(E)}, \mathbf{feed(D)} + \mathbf{feed(E)}]$
 b. $\{\langle \mathbf{A}, \mathbf{feed(C)} + \mathbf{feed(D)} \rangle, \langle \mathbf{B}, \mathbf{feed(C)} + \mathbf{feed(E)} \rangle\}, \{\langle \mathbf{A}, \mathbf{feed(C)} + \mathbf{feed(E)} \rangle, \langle \mathbf{B}, \mathbf{feed(D)} + \mathbf{feed(E)} \rangle\}, \dots$

Now, each such assignment gives us an element of our final plural set in the following way. For each pair in the assignment, we apply all the elements of the predicate plurality to the individual and sum up the results. We then sum up the results over all individuals, yielding a single plurality of propositions. Finally, all of these

proposition pluralities – corresponding intuitively to different assignments that map every girl to two pets – are collected into a set, as indicated in (50).

$$(50) \quad \llbracket \text{every girl} \rrbracket((49a)) = [\mathbf{feed(C)(A)} + \mathbf{feed(D)(A)} + \mathbf{feed(C)(B)} + \mathbf{feed(E)(B)}, \\ \mathbf{feed(C)(A)} + \mathbf{feed(E)(A)} + \mathbf{feed(C)(B)} + \mathbf{feed(D)(B)}, \\ \mathbf{feed(C)(A)} + \mathbf{feed(E)(A)} + \mathbf{feed(D)(B)} + \mathbf{feed(E)(B)}, \dots]$$

The crucial novelty here is that for each girl, some predicate sum must be applied ‘distributively’ to that girl, i.e. the girl must satisfy all predicates in the sum. This condition, implemented by the operator \mathcal{D} in (47a), ensures that each individual girl is related to two pets. Therefore, we don’t get pluralities like $\mathbf{feed(Dean)(Ada)} + \mathbf{feed(Carl)(Bea)}$, and the cumulative reading of (48a) is blocked.

Next, consider (48b), where the *every* DP occurs in object position. First, we apply $\llbracket \text{every pet} \rrbracket$ to the singleton set $[\mathbf{feed}]$. In this case, there is only one assignment of predicate sums to the individual pets, since we have only one predicate to assign. Definition (47) therefore yields a singleton set, shown in (51a). Importantly, since this set contains a plurality, it can combine with the subject plurality via Cumulative Composition. We end up with a set, partially shown in (51b), which contains all sums of propositions of the form $\mathbf{fed}(x)(y)$ that ‘cover’ every pet and also ‘cover’ both Ada and Bea – exactly what we need for the cumulative reading.

$$(51) \quad \text{a. } \llbracket \text{every pet} \rrbracket(\llbracket \text{fed} \rrbracket) = \llbracket \text{every pet} \rrbracket([\mathbf{fed}]) = [\mathbf{feed(C)} + \mathbf{feed(D)} + \mathbf{feed(E)}] \\ \text{b. } \mathcal{C}((51a), [\mathbf{A} + \mathbf{B}]) = [\mathbf{feed(C)(A)} + \mathbf{feed(D)(A)} + \mathbf{feed(E)(B)}, \mathbf{feed(C)(B)} \\ + \mathbf{feed(D)(A)} + \mathbf{feed(E)(A)}, \mathbf{feed(C)(B)} + \mathbf{feed(D)(A)} + \mathbf{feed(E)(B)}, \dots]$$

Thus, the plural projection framework allows us to define a denotation for *every* that predicts cumulativity asymmetries. It remains to be shown that the analysis can also deal with the interaction between distributivity and cumulativity in Schein sentences.

4.2 Schein sentences

Recall that the sentence in (52) (= (11)) is true in scenario (10) above.

$$(52) \quad \text{Ada and Bea taught every dog two new tricks.} \quad \mathbf{true} \text{ in (10)}$$

The predicate *taught two new tricks* denotes the plural set in (53), due to Cumulative Composition and our analysis of plural indefinites. When we combine this with *every dog*, we must consider all possible assignments that map each dog to a plurality from that set. Crucially, Carl and Dean may be mapped to different elements of (53), which accounts for the ‘distributive’ interpretation of *every dog* relative to *two new tricks*. For each assignment of predicate pluralities to the two dogs, the results of functional application are summed up, yielding the plural set indicated in (54).

- (53) $\llbracket \textit{taught two new tricks} \rrbracket = \mathcal{C}(\llbracket \textit{taught} \rrbracket, \llbracket \textit{two new tricks} \rrbracket) = [\textit{taught}(\mathbf{T1}) + \textit{taught}(\mathbf{T2}), \textit{taught}(\mathbf{T1}) + \textit{taught}(\mathbf{T3}), \textit{taught}(\mathbf{T2}) + \textit{taught}(\mathbf{T3})]$
- (54) $\llbracket \llbracket \textit{every dog} \rrbracket \textit{taught two new tricks} \rrbracket = [\textit{taught}(\mathbf{T1})(\mathbf{C}) + \textit{taught}(\mathbf{T2})(\mathbf{C}) + \textit{taught}(\mathbf{T2})(\mathbf{D}) + \textit{taught}(\mathbf{T3})(\mathbf{D}), \textit{taught}(\mathbf{T1})(\mathbf{D}) + \textit{taught}(\mathbf{T2})(\mathbf{D}) + \textit{taught}(\mathbf{T2})(\mathbf{C}) + \textit{taught}(\mathbf{T3})(\mathbf{C}), \dots]$

The plural set in (54) then combines via Cumulative Composition with **[Ada + Bea]**. So the sentence is true if there is a predicate plurality P in (54) such that Ada and Bea each satisfy at least one atomic part of P , and each atomic part of P is satisfied by Ada or Bea. This corresponds to our basic idea for Schein sentences from Section 2.

4.3 Interim summary

We showed how the plural projection system accounts for the data pattern described in Section 1. The main ingredient is a lexical entry for *every* that allows *every* DPs to manipulate plural sets of predicates. Each atom in the NP denotation is assigned a predicate plurality. While these predicate pluralities apply ‘distributively’ to the respective individuals, the results of these applications are collected into sums again, resolving the tension between distributivity and cumulativity in Schein sentences.

5 Conclusion and open problems

We presented a new analysis of what we call ADUs – elements that can only cumulate with syntactically ‘higher’ plural expressions – and spelled out the details of this analysis for English *every* DPs. The system it is based on derives cumulativity in a ‘step-by-step’ process, without appealing to syntactically derived cumulative relations. It does so by (i) including ‘higher-type’ pluralities and sets thereof, (ii) assuming that nodes containing semantically plural expressions can inherit the part structure of the embedded plurality and (iii) encoding cumulativity in this ‘projection’ rule. *Every* DPs combine with sets of predicate pluralities and apply these predicate pluralities to the individuals in their restriction in a ‘distributive’ manner. Crucially, the resulting values are pluralities that are accessible for further cumulative composition. This derives the basic cumulativity asymmetries, but also the behavior of ADUs in Schein sentences. We argued that the analysis presented here is more general in its scope than previous analyses of cumulativity asymmetries.

At this point, however, the system does not extend to collective predicates or (like previous proposals) to non-upward-monotonic quantifiers. On the empirical side, more research is needed regarding the basic asymmetries, which we here took to be tied to scope (following [Champollion 2010](#)), but which, at least in some languages, seem to be subject to more complex restrictions (see [Flor 2017](#)).

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Nina Haslinger
 Universität Wien
 Universitätsring 1
 1010 Wien/Vienna, Austria
ninamhaslinger@gmail.com

Viola Schmitt
 Universität Wien
 Universitätsring 1
 1010 Wien/Vienna, Austria
viola.schmitt@univie.ac.at