Reciprocals and Cumulation

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1. Reciprocal Readings

A reciprocal sentence like (1), where the antecedent of the reciprocal denotes a group with just two members, has only one interpretation: In order for (1a) to be true, the first member of the antecedent group likes the second and vice versa.

(1)  
   a.  Mary and John like each other.
   b.  Mary likes John and John likes Mary.

Things get a little more complicated when we look at larger antecedent groups. It seems that in order for (2a) to be true, every member of the antecedent group has to like every other member of the antecedent group.

(2)  
   a.  The children like each other.
   b.  Each child likes every other child.

This suggests a twofold universal quantification over members of the antecedent group. This reading is known in the literature as a strongly reciprocal interpretation (Langendoeen's (1978) name for Fiengo and Lasnik's (1973) each-the-other relation). I give the abstract schema for strong reciprocity in (3).

(3)  strong reciprocity:
     \[ \forall x \in A: \forall y \in A \ [ y \neq x \rightarrow xRy ] \]

A is the meaning of the antecedent of the reciprocal and R is the reciprocal relation (like in the example in (2)).

While strong reciprocity (SR) accurately describes the truth conditions of (2a), there are examples where SR is too strong. That is, there are reciprocal sentences that are judged to be true even though SR cannot be met. (4a) is an example. Imagine that the children are standing in a long line, or in a circle. It is not the case that everyone is touching everyone else. The intuitive truth conditions are stated in (4b).

(4)  
   a.  The children are touching each other.
   b.  Each child touches, and is touched by, at least one other child.

This reading is also well-known in the literature, under the name of weak reciprocity (WR) (Langendoeen (1978)). The abstract schema for the truth conditions of a weakly reciprocal interpretation are given in (5).

(5)  weak reciprocity:
     \[ \forall x \in A: \exists y \in A [ xRy \land x \neq y ] \land \forall y \in A: \exists x \in A [ xRy \land x \neq y ] \]

I will assume that both SR and WR are genuine readings of reciprocal sentences. I take it that both (2) and (4) are in principle ambiguous, and pragmatic factors decide

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which reading is favoured. For the time being, I will ignore any other interpretations of reciprocal sentences suggested in the literature (compare e.g. Dalrymple et al. (1998)). The goal of this paper is to develop a theory of how the two readings are compositionally derived. This involves decisions on the semantic contribution of the reciprocal itself and on how this interacts with plural predication. I first discuss the Heim, Lasnik and May (1991a,b) (henceforth HLM) approach, which accounts for SR but not WR (section 2). There is a recent proposal by Sternefeld (1998) that derives WR. This proposal as well as some problems it runs into are discussed in section 3. Section 4 proposes an analysis that combines aspects of HLM with aspects of Sternefeld's story and, I argue, arrives at a coherent overall picture of the semantics of reciprocals.

2. Strong Reciprocity

2.1. Heim, Lasnik and May

The version of an HLM analysis I will introduce here departs in some ways from HLM (1991a). It doesn't associate either the antecedent or the reciprocal with any quantificational force. Rather, general mechanisms of plural predication are blamed for the two-way universal quantification we observe in SR. In this, the analysis I will introduce is closer to HLM (1991b), and also to Heim (1994) and Sauerland (1998). I take the liberty to rephrase the analyses presented there somewhat to suit my intensions in this paper better. The underlying intuition about how semantic composition works remains the same.

Let us first note some general facts about plural predication, i.e. about how we attribute properties to groups, and when we can say that a relation holds between two groups. Consider (6).

(6) a. The children are asleep.
   b. Each of the children is asleep.

A predicate like asleep can only be a property of singular individuals, not of groups. When we attribute asleep of a group, this can only mean that all members of that group are asleep. Similarly, (7) has a reading in which the sentence is true iff each member of the subject group read each member of the object group.

(7) a. Sue and Amy read 'Fried Green Tomatoes' and 'The L-Shaped Room'.
   b. Each of Sue and Amy read each of FGT and L.

This is the phenomenon of distributivity. I will use the familiar * operator from Link's (1983) work to account for distributivity. The operator is defined in (8).

(8) *distribution:
* is that f: D<e,t> -> D<e,t> such that for any h in D<e,t> and any x in D:
   [*f](x)=1 iff [f(x) =1 or \exists u,v[x=u\&v \& [*f](u) \& [*f](v)]]

Note that my notation for group formation is '&'. I will assume the kind of algebraic structure that Link (1983) suggested for groups, but remain neutral with respect to the sets vs. mereological sums discussion (compare e.g. Schwarzschild (1996) for discussion). I will informally talk about groups as sets of individuals, though. Now back to distributivity and (6). (6) will be assumed to have a Logical
Form as in (9a). This translates to (9b). If we assume that in the basic extension of the predicate asleep, there are only singular individuals, (9b) is equivalent to (9c).

(9)  
   a.  \[(\text{the children} \ [ \ * \ [ 1 \ [ t1 \ is \ asleep]]])\]  
   b.  \[C \in \ *\lambda x[x \ is \ asleep]\]  
   c.  \[\forall x[x \in C \rightarrow x \ is \ asleep]\]  

(7) is analogous, except for the complicating factor that we have to distribute over the object argument as well as the subject argument. I will assume that the sentence under the reading described has a Logical Form as indicated in (10a). A translation is given in (10b). Under the assumption that only singularities are in the basic extension of read, this is equivalent to (10c).

(10)  
   a.  \[(\text{Sue and Amy} \ [ \ * \ [ 1 \ ([\text{FGT and L} \ [ \ * \ [ 2 \ [ t1 \ read \ t2]])]]])\]  
   b.  \[S\&A \in \ *\lambda x[ \ FGT\&L \in \ *\lambda y[ x \ read \ y]]\]  
   c.  \[\forall x[x \in S\&A \rightarrow \forall y[y \in FGT\&L \rightarrow x \ read \ y]]\]  

Thus, distribution accounts for the readings we observe for (6) and (7), and in particular is the source of the universal quantification we observe in the paraphrases we give to such sentences. In (11a), I state the schema for the truth conditions of a sentence under a doubly distributive reading. The analysis of this in terms of the * operator is given abstractly in (11b).

(11)  
   a.  \[\forall x[x \in A \rightarrow \forall y[y \in B \rightarrow xRy]]\]  
   b.  \[A \in \ *\lambda x[ B \in \ *\lambda y[ xRy]]\]  

Accordingly, we will assume that the universal quantifiers that occur in the schema for SR have the same source. That means that both the reciprocal and the antecedent denote groups, which we then distribute over. The reciprocal must denote a group in dependence on the antecedent. HLM suggest that this group is 'the other ones among them', where them is coreferential with the antecedent. That is, their theory is guided by the paraphrase in (12b). If we add to this the effects of double distribution, the paraphrase amounts to (12c,d), strong reciprocity.

(12)  
   a.  Mary, Sue and Bill saw each other.  
   b.  Mary, Sue and Bill saw the other ones among Mary, Sue and Bill.  
   c.  Each of Mary, Sue and Bill saw every other one of Mary, Sue and Bill.  
   d.  \[\forall x \ [x \in MSB \rightarrow \forall y \ [y \in MSB \& y \neq x \rightarrow x \ saw \ y]]\]  

The idea is, then, that the reciprocal denotes a group that contains all the members of the antecedent, minus the individual we are looking at in terms of distribution (Mary must have seen everyone among Mary, Sue and Bill minus Mary, Sue everyone minus Sue etc.). The reciprocal incorporates two anaphoric dependencies: one is coreference with the antecedent, the other is dependence on the variable bound in the distribution over the antecedent. If x1 is the variable and Pro3 is the hidden pronoun coreferent with the antecedent, this can be written as in (13).

(13)  each other = the other one(s) among them  
\[\max(*\lambda z[z \neq x1 \& z \leq x3 \& \text{person(z)}])\]
I represent this as (14).

(14)  \[
\text{max} \left[ \left[ \text{other } x_1 \right] \left( \text{of } \text{Pro3} \right) \right]
\]

If this is the representation of the reciprocal, compositional interpretation will yield the group meaning that we need: \textit{other} denotes non-identity, which applied to \( x_1 \) is the set of all objects different from \( x_1 \). This combines with the set of parts of the antecedent of the reciprocal intersectively. HLM suggest that there is a silent \textit{ones} in the reciprocal, which could plausibly pick up a meaning like \textit{person} in our example. We again intersect, and then look for the largest group each member of which has all three properties. The definition of the maximality operator is given in (15).

(15)  \[
\text{Let } S \text{ be a set ordered by } \leq \text{ Then } \text{max}(S) = \{ s \in S \& \forall s' \in S \left[ s \leq s' \right] \}
\]

The maximality operator captures the plural definite in the paraphrase, following Sharvy (1980) and Link (1983). According to these assumptions, the LF for (12) will be (16):

(16)  \[
\left[ \text{Mary, Sue and Bill} \right]^3
\left[ \left[ \text{1} \left[ \text{max} \left[ \left[ \text{other } x_1 \right] \left( \text{of } \text{Pro3} \right) \right] \left[ \text{2} \left[ t_1 \text{ saw } t_2 \right] \right] \right] \right] \right]
\]

This LF straightforwardly translates to (17a). The maximum operator takes as its argument the set of all person parts of Mary, Sue and Bill that are not identical to \( x \). That set itself would have two members who are individual people, hence it would not have a maximum. The star operator corresponds to closure under group formation, hence the *-ed set will also contain the group that has those two people as its members. This is the maximum. I will write that group as as MSB-*x ((17b)). Suppose that in the basic extension of see, we only have singularities. Then (17b) will amount to (17c), the desired strongly reciprocal interpretation of the sentence.

(17)  \[
a. \quad \text{MSB} \in \lambda x \left[ \text{max}\left( \lambda z \left[ z \neq x \& z \leq \text{MSB} \& \text{person}(z) \right] \right) \right] \in \lambda y \left[ x \text{ saw } y \right]
\]
\[
b. \quad \text{MSB} \in \lambda x \left[ \text{MSB} -x \in \lambda y \left[ x \text{ saw } y \right] \right]
\]
\[
c. \quad \forall x \left[ x \in \text{MSB} \rightarrow \forall y \left[ y \in \text{MSB} \& y \neq x \rightarrow x \text{ saw } y \right] \right]
\]

A nice feature of this analysis is that it reduces the properties of the reciprocal construction as far as possible to properties of plural predication. The only aspect specific to reciprocal interpretation comes from the semantic contribution of the reciprocal itself.

In the next subsection, we will refine this analysis with respect to the role of the * operator. One desirable effect will be that we don't need any further suppositions to make (17b) equivalent to (17c).

2.2. \textit{Heim, Lasnik and May plus Covers}

Schwarzschild (1996) argues that we can only distribute down to subgroups that are contextually salient. That is, there is no such thing as our unrestricted distributivity operator from the last subsection. Instead, distribution is sensitive to a partition of the universe of discourse into salient subpluralities, and is restricted to those. I will first give some motivation for this and then choose a particular way of incorporating Schwarzschild's idea. Consider (18).
(18)  
  a. The cows and the pigs filled the barn to capacity.
  b. The female animals and the male animals filled the barn to capacity.

(18a) can mean that the cows filled the barn to capacity, and so did the pigs. This is a distributive reading of the sentence. (18b) cannot normally be understood in this way, i.e. it is not naturally taken to mean that the cows filled the barn to capacity and so did the pigs. Actually, this is unexpected. Suppose that the cows and the pigs are all the animals there are, and this is the same group as the female and the male animals. The two subjects in (18a) and (18b) then refer to the same group. The two VPs are obviously identical. But then, it is rather mysterious where the interpretational difference between the two should come from. Notice that the difference is which subgroups we distribute down to. (18b) is much more naturally taken to mean that the female animals filled the barn to capacity, and so did the male animals. That is, we divide up the group denoted by the subject NP in different ways, and how we divide up seems to depend on how we mention the group.

Schwarzschild suggests therefore that distribution is sensitive to a division of the universe of discourse into salient subgroups, a cover. The definition of cover is given below.

(19)  
  C is a cover of P iff
  C is a set of subsets of P
  Every member of P belongs to some set in C
  {} is not in C

Since I implement distribution via *, I propose to restrict the * operator by the contextually salient cover. This is also proposed in Heim (1994) and Sauerland (1998). A free variable ranging over covers will appear in my LFs from now on whenever there is a * operator, representing the sensitivity to context. The LF for (18) is given in (20). The free variable Cov occurs in the sister of the * operator. It is interpreted intersectively with the predicate 'fill the barn to capacity'. The result is the set of all salient groups that filled the barn to capacity. Hence, the new *-ed predicate will apply to the cows and the pigs just in case these animals can be divided into salient subgroups that filled the barn to capacity.

(20)  
[[The cows and the pigs] [ * [ Cov [ filled the barn to capacity]]]]

Obviously, what exactly this means depends on what the context assigns as an interpretation to the free cover variable. Suppose the salient cover is as in (21a). Then (20) means that the cows filled the barn to capacity, and so did the pigs. Suppose on the other hand that the contextually salient cover was as in (21b). We would then get the reading that is prominent in (18b).

(21)  
  a. Cov\{[the animals]\} = {[[the cows]], [[the pigs]]}
  b. Cov\{[the animals]\} = {[[the female animals]], [[the male animals]]}

Assume that mentioning the cells in a particular cover explicitly is enough to make that cover salient. We then have an account of the interpretational difference in (18): The way we mention the subject group makes different partitions into subgroups salient. Distribution is to those salient subgroups, because the * operator is restricted to salient covers.

Restricting distribution to salient subpluralities is useful even when there are no such interesting subpluralities around as the cows and the pigs. Consider (22a) (similar data are discussed for example in Lasersohn (1995) and Schwarzschild (1996)).
(22)  
   a.  Jim, Ed and Sue make $7000.-.
   b.  [[[Jim, Ed and Sue] [ * [ make $7000.- ]]]]

Suppose that Jim and Ed together make $7000.-, and Sue by herself also makes $7000.-. (22a) is not a very good way to describe this situation - most people judge the sentence to be false. Yet, if we use the unrestricted * operator to account for distributivity, we predict the sentence to be true in the situation described. This is because there is a way to divide up Jim, Ed and Sue into subgroups each of which is in the extension of 'make $7000.-'. Suppose on the other hand that the sister of * contains a restriction to salient subpluralities, a free cover variable, as in (23a).

(23)  
   a.  [[[Jim, Ed and Sue] [ * [ Cov [ make $7000.- ]]]]
   b.  Cov1JES = \{\{J\}, \{E\}, \{S\}\}
       Cov2JES = \{\{JES\}\}
   c.  \forall x \ x \in JES -> x makes $7000.-
       \forall x \ x \in \{JES\} -> x makes $7000.-

(23a) says that Jim, Ed and Sue can be divided up into salient subpluralities that make $7000.-. Which subpluralities can we expect to be salient? There is no indication of any interesting way to divide up our group. Schwarzschild suggests, and we follow him here, that in this case, two trivial covers of a plurality are salient. One divides it up into singularities, and the other cover is a one-membered set whose only member is the group itself. If these are assigned to the free variable in (23a) we get the collective reading and a reading that distributes to singular individuals, respectively. Those are the intuitively available readings. Thus, if there is no indication that non-trivial subpluralities are relevant, such subpluralities cannot be used to make a plural predication come out true.

We will henceforth assume that the cover restriction associated with each * operator is a set of singularities (collective readings are irrelevant, and we will not discuss interesting subgroups). A more complete LF for (16) for example will look like (24):

(24)  
   a.  Mary, Sue and Bill saw each other.
   b.  [[[Mary, Sue and Bill]3 *[Cov[ 1
       [ [max[*[Cov[[other x1] (of) Pro3]] *[Cov[ 2 [t1 saw t2]]]]]]]
   c.  MSB ∈ *λx[Cov(x) &
       max(*λz[z≠x & z≤MSB & Cov(z)]) ∈ *λy[Cov(y) & x saw y]]

Since both x and z range over singularities only (given our assumptions about the cover), we get (25a). Now the distribution will be guaranteed to be down to singularities, and (25a) is equivalent to (25b).

(25)  
   a.  MSB ∈ *λx[Cov(x) & MSB-x ∈ *λy[Cov(y) & x saw y]]
   b.  \forall x \ x \in MSB -> \forall y \ y \in MSB & y≠ x -> x saw y]

This is the slightly updated HLM story I want to tell about SR. The logical structure of strong reciprocity as I state it is given in (26a). If we presuppose that x and y are singularities, this is the same as (26b), and the same as SR as stated in section 1.
(26) a. \( A \in \lambda x[\text{Cov}(x) \& A - x \in \lambda y[\text{Cov}(y) \& xRy]] \)
   b. \( A \in \lambda x[\text{A-x \in \lambda y[ xRy]]} \)
   c. \( \forall x \in A : \forall y \in A \ [y \neq x \rightarrow xRy] \)


3.1. Weak Reciprocity and Cumulation

Sternefeld (1998), following Langendoen (1978), observes that deriving an SR interpretation for reciprocal statements is not sufficient. We need to derive the truth conditions of WR discussed in section 1 for data like (27).

(27) a. The children are touching each other.
   b. \( \forall x \in C : \exists y \in C [x \text{ touch } y \& x \neq y] \& \forall y \in C : \exists x \in C [x \text{ touch } y \& x \neq y] \)

A key observation for Sternefeld's analysis of WR is Langendoen's insight that similarly to SR being parallel to double distribution, there is an analogue to WR when we look at relational plurals. Let us reconsider (28a). The doubly distributive reading of this sentence which we described in the last section is not the only possible interpretation of the sentence. It can also be judged true under weaker conditions, the reading paraphrased in (28b). Such a reading is brought out unambiguously if we add a respectively to the sentence.

(28) a. Sue and Amy read 'Fried Green Tomatoes' and 'The L-Shaped Room'.
   b. Each of the women read one of the books, and each of the books was read by one of the women.

In abstract terms, a relational plural sentence allows for an interpretation schematized in (29). I will refer to this with Sternefeld as the cumulative reading. Sternefeld (following Krifka (1986)) introduces a new pluralization operation that pluralizes two argument slots simultaneously - cumulation as defined in (30).

(29) \( \forall x \in A : \exists y \in B: xRy \& \forall y \in B : \exists x \in A : xRy \)

(30) - cumulation:
   ** is that function: \( D<e,\leq,e,\geq,t> \rightarrow D<e,\leq,e,\geq,t> \) such that for any R:
   \( \exists x_1 x_2 y_1 y_2 [x=x_1 \& x_2 \& y=y_1 \& y_2 \& \text{R}(y_1)(x_1) \& \text{R}(y_2)(x_2)] \)
   The reading of (28a) we discovered in (28b) can now be derived via an LF as in (31a). This translates to (31b).

(31) a. \( \text{[S & A]} \ [\text{[**read]} \ [	ext{FGT} \& \text{L}]]] \)
   b. \( \text{**y}\lambda x[x \text{ read } y] \) (FGT&L) (S&A)

(31b) amounts to a cumulative reading: we collect into a group things that occur in the domain of the read relation. Then we collect into a group those things that occur in the range of the read relation with them. The cumulated relation **read holds
between those groups. That means that for everything in the domain group, there must be something in the range group that it stands in the original read relation to, and vice versa. Hence we get the funny \( \forall \exists \) quantificational effect.

Compare the abstract truth conditions for a cumulative reading to those of WR. Once more, there is a clear parallel in how we quantify over parts of the two pluralities we are talking about in the sentence, in this case, the \( \forall \exists \) effect. We blame pluralization operations for the quantifiers that occur in such paraphrases. Hence, Sternefeld suggests, we ought to derive WR via the workings of \( \pi \). He proposes an LF for (27a) that looks essentially like (32a).

\[
\begin{align*}
&\text{a. } \left[ [\text{The children}] \ [\pi [\text{other touch}] [\text{Pro} \ (\text{other})] ] \right] \\
&\text{b. } \pi\lambda y\lambda x [x \text{ touches } y \land x \neq y] (C) (C)
\end{align*}
\]

(32) incorporates a particular view of the role of the reciprocal besides using \( \pi \). Notice that the cumulated relation holds, not between two independent groups, but between the antecedent group and that same group. This reflects the anaphoric nature of the reciprocal. Let us now look more closely at the reciprocal relation, the relation that cumulates. Obviously, this must include touch; however, we want to cut out the reflexive part of the touch relation. That is, the reciprocal statement is not made true by every child touching herself or himself. Hence we combine a non-identity statement with the original relation and get something like 'other-touch'.

On this view, then, the reciprocal makes two independent semantic contributions. One is anaphoricity, and the other is a non-identity condition which must be combined with the reciprocal relation by way of some kind of generalized predicate modification that allows 'intersection' of two relations. Accordingly, the reciprocal is split up at LF into these two components, which are interpreted independently. Compare this to the HLM view of the role of the reciprocal: there, too, the reciprocal contributed non-identity and anaphoricit, but those two contributions were combined within one constituent and led to the description of a particular group.

(33) schematizes the Sternefeld analysis of WR:

\[
\begin{align*}
&A, A \in \pi \lambda y \lambda x [x \neq y \land x \text{ touches } y] &
\end{align*}
\]

Next, we will look at some problems for this approach.

3.2. Problems for Sternefeld

While I like the reduction of WR to \( \pi \) very much, I see two kinds of problems for this particular way of doing this. One kind has to do with the range of reciprocal readings predicted, and the other has to do with the interaction of the non-identity statement with other operators. Both in effect concern the treatment of the non-identity statement proposed by Sternefeld. We will examine the two cases in turn.

Notice first that the whole NP each other does not receive a denotation here - there is no such thing as a group denoted by the reciprocal (the group we informally called A-x above). If we are right about how SR is derived (via double distribution), this group is needed. This argument is only as strong as your belief in the analysis of SR presented above, obviously. Alternatives are conceivable. But I think the point regarding the existence of A-x can be made stronger. Consider (34):
(34)  a.  Our committees are made up of each other.
    b.  For each \( x \), \( x \) is one of us: \( x \)'s committee consists of the other ones among us.
    c.  \( \text{our}_3 \left[ \ast \text{Cov}_4 \left[ t_4 \text{\'s committee is made up of} \right] \right] \)
        \( \left[ \max \left[ \ast \text{Cov}_4 \left[ \text{other} x_4 \text{ (of) Pro}_3 \right] \right] \right] \)
    d.  \( \forall x \in [[\text{we}]] : x \text{\'s committee is made up of } [[\text{we}]] - x \)

(34a) exhibits a reading that is collective with respect to the group denoted by the reciprocal. That reading is paraphrased in (34b) and derived via the LF in (34c). There is no distribution over the object argument slot here at all, the reciprocal enters composition as a group directly. That group must be, essentially, 'the rest of us', in this example. Other data that illustrate this collective reading of reciprocals, taken from Dalrymple et al. (1998), are given in (35).

(35)  a.  The satellite, called Windsock, would be launched from under the wing of a B-52 bomber and fly to a 'liberation point' where the gravitational fields of the Earth, the Sun and the Moon cancel each other out.
    b.  The forks are propped against each other.

For (35b), for instance, imagine three forks. Each one is jointly supported by the other two, but not by any single other fork. (36) presents the abstract truth conditions for this collective reading of the reciprocal. We only distribute over the antecedent argument slot, and don't pluralize the argument slot of the reciprocal in any way at all.

(36)  - collective reading
    a.  \( \forall x \in A : xR(A-x) \)
    b.  \( A \in \ast \lambda x \left[ xR(A-x) \right] \)

These data show that the group \( A-x \) is needed and must be the meaning or one possible meaning of the reciprocal. Sternefeld's analysis of the reciprocal does not provide us with a natural way of getting that group. He does make a suggestion for how to derive such a meaning, which I do not want to discuss in any detail. I think it amounts to an ambiguity hypothesis: the reciprocal can either be treated in the Sternefeld fashion or as a HLM reciprocal, with various representational and semantic differences between the two. This is a little unsatisfactory. There ought to be a natural relation between all readings that reciprocals allow for.

The second kind of problem concerns the interaction of reciprocity with scope bearing elements like negation and quantifiers. A relevant example is (37).

(37)  They don't like each other.

Sternefeld would presumably predict that negation can take either wide or narrow scope relative to cumulation. The wide scope reading of negation is given in (38a). For the narrow scope reading, two LFs are conceivable: (38b) and (38c).

(38)  a.  \( \neg \left[ <[[\text{they}]],[[\text{they}]]> \in **\lambda y \lambda x [x \neq y \land x \text{ likes } y] \right] \)
    b.  \( <[[\text{they}]],[[\text{they}]]> \in **\lambda y \lambda x [\neg [x \neq y \land x \text{ likes } y] \)
    c.  \( <[[\text{they}]],[[\text{they}]]> \in **\lambda y \lambda x [x \neq y \land \neg [x \text{ likes } y]] \)
Suppose we just have a two-membered group, for simplicity. Then (38a) is compatible with one person liking the other, but denies that this is mutual. The narrow scope reading is stronger, it describes mutual dislike. I think the latter reading is the more plausible interpretation of (37). The formula that accurately represents the stronger reading is (38c). (38b), actually, is a tautology. It says that the referent of they can be divided up into subgroups x and y that make the formula \(-[x \neq y \& x \text{ likes } y]\) true; or equivalently: the referent of they can be divided up into subgroups x and y that make the formula 'x=y' true or 'x likes y' false. For this it is enough to choose identical subgroups of [[they]]. This is always possible. Notice that in (38b) the non-identity condition is interpreted in its overt position. This is what we assumed above when we discussed Sternefeld's analysis: we first create a relation 'other-like' and then combine that with the rest. While he could presumably raise the non-identity statement out of the scope of negation to derive (38c), we would in addition need some stipulation that excludes an LF corresponding to (38b).

Notice that it is not the case that the reciprocal necessarily takes wide scope. (39a) is ambiguous: It might mean either (39b) or (39c).

(39) a. Mary and Sue introduced no one to each other.
   b. There is nobody such that Mary introduced him to Sue and Sue introduced him to Mary.
   c. Mary didn't introduce anybody to Sue and Sue didn't introduce anybody to Mary.

(40a) and (40b) are the semantic representations associated with these two readings on the Sternefeld analysis:

(40) a. \(-\exists z <\text{M&S, M&S}> \in **\lambda y \lambda x [x \neq y \& x \text{ introduced } z \text{ to } y]\)
   b. <\text{M&S, M&S}> \in **\lambda y \lambda x [x \neq y \& \exists z [x \text{ introduced } z \text{ to } y]]
   c. <\text{M&S, M&S}> \in **\lambda y \lambda x \neg \exists z [x \neq y \& x \text{ introduced } z \text{ to } y]

(40c) is once more a tautological reading we mistakenly predict if the non-identity condition is in the scope of the negation. The point of the example is that there is interaction between the interpretation of the reciprocal and the quantifier (a claim to the contrary is found in Moltmann (1992), but I think that that is not true, in the light of data like (40)). In particular, the reciprocal can end up inside or outside the scope of a negative operator. But the non-identity statement by itself doesn't seem to be able to be inside the scope of negation while the rest of the reciprocal is outside.

The same ambiguity is found in (41).

(41) a. Mary and Sue only introduced BILL to each other.
   b. There is no x other than Bill such that Mary introduced x to Sue and Sue introduced x to Mary.
   c. Mary introduced Bill and no one else to Sue, and Sue introduced Bill and no one else to Mary.

(41) also presents the same problem to the current analysis of WR. Other downward monotonic operators will have a similar effect. This might make us question Sternefeld's treatment of the non-identity condition. Just looking at downward monotonic expressions, though, leaves the possibility open that it is simply so implausible to interpret an utterance as a tautology when there are other
possibilities, that one will always disregard the tautological interpretation in favour of less trivial readings. Therefore, let me provide a slightly less obvious example in which the truth conditions wrongly predicted by leaving the non-identity condition in situ are not tautological - (42a) below:

(42) a. The four professors introduced exactly two students to each other.
   b. There are exactly two students that the professors introduced to each other.
   c. The four professors stand in a reciprocal relation of introducing exactly two students.

(42a) has the two readings paraphrased in (42b,c). We are interested here in the narrow scope reading of 'exactly two students', hence in the correct formalization of (42c). (43) is the Sternefeld representation of this reading in which we leave the non-identity condition in situ:

(43) \(<P,P> \in **\lambda y \lambda x[\text{card}(\lambda z[x\neq y & \text{student}(z) & x \text{ introduced } z \text{ to } y])=2]\)

(43) says that there must be a division of P (the group referred to by 'the four professors') into subgroups that makes (44) true:

(44) \(\text{card}(\lambda z[x\neq y & \text{student}(z) & x \text{ introduced } z \text{ to } y])=2\)

That is, there must be a division of P into subgroups such that exactly two objects z make (45) true:

(45) \(x\neq y & \text{student}(z) & x \text{ introduced } z \text{ to } y\)

Imagine the following situation: the four professors are a, b, c and d; a introduced exactly one student to b and vice versa; c introduced exactly one student to d and vice versa. Since all in all four students were introduced, (42a) is intuitively false. Yet, we can find a division of a&b&c&d such that there are exactly two z that make (45) true. Here is how: we divide <P,P> into <a&b,a&b> and into <c&d,c&d>, and then further into <a,a>, <b,b>, <c,d> and <d,c>. Only the c,d - pairs will make (45) true, hence exactly two students z fit the description in (45).

This shows that the truth conditions assigned in this way are too weak. Yet the statement expressed in (43) is not tautological: it is false, for example, if fewer than two students were introduced. A correct formalization in Sternefeld's framework once more puts the non-identity statement outside the scope of the other operator:

(46) \(<P,P> \in **\lambda y \lambda x[x\neq y & \text{card}(\lambda z[\text{student}(z) & x \text{ introduced } z \text{ to } y])=2]\)

Obviously, similar problems arise with other operator that require us to count. The generalization over these data seems to be the following: the non-identity condition 'x\neq y' must be in the immediate scope of the binders of x and y; that is, there must be no other intervening operator between the abstraction over x and y and cumulation, and the non-identity condition. Intuitively, we always want to divide up the antecedent of the reciprocal into non-identical subgroups that make the reciprocal relation true. I will refer to this as the 'non-identical subgroups effect'. Sternefeld's analysis does not seem to provide a natural explanation for the non-
identical subgroups effect, given that the relation of non-identity could be interpreted in various places.

Let me summarize the discussion in this section. We saw a very smooth analysis of the phenomenon of WR in terms cumulation. A very attractive feature of Sternefeld's analysis is that it reduces the semantic effects of reciprocity largely to the semantics of pluralization. On the other hand, on this particular implementation of this idea, the relation to the HLM meaning of the reciprocal is unclear. The question arises whether we could not save the idea that WR comes about via cumulation, but in some way use HLM still. Since we need the HLM meaning anyway, this seems a natural thing to look for. The next section proposes an analysis of WR in terms of cumulation on the basis of HLM. We will see that the non-identical subgroups effect finds a natural explanation that way.

4. A Heim, Lasnik and May Analysis with Cumulation

4.1. WR by QR

Let's start by figuring out what exactly the problem is for the HLM reciprocal when we try to use it for WR. Here is one way of looking at it: as Sternefeld shows, we want a cumulated relation to hold between the group denoted by the antecedent, and that same group. The reciprocal on the HLM story denotes, not the antecedent group A, but a group we called A-x. Obviously, we can't have a cumulated relation between A and A-x because the variable x would remain unbound.

However, notice that we do have a second expression referring to the antecedent group on the HLM proposal. In (47b), it is the hidden pronoun Pro3 that shows up as *them* in the paraphrase 'the other ones among them'. So perhaps we could cumulate between this *them* and the antecedent? This idea is implemented in (48).

(47) a. The children like each other.
   b. The children3 like [ max [* [ Cov [[other x1] (of) Pro3]]]]

(48) a. [Pro3 [[the children]3
   [**[ 1 [Cov[ 2 [t1 [Cov [like [ max [ * [[other x1] (of) t2]]]]]]]
   b. <C,C>ε **λxλy[Cov(x) & Cov(y) & x like max(*λz[Cov(z) & z≠x & z≤y])]}
   c. <C,C>ε **λxλy[ x like max(*λz[z≠ x & z ≤y])]

Imagine we assign to (47a) the LF in (48a). We QR both the subject and the covert pronoun that is anaphoric with it. We cumulate the resulting relation. This will yield the translation in (48b), which is simplified to (48c) assuming that x, y and z all range over singularities only. This is guaranteed by the cover, which in this case restricts cumulation.

It is by no means obvious that (48c) denotes anything useful. Let's approach the problem of figuring out what it means by comparing the relation we cumulate to the relation that Sternefeld would want to cumulate for this example. Since Sternefeld predicted the right truth conditions for this type of example, we want to know whether the relation he cumulates is the same as ours. If they are, then we predict the same truth conditions. The two relations are given in (49).
(49) a. $\lambda y \lambda x[y \neq x \& L(x, y)]$ (b) (a)
b. $\lambda y \lambda x[L(x, \max(*\lambda z[z \neq x \& z \leq y])]]$ (b) (a)

Suppose we choose a and b such that $a = b$. What is the set of all singularities that are not identical to a and a part of b? Since both a and b are singularities, if they are the same singularity, this is the empty set. Closure under group formation (that is application of the * operator) will still result in the empty set. The maximum of the empty set is undefined. Hence, (49b) presupposes that $a \neq b$. What if that is the case, i.e. choose a and b such that $a \neq b$? Then the maximum of the set of things that are a part of b and not identical to a will be b. Hence, (49b) presupposes that $b \neq a$ and asserts that $L(a, b)$. (49a) asserts that $b \neq a$ and $L(a, b)$. Thus (49a,b) are true of the same pairs $<a, b>$. The only difference is that (49b) may be undefined when (49a) is false. This looks promising.

Let me rewrite (50a) as (50b), which I find somewhat more readable:

(50) a. $\lambda y \lambda x[L(x, \max(*\lambda z[z \neq x \& z \leq y])]]$
b. $\lambda y \lambda x[L(x, y) \& @(x \neq y)]$
c. $\lambda y \lambda x[L(x, y) \& x \neq y]$

The notation means that the argument of the @ is a presupposition rather than part of the assertion; a similar notation is found in Beaver (1995). What we are really interested in is (51a) compared to (51b):

(51) a. $**\lambda y \lambda x[L(x, y) \& @(x \neq y)]$ (A)(A)
b. $**\lambda y \lambda x[L(x, y) \& x \neq y]$ (A)(A)

(51a) poses an interesting question about presupposition projection. Notice that we would like to know what happens to the non-identity presupposition $x \neq y$ when $x$ and $y$ get bound. For this, recall the definition of **:

(52) ** is that function: $D<e, e, t, t> \rightarrow D<e, e, t, t>$ such that for any R: $[**R](y)(x) = 1$ iff $R(y)(x)$ or $\exists x1x2y1y2[x = x1 \& x2 \& y = y1 \& y2 \& **R(y1)(x1) \& **R(y2)(x2)]$

A pair $<A,A>$ can never get into a reciprocal cumulated relation like (51a) via the first clause of the disjunction since $A = A$, but the basic relation presupposes non-identity. Hence $<A,A>$ must get in there via the second clause. Thus we must be able to divide up A into non-identical parts $y1$ and $x1$ for this to come out true. For $A$ to be divisible into non-identical parts means that $A$ is a plurality. I want the non-identity condition to project as a presupposition of plurality.

It is not clear, however, that it will project as a presupposition in that way. Notice we could divide up a group that consists of, say, Fred, into parts Fred and Amy. This will of course make the first conjunct in the second clause of (52) false; however, the whole expression will be perfectly well defined. It could only be true if the antecedent has more than one part, but as far as I can see, it does not carry any presupposition. So this would be identical to Sternefeld's truth conditions.

However, I think it is straightforward to improve on that a little, with the present analysis. I will make the following assumption:
(53) **pluralized partial functions:**

[*g*](x) is undefined if *g*(x) is undefined and *x* cannot be divided into parts for which *g* is defined.

[**](f) (y)(x) is undefined if *f*(y)(x) is undefined and *x* and *y* cannot be divided into parts for which *f* is defined.

The motivation for this assumption comes from data like (54):

(54) a. Agatha and Gwendolyn stopped smoking.
   *A* [x stopped smoking] (A&G)

b. Agatha and Gwendolyn stopped seeing Hercule and Marsellus (respectively).
   **A** [y [x stopped seeing y]] (H&M)(A&G)

Imagine that Agatha has never smoked. Then (54a) should be a presupposition violation just like 'Agatha stopped smoking'. This will only come out right if we prevent that the group Agatha and Gwendolyn is divided up into, say, Gwendolyn and Tom, both of whom used to smoke. In other words: you do not escape a presupposition failure by choosing non-parts of the groups you are looking at. Analogous reasoning holds for (54b) and cumulation.

If we adopt the assumption in (53), (51a) is only defined if *A* has two distinct parts. This is the presupposition that the antecedent is a group. I think this presupposition is a good thing. Consider (55).

(55) These pants resemble each other.

The interpretations assigned to (55) under Sternefeld's analysis and under my analysis are given in (56a) and (56b) respectively.

(56) a. **A** [y [x resemble(x, y) & y ≠ x]] (p)(p)
   \[\forall x[y ≤ p -> \exists y[y ≤ p & resemble (x, y) & x ≠ y]]\]

b. **A** [y [x resemble(x, y) & @ (y ≠ x)]] (p)(p)
   \[\forall x[x ≤ p -> \exists y[y ≤ p & resemble (x, y) & @ (x ≠ y)]]\]

Suppose there is only one pair of pants. Then (56a) will be false, and (56b) will be undefined. I think undefined is better, because (57) is still inappropriate, rather than true:

(57) These pants do not resemble each other (because there is only one of them).

Hence, this analysis predicts that reciprocals introduce a presupposition that their antecedent is a plurality. Notice that we already made this prediction for SR: The HLM representation of the reciprocal 'the other one(s) among them' will only be defined if the antecedent has at least two distinct subgroups. We now predict that there is always such a presupposition, in cases of a WR interpretation as well as SR.

To summarize the discussion of this subsection: In these simple cases, the HLM + QR analysis I suggest is very similar to the Sternefeld analysis, except that it gives us a straightforward way to capture a presupposition of plurality. Hence I think that the semantic result of the QR operation in (48) is, actually, slightly better.
than the original Sternefeld analysis. (59) states the abstract truth conditions of a weakly reciprocal statement under this analysis.

\[(59)\]
\[\begin{align*}
  & \text{a. } <A,A> \in \exists \forall \lambda y \lambda x [\text{Cov}(x) & \text{Cov}(y) & \\
  & \text{R}(x, \text{max}(\lambda \lambda z [\text{Cov}(z) & z \neq x & z \leq y])]
  \\
  & \text{b. } <A,A> \in \exists \forall \lambda y \lambda x [\text{R}(x, \text{max}(\lambda \lambda z [z \neq x & z \leq y])]
  \\
  & \text{c. } <A,A> \in \exists \forall \lambda y \lambda x [\text{R}(x,y) & \forall (y \neq x)]
\end{align*}\]

4.2. Non-identity as Presupposition

Let us now look at the non-identical subgroups effect and what a presuppositional analysis has to say about it. Remember that the effect seemed to be that the non-identity condition 'x \neq y' must be in the immediate scope of the binders of x and y; that is, there must be no other intervening operator between the abstraction over x and y and cumulation, and the non-identity condition. In (60) below, I repeat some of the problematic data as well as their new semantic analyses. The non-identity condition is marked as a presupposition.

\[(60)\]
\[\begin{align*}
  & \text{a. They don't like each other.}
  \\
  & \text{a. } \neg [<[\text{they}],[[\text{they}]> \in \exists \forall \lambda y \lambda x [@(x \neq y) & x \text{ likes } y]]
  \\
  & \text{b. }<[\text{they}],[[\text{they}]> \in \exists \forall \lambda y \lambda x [\neg [@(x \neq y) & x \text{ likes } y]
  \\
  & \text{c. }<[\text{they}],[[\text{they}]> \in \exists \forall \lambda y \lambda x [x \neq y & \neg [x \text{ likes } y]
\end{align*}\]

Since the non-identity condition is a presupposition, it will project up to the point where the two variables contained in it get bound. If we ignore the plurality presupposition (which is how the presuppositionality of the reciprocal projects after that), (60b) amounts to (60c) which was the desired interpretation. Hence, on this analysis, we do not need to exclude LFs that leave the remnant of the reciprocal in situ. Presuppositionality suffices to derive the non-identical subgroups effect.

(61) is parallel. The interesting case is reading (61c) and representation (62b). Once more, (62b) is the same as the desired (62c) modulo plurality presupposition.

\[(61)\]
\[\begin{align*}
  & \text{a. Mary and Sue introduced no one to each other.}
  \\
  & \text{b. There is nobody such that Mary introduced him to Sue and Sue introduced him to Mary.}
  \\
  & \text{c. Mary didn't introduce anybody to Sue and Sue didn't introduce anybody to Mary.}
\end{align*}\]

\[(62)\]
\[\begin{align*}
  & \text{a. } \neg \exists z [<M&S,M&S> \in \exists \forall \lambda y \lambda x [@(x \neq y) & x \text{ introduced } z \text{ to } y]]
  \\
  & \text{b. }<M&S,M&S> \in \exists \forall \lambda y \lambda x [\neg [@(x \neq y) & x \text{ introduced } z \text{ to } y]
  \\
  & \text{c. }<M&S,M&S> \in \exists \forall \lambda y \lambda x [x \neq y & \neg \exists z [x \text{ introduced } z \text{ to } y]]
\end{align*}\]

I provide (63) below for completeness - it does not add anything new at this point.

\[(63)\]
\[\begin{align*}
  & \text{a. The four professors introduced exactly two students to each other.}
  \\
  & \text{b. }<P,P> \in \exists \forall \lambda y \lambda x [\text{card}(\lambda z [@(x \neq y) & \text{student}(z) & \\
  & x \text{ introduced } z \text{ to } y])=2]
\end{align*}\]
c. \(<P, P> \in **\lambda y \lambda x [x \neq y \& \text{card}(\lambda z [\text{student}(z) \& x \text{ introduced } z \text{ to } y]) = 2]\>

The non-identical subgroups effect, on this analysis, has nothing to do with scope, or with the LF position of the non-identity condition, but rather with its presuppositional nature. The presupposition is projected up to the point where the variables get bound. Hence the tie between the non-identity requirement on subgroups and cumulation finds a logical explanation. The presupposition is something we inherit from the HLM analysis by virtue of the definiteness of the reciprocal. I conclude that there is independent motivation for taking the definite paraphrase ‘the other ones among them’ as a guideline.

4.3. Cumulation and QR

Let me finally address the following question (which might have worried some people ever since the beginning of this section): what sort of theoretical gymnastics does this analysis force us to do, that we wouldn’t otherwise have to engage in? The main worry one could have, I take it, is about the role of QR in deriving the relation-denoting constituent that is the sister of **. One might wonder whether this is legal, i.e. whether syntax can create that constituent and whether cumulation can operate on it. I will try to convince you that we do not need to assume anything that we wouldn’t have to assume for independent reasons anyway.

Notice that my story is incompatible with what one might call a lexicalist approach to pluralization. The relation I need to cumulate is one that is not the meaning of any lexical item, or indeed of any constituent that exists in overt syntax. That constituent comes into existence only at LF, via a process I called QR. I think that we need to assume that this possibility exists anyway. Consider (64):

\[
\begin{align*}
(64) & \quad a. \quad \text{Jim and Frank want to marry two dentists.} \\
& \quad b. \quad \text{Jim and Frank want to marry Sue and Amy.} \\
& \quad c. \quad **\lambda y \lambda x [x \text{ wants to marry } y] \\
& \quad d. \quad [\text{[Jim and Frank]} [\text{[Sue and Amy]} \\
& \quad \quad [**[ 2 [1 \text{ [t1 wants to marry t2]]]]]]]
\end{align*}
\]

The relation that cumulates in the most plausible interpretations of (64a) and (64b) is the one in (64c). This is not the meaning of any lexical item. We can create the appropriate constituent as shown in (64d). A similar example is (65):

\[
\begin{align*}
(65) & \quad a. \quad \text{Two girls gave two boys a flower.} \\
& \quad b. \quad **\lambda y \lambda x \exists z [\text{flower}(z) \& \text{give}(x, y, z)] \\
& \quad c. \quad [\text{[two girls]} [\text{two boys}] [**[ 2 [1 \text{ [t1 gave t2 a flower]]]]]]
\end{align*}
\]

Examples of this kind have been observed by Sauerland (1998) and in a slightly different connection by Winter (1997). Notice also that example (39) above similarly requires cumulation of a non-lexical constituent: the reciprocal relation on reading (39c) is ‘x introduced nobody to y’. Sauerland in fact argues that the required constituent is created by QR. To support this claim, he provides examples like (66):

\[
\begin{align*}
(66) & \quad a. \quad \text{The lawyers have pronounced the proposals to be against the law.} \\
& \quad b. \quad \# \quad \text{The lawyers have pronounced that the proposals are against the law.}
\end{align*}
\]
A cumulative reading is possible in (66a) but not in (66b). The reason is that the required QR operation is possible out of an infinitival clause, but not a finite clause. I add some further examples that support the same conclusion. A cumulated reading is possible just in case QR can generate the required constituents. It is impossible out of scope islands.

(67) a. #Sue and Amy talked to a man who lived in two European countries.
b. *Sue and Amy talked to a man who lived in Storrs and Danbury, respectively.
c. **\( \lambda y \lambda x [x \text{ talked to a man who lived in } y] \)

(68) a. Sue and Amy saw a premiere of two new operas.
b. Sue and Amy saw a premiere of Oklahoma! and Cats respectively.
c. **\( \lambda y \lambda x [x \text{ saw a premiere of } y] \)

Most interesting for the present purposes are data that illustrate QR out of NPs. I have tried below to find NPs whose structure can be assumed to be similar to the hidden structure of the reciprocal NP according to HLM. Cumulation is possible.

(69) a. Sue and Amy drank most of the beer and the wine, respectively.
b. **\( \lambda y \lambda x [x \text{ drank most of } y] \)

(70) a. Sue and Amy hate the other ones among the two groups.
b. **\( \lambda y \lambda x [x \text{ hates the other ones among } y] \)

(71) a. Sue and Amy talked to many of the citizens of two of these communities.
b. **\( \lambda y \lambda x [x \text{ talked to many of the citizens of } y] \)

(72) a. Sue and Amy compared many of the children from the two groups (and both said that the developments of the children in their group had been fairly homogeneous).
b. **\( \lambda y \lambda x [x \text{ compared many of the children from } y] \)

I conclude that the structures I need to generate for my account of WR are expected to exist independently of reciprocals. Cumulation must be able to find these LF constituents. Hence in terms of the theory of pluralization, we get WR for free.

Let me summarize what I like about the present proposal. The only special aspect of reciprocal sentences is located in the internal structure and meaning of the reciprocal itself. The rest follows from the way the Logical Form and semantics of pluralization works. This is essentially Sternefeld's plot. I have made some small improvements with respect to the details of the analysis. The major improvement, as I see it, is that the reciprocal is (always) recognized as a presuppositional expression. We have seen the semantic effects of presuppositionality. We derive them from HLM, thus achieving uniformity of the analysis of the reciprocal across the different reciprocal interpretations. Variation in interpretation is derived via variation of the LFs assigned to reciprocal sentences. We find the same possibilities for LFs with relational plurals.

I have not discussed the role of the cover restriction on subpluralities in richer contexts. Obviously, more interesting covers will lead to different readings. I
will leave a comparison of what readings I expect with the readings discussed, e.g., in Fiengo and Lasnik (1973) and Dalrymple et al. (1998) for another time.

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