Numeral *any*: the view from Farsi*

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Abstract This paper documents a dimension of crosslinguistic variation among Universal Free Choice Items. In English, the distribution and interpretation of *any* DPs containing a numeral (‘numeral *any*’) differs from that of *any* DPs with no numeral (Dayal 2005, 2013; Chierchia 2013). In contrast, the Farsi counterparts of *any* and numeral *any* mirror each other. Two competing analyses of the contrast between *any* and numeral *any* are assessed against the Farsi data—the Wide Scope Constraint Analysis (Chierchia 2013) and the Viability Constraint Analysis (Dayal 2013). The paper shows that, with minimal extensions, either analysis can capture the behavior of the Farsi counterpart of numeral *any* with distributive predicates. The situation changes, however, when the minimally modified analyses are assessed with respect to sentences with collective predicates: the extended Wide Scope Constraint Analysis captures the attested interpretation of those sentences, but the extended Viability Constraint Analysis rules them out, undergenerating.

Keywords: free choice items, numeral *any*, plurality, collective predicates, Farsi

1 Introduction

The distribution of English *any* DPs is restricted. Moving beyond downward entailing environments, *any* DPs in declarative sentences are limited to only some modal environments. In the absence of modification (‘subtrigging’) (Legrand 1975), *any* DPs are deviant in episodic sentences, and in sentences containing a necessity modal, as seen in (1), but licensed in sentences containing a possibility modal, as (2) shows.

\[(1) \quad \begin{align*}
    &a. \quad \text{* Bill read any book.} \\
    &b. \quad \text{* Bill must read any book.}
\end{align*} \]

\[(2) \quad \text{Bill can read any book.} \]

*Any* DPs containing NPs modified by numerals (‘numeral *any*’) depart from this pattern. Numeral *any* is deviant in episodic sentences, as seen in (3), but licensed in

\[(3) \quad \text{Any book can be read by Bill.} \]

* We are grateful to the reviewers and conference participants (especially Danny Fox, Amir Ahmad Anvari, and Fereshteh Modarresi) and to Jonathan Palucci for insightful discussion. This project was funded by the Social Sciences and Humanities Research Council of Canada through an Insight Grant (Modality across Categories: Modal Indefinites and the Projection of Possibilities, 435-2018-0524, PI: Alonso-Ovalle). Our names are listed in alphabetical order. As usual, all errors are our own.

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sentences containing both possibility and necessity modals, even in the absence of modification, as (4) shows (Dayal 2005, 2013; Chierchia 2013).

(3) * Bill read any two books. 
(4) a. Bill can read any two books. 
   b. Bill must read any two books.

Recent analyses of universal free choice items (Dayal 2005, 2013; Chierchia 2013) seek to capture the contrast between *any and numeral any. This paper shows that the contrast is not universal. As we will see, the distribution of the counterpart of numeral any in Farsi mirrors that of the counterpart of any in that language.

This observation poses the question of the extent to which the existing analyses of the contrast between English *any and numeral any can capture the attested crosslinguistic variation. The paper addresses this question by assessing two competing analyses of the contrast between *any and numeral any—the Wide Scope Constraint Analysis (Chierchia 2013) and the Viability Constraint Analysis (Dayal 2013)—against the Farsi data. We will see that, with minimal extensions, either analysis can capture the Farsi pattern when the counterpart of *any and numeral any combine with distributive predicates. The predictions of the minimally modified analyses differ, however, when we consider sentences containing collective predicates: the minimally modified Wide Scope Constraint Analysis predicts these sentences to be acceptable, and captures their attested interpretation; in contrast, the minimally modified Viability Constraint Analysis rules these sentences out, and, therefore, undergenerates.

The paper is organized as follows: Section 2 presents the behavior of the counterparts of *any and numeral any in Farsi, Section 3 reviews how the Wide Scope Constraint Analysis and the Viability Constraint Analysis capture the contrast between *any and numeral any, Section 4 discusses how a minimal extension to each theory can capture the Farsi pattern, and Section 5 assesses the different predictions made by each theory with respect to examples containing collective predicates. Section 6 concludes with a summary and open issues.

2 Farsi *har -i DPs

*Har -i DPs are DPs headed by the determiner har that contain NPs marked with the enclitic -i, which is associated with other free choice items. Alonso-Ovalle & Moghiseh (2019) show that these DPs have all core properties of universal free choice items. As seen in (5) and (6) below, *har -i DPs are ungrammatical in positive episodic sentences, unless they are modified. When licensed by modification, *har -i DPs have universal force, just like English *any DPs: (6) conveys that Roya read all books that were on her desk.
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(5) * Roya har ketab-i xund.
   Roya HAR book-IND read-3.SG
   (Alonso-Ovalle & Moghiseh 2019: 690)

(6) Roya har ketab-i ke roo miz-esh boode bashe xund-e.
    Roya HAR book-IND that on table-POSS.3SG was SUBJ read-3.SG
    ‘Roya read any book that was on her desk.’
    (Alonso-Ovalle & Moghiseh 2019: 690)

Like *any* DPs, *har* -i DPs are deviant in sentences containing necessity modals, as seen in (7), unless they are modified, as in (8), in which case, again, they have universal force—(8) conveys that Roya must read all the books that she finds.

(7) * Roya bayad har ketab-i bexun-e.
    Roya must HAR book-IND read-3.SG
    (Alonso-Ovalle & Moghiseh 2019: 690)

(8) Roya bayad har ketab-i ke peyda mikon-e bexun-e.
    Roya must HAR book-IND that find does-3.SG read-3.SG
    ‘Roya must read any book that she finds.’
    (Alonso-Ovalle & Moghiseh 2019: 690)

Finally, *har* -i DPs are licensed in sentences with possibility modals, and, like their English counterparts with *any*, these sentences convey a free choice component: the sentence in (9) is false in the scenario in (10) because it conveys that all books are permitted options for Roya.

(9) Roya mitun-e har ketab-i bexun-e.
    Roya can-3.SG HAR book-IND read-3.SG
    ‘Roya can read any book.’
    (Alonso-Ovalle & Moghiseh 2019: 690)

(10) * Scenario:
     There are five books \{\(b_1, \ldots, b_5\}\). Roya is not allowed to read \(b_4\) or \(b_5\).
     (Alonso-Ovalle & Moghiseh 2019: 689)

Like *any* DPs, *har* -i DPs can contain NPs with numerals, as seen in (11). We will refer to those *har* -i DPs containing an NP with a numeral as ‘numeral *har*.

(11) Roya mitun-e [DP har do ta doone ketab-i ] bexun-e.
    Roya can-3.SG har two CL CL book-IND read-3.SG
    ‘Roya can read any two books.’

Unlike in the case of *any*, the presence of a numeral within a *har* -i DP does not affect its distribution. Numeral *har* is not licensed in positive episodic sentences
or in sentences with necessity modals, as (12) and (13) show, but it is licensed in sentences containing possibility modals, as seen in (14).  

(12) * Roya har do ta doone ketab-i xund.  
Roya HAR two CL CL book-IND read-3.SG  
(13) * Roya bayad har do ta doone ketab-i bexun-e.  
Roya must HAR two CL CL book-IND read-3.SG  
(14) Roya mitun-e har do ta doone ketab-i bexun-e.  
Roya can-3.SG har two CL CL book-IND read-3.SG  
‘Roya can read any two books.’  

Like their English counterparts with numeral *any*, the interpretation of sentences with possibility modals containing numeral *har* convey a free choice component: (14) claims that Roya is permitted to read any group of two books, and is therefore false in the scenario in (15).  

(15) The Spanish course syllabus lists five groups of two books for students to read ($\{G_1, \ldots, G_5\}$). Each group corresponds to a proficiency level. Students are permitted to read any group of books that is at or below their current proficiency level. Roya is in level 3, so she cannot read $G_4$ or $G_5$.  

What determines the contrast between Farsi and English? To start making progress in answering this question, we will review in the next section two analyses of the contrast between *any* and numeral *any*. We will then come back to the Farsi data in sections 4 and 5 to discuss how these analyses can be extended to capture the difference between English and Farsi.  

3 Two analyses of the contrast between *any* and numeral *any*  

In this section, we will review the main components of two recent analyses of English *any* that aim to capture the contrast with numeral *any*: the Wide Scope Constraint Analysis (Chierchia 2013) and the Viability Constraint Analysis (Dayal 2013).  

3.1 The Wide Scope Constraint Analysis (Chierchia 2013)  

Within the alternative-based theory presented in Chierchia 2013, free choice items are analyzed as existential quantifiers. The LF for the sentence in (1a), for instance,  

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1 The contrast between types of modals that we report here was confirmed in a small pilot acceptability rating questionnaire run with nine monolingual Farsi native speakers, belonging to different age groups (from 28 to 51), all from Tehran. In the examples above, the numeral *do* (‘two’) combines with two classifiers: *ta* and *doone*. The first classifier is required, but the second is optional. The examples in the questionnaire didn’t contain *doone*.  

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contains the constituent in (16), whose interpretation conveys that Bill read at least one book.\(^2\)

\[
\begin{align*}
\text{any book}_D & \lambda_1 \text{ Bill read } t_1 \notag \\
\end{align*}
\]

\(16\) \[\boxed{\text{any book}_D \lambda_1 \text{ Bill read } t_1} \equiv R(a) \lor R(b) \quad (g(D) = \text{book}^w = \{a, b\})\]

On top of this, any DPs introduce into the semantic derivation two types of alternatives that end up being propositional (‘pre-exhaustified’ domain alternatives and scalar alternatives). These alternatives are used by corresponding exhaustification operators (\(O_{\text{exh-D}}\) and \(O_{\sigma}\) respectively).\(^3\) An exhaustification operator applies to a propositional constituent \(\phi\) and (possibly) strengthens the proposition that this constituent denotes by excluding all alternatives to \(\phi\) of the relevant type that are not entailed by \(\phi\).

\[
\begin{align*}
\text{any book}_D & \lambda_1 \text{ Bill read } t_1 \\
\end{align*}
\]

\(17\) \[\boxed{\text{O}_x \phi} = \lambda w.\phi(w) = 1 \land \forall p \in \phi^{\text{ALT-}x} [p(w) = 1 \rightarrow \phi \subseteq p]\]

In positive episodic sentences, exhaustification yields a contradiction. Consider, as illustration, the LF in (18) below:

\[
\begin{align*}
\text{O}_{\text{exh-D}} &\text{ O}_{\sigma} \text{ any book}_D \lambda_1 \text{ Bill read } t_1 \\
\text{The set of scalar alternatives to the argument of O}_{\sigma} &\text{ is the set in (19), the singleton that contains the proposition that is true in a world } w \text{ in case Bill read in } w \text{ all books in the domain.} \\
\text{The proposition in the set in (19) is stronger than the proposition expressed by the argument of O}_{\sigma}, &\text{ and, therefore, it gets excluded, deriving (20).} \\
\text{The domain alternatives to the argument of O}_{\text{exh-D}}, &\text{ in (21a), correspond to the proposition that this argument expresses when the domain of quantification of any is restricted to any subset of its original domain. The set of pre-exhaustified domain alternatives, in (21b), is the set containing for any domain alternative } p, \text{ the result}\]
\]

\(20\) \[\boxed{\text{O}_{\sigma} \text{ any book}_D \lambda_1 \text{ Bill read } t_1} = [R(a) \lor R(b)] \land \neg[R(a) \land R(b)]\]

\(3\) See Moghiseh 2020 for a case study, based on Farsi, that shows the need to have different exhaustification operators targeting different types of alternatives.

\(2\) Notation: We assume an interpretation function relativized to a world (and a variable assignment function), mapping IPs to truth values, and use \(\llbracket \alpha \rrbracket\) (for a node \(\alpha\) of type \(t\)) to refer to \(\lambda w.\llbracket \alpha \rrbracket^w\). For ease of presentation, ‘\(R(a)\)’ stands for the function named by the expression ‘\(\lambda w.\text{READ}_w(Bill)(a)\)’ (naming the proposition that is true in any world \(w\) in case Bill read book \(a\) in \(w\)). ‘\(R(a) \lor R(b)\)’ stands for the function named by ‘\(\lambda w.\text{READ}_w(Bill)(a) \lor \text{READ}_w(Bill)(b)\)’, and, finally, ‘\(R(a) \land R(b)\)’ for the function named by ‘\(\lambda w.\text{READ}_w(Bill)(a) \land \text{READ}_w(Bill)(b)\)’. ‘\(\llbracket \alpha \rrbracket^{\text{ALT-}x}\) refers to the set of propositional alternatives to \(\llbracket \alpha \rrbracket\) of type \(x\) (scalar / domain / pre-exhaustified domain alternatives.).
of strengthening \( p \) with the exclusion of any other proposition in the set of domain alternatives that is ‘innocently excludable.’

\[ (21) \]
\begin{align*}
\text{a. } \{ R(a), R(b) \} & \quad \text{(domain alternatives)} \\
\text{b. } \{ R(a) \land \lnot R(b), R(b) \land \lnot R(a) \} & \quad \text{(pre-exhaustified domain alternatives)}
\end{align*}

The pre-exhaustified domain alternatives to the argument of \( O_{EXH-D} \), in (21b), are also stronger than the proposition that this argument expresses, and, so, they are excluded. The negation of these alternatives are equivalent to the conditionals in (22). Excluding both alternatives in (21b), then, contributes the biconditional in (23), which is inconsistent with (20), thus deriving a contradiction. The derivation of a contradiction correlates with the unacceptability of any.

\[ (22) \]
\[ \{ R(a) \to R(b), R(b) \to R(a) \} \]

\[ (23) \]
\[
\left[ O_{EXH-D} O_{\sigma} \ \text{any bookD} \ \lambda_1 \ \text{Bill read t}_1 \right] = [R(a) \lor R(b)] \land \lnot[R(a) \land R(b)] \land [R(a) \leftrightarrow R(b)] \iff \bot
\]

The Wide Scope Constraint Analysis assumes that any must scope over modals. Notice that, given the current setup, a contradiction is expected to be derived irrespective of the type of property that any combines with. As long as that property is kept constant in the assertion, pre-exhaustified, and scalar alternatives, a contradiction is derived. This means that exhaustification is also expected to yield a contradiction in modal sentences containing any, as illustrated in (24) and (25) below: in both cases, the existential component (the first conjunct in the formulas below) together with the domain implicature (the third conjunct) entails the scalar component (the second conjunct), as long as the domain of the modal is kept constant.

\[ (24) \]
\[
\left[ O_{EXH-D} O_{\sigma} \ \text{any bookD} \ \lambda_1 \ \text{mustC} \ \text{Bill read t}_1 \right] = [\Box C R(a) \lor \Box C R(b)] \land \lnot[\Box C R(a) \land \Box C R(b)] \land [\Box C R(a) \leftrightarrow \Box C R(b)] \iff \bot
\]

\[ (25) \]
\[
\left[ O_{EXH-D} O_{\sigma} \ \text{any bookD} \ \lambda_1 \ \text{canC} \ \text{Bill read t}_1 \right] = [\Diamond C R(a) \lor \Diamond C R(b)] \land \lnot[\Diamond C R(a) \land \Diamond C R(b)] \land [\Diamond C R(a) \leftrightarrow \Diamond C R(b)] \iff \bot
\]

The derivation of a contradiction doesn’t align properly with the acceptability of any, then, since any is acceptable with possibility modals. Notice, however, that modals are context-sensitive: the property that any combines with in (24) and (25)

4 Throughout the paper, we will consider only those domain alternatives that correspond to proper subsets of the domain of quantification. A proposition \( q \) is an alternative to \( p \) that is innocently excludable, in case every way of conjoining \( p \) with as many negated alternatives to \( p \) as consistency with \( p \) allows for entails the negation of \( q \) (Fox 2007; Alonso-Ovalle 2008). In the case at hand, there is only one alternative to any domain alternative, hence, a pre-exhaustified domain alternative is the result of conjoining any domain alternative \( p \) with the negation of the other member of the set of domain alternatives.
depends on the value of a free variable (C) determining the domain of quantification of the modal. To account for the distribution of any, the Wide Scope Constraint Analysis exploits this context sensitivity and assumes the constraint in (26):

(26) **Modal Containment**: the modal base in the scalar implicature must be a proper subset of the modal base in the domain implicature.

(Chierchia 2013: 314)

Modal Containment avoids the derivation of a contradiction, but only in the possibility modal case. Consider (24). The first conjunct in (27) collapses the existential component and the domain implicature in (24). In the second conjunct, we use a different variable (C') for the modal base in the scalar component, which we assume ranges over subsets of the value of C. If the first conjunct in (27) is true, Bill reads all books in all accessible worlds in C. This entails the negation of the second conjunct, regardless of the value of C'.

(27) \[\square_C R(a) \land \square_C R(b) \land \neg \square_{C'} R(a) \land \square_{C'} R(b)\]

The situation is different in (25). As before, the first conjunct in (28a) collapses the existential component and the domain implicature in (25). In this case, the first conjunct can be consistent with the second when Modal Containment is obeyed. To see this, consider, for instance, the worlds in (28b), and assume that the value of C is \{w_1, w_2\} and the value of C' is \{w_1\}.

(28) a. \[\diamond_C R(a) \land \diamond_C R(b) \land \neg \diamond_{C'} R(a) \land \diamond_{C'} R(b)\]

b. \[w_1 \ R(a) \land \neg R(b) \]

w_2 \neg R(a) \land R(b)

Modal Containment can then avoid the derivation of a contradiction with possibility modals, but not with necessity modals. This correlates with the acceptability of any with possibility modals but not with necessity modals, and derives the basic distribution pattern.

We turn now to numeral any. We will assume that numerals express properties of individuals, as in (29), where ‘|x|’ corresponds to the number of atomic individuals that x consists of.

(29) \[[\text{two}] = \lambda x. |x| \geq 2\]

The LF of the sentence in (3) contains the constituent in (30), which expresses the proposition that Bill read two or more books.

(30) \[[\text{any two books}] \lambda_1 \text{Bill read } t_1]^g =
\begin{align*}
R(a \oplus b) & \lor R(b \oplus c) \lor R(a \oplus c) \lor R(a \oplus b \oplus c) \\
(\Leftrightarrow R(a \oplus b) & \lor R(b \oplus c) \lor R(a \oplus c))
\end{align*}

\(g(D) = [\text{books}]^w = \{a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}\)
The scalar alternative to (30) is in the set in (31). This proposition (the proposition that Bill read a group containing at least three books in D) is determined by considering a higher value for the numeral.

(31) \ \{R(a \oplus b \oplus c)\} \quad \text{(scalar alternative)}

The scalar alternative in (31) is stronger than (30), so $$O_\sigma$$ excludes it.

\[
(32) \quad [O_\sigma \text{any two books}_D \lambda_1 \text{Bill read } t_1] = \\
[R(a \oplus b) \vee R(b \oplus c) \vee R(a \oplus c) \wedge \neg R(a \oplus b \oplus c)]
\]

The domain alternatives to (32), in (33), yield the pre-exhaustified domain alternatives in (34). These alternatives are stronger than (32) and get excluded. Since the alternatives in (34) are related by entailment, we only need to consider the weaker ones (because their negation will entail the negation of the stronger ones.) The negation of the weaker alternatives in (34) are equivalent to the conditionals in (35). Together with the existential component, these conditionals derive the first conjunct in (36a). Assuming that the extension of the predicate is cumulative (that whenever $$R$$ is true of $$x, y$$, $$R$$ will be true of the sum of $$x$$ and $$y$$), the first conjunct in (36a) entails (36b), yielding a contradiction.

\[
(33) \quad \begin{cases} 
R(a \oplus b), R(a \oplus c), R(a \oplus c), R(a \oplus b \oplus c), \\
R(a \oplus b) \wedge \neg (R(a \oplus b \oplus c)), \ldots \\
\quad [R(a \oplus b) \vee (R(b \oplus c) \wedge \neg R(a \oplus b \oplus c)) \ldots 
\end{cases}
\]

\[
(34) \quad \begin{cases} 
R(a \oplus b) \wedge \neg R(a \oplus c) \wedge \neg R(b \oplus c), \\
R(a \oplus c) \wedge \neg R(b \oplus c) \wedge \neg R(a \oplus b), \\
R(b \oplus c) \wedge \neg R(a \oplus c) \wedge \neg R(a \oplus b), \\
\quad [R(a \oplus b) \vee R(a \oplus c) \wedge \neg R(b \oplus c)], \\
\quad [R(a \oplus c) \vee R(b \oplus c) \wedge \neg R(a \oplus b)], \\
\quad [R(b \oplus c) \vee R(a \oplus b) \wedge \neg R(a \oplus c)]
\end{cases}
\]

\[
(35) \quad \begin{cases} 
[R(a \oplus b) \vee (R(a \oplus c))] \rightarrow R(b \oplus c), \\
[R(a \oplus c) \vee (R(b \oplus c))] \rightarrow R(a \oplus b), \\
[R(b \oplus c) \vee (R(a \oplus b))] \rightarrow R(a \oplus c)
\end{cases}
\]

(36) a. \ [O_{EXH-D} O_\sigma \text{any two books}_D \lambda_1 \text{Bill read } t_1] = \\
[R(a \oplus b) \wedge R(b \oplus c) \wedge R(a \oplus c) \wedge \neg R(a \oplus b \oplus c)]

5 We ignore any subdomain containing only atomic individuals, since these domains derive a contradiction, which the exhaustifier will negate to no effect. Similarly, the 'mixed domains', containing both atomic and non-atomic individuals can be disregarded, since they yield alternatives that are equivalent to those coming from domains containing only plural individuals. In the rest of the paper, we will only consider subdomains closed under sum formation. The alternatives in the first line correspond to the singleton subdomains containing one plural individual. In those cases, the scalar alternative is equivalent to the assertion, and therefore, scalar exhaustification is trivial.
b. $R(a \oplus b \oplus c)$

This setup also derives a contradiction in cases where numeral *any* combines with a necessity modal. To illustrate, consider the sentence in (4b), with the LF in (37a). Collapsing the existential component and the domain implicature that this LF derives gives us the first conjunct in (37a), which entails (37b), contradicting the scalar implicature.

(37) a. \[
\begin{aligned}
O_{EXH-D} \ O_{\sigma} \ \text{any two books}_D \ \lambda_1 \ \text{must}_C \ \text{Bill read}_t \\
&= [\Box_C R(a \oplus b) \land \Box_C R(b \oplus c) \land \Box_C R(a \oplus c)] \land \neg \Box_C R(a \oplus b \oplus c)
\end{aligned}
\]

b. $\Box_C R(a \oplus b \oplus c)$

When numeral *any* combines with possibility modals, no contradiction is derived. This time, the first conjunct in (38a) does not entail (38b): the proposition expressed by the first conjunct when $g(C) = \{w_1, w_2, w_3\}$ is true in the scenario in (38c), where (38b) is false.

(38) a. \[
\begin{aligned}
\begin{array}{c|c|c}
O_{EXH-D} \ O_{\sigma} \ \text{any two books}_D \ \lambda_1 \ \text{can}_C \ \text{Bill read}_t \\
&= [\Diamond_C R(a \oplus b) \land \Diamond_C R(b \oplus c) \land \Diamond_C R(a \oplus c)] \land \neg \Diamond_C R(a \oplus b \oplus c)
\end{array}
\end{aligned}
\]

b. $\Diamond_C R(a \oplus b \oplus c)$

c. $w_1$ \quad $R(a \oplus b) \land \neg R(a \oplus b \oplus c)$

c. $w_2$ \quad $R(b \oplus c) \land \neg R(a \oplus b \oplus c)$

c. $w_3$ \quad $R(a \oplus c) \land \neg R(a \oplus b \oplus c)$

The derivation of a contradiction does not align with the distribution of numeral *any*, then: wide scope numeral *any* yields a contradiction in episodic sentences, where it is not licensed, and a contingent meaning in sentences containing a possibility modal, where it is licensed; but it also yields a contradiction in sentences containing a necessity modal, where it is licensed.

To explain the attested distribution, the analysis assumes that another interpretation constraint can override the Wide Scope Constraint. Consider again (36a), repeated below as (39a):

(39) a. \[
\begin{aligned}
\begin{array}{c|c|c}
O_{EXH-D} \ O_{\sigma} \ \text{any two books}_D \ \lambda_1 \ \text{Bill read}_t \\
&= [R(a \oplus b) \land R(b \oplus c) \land R(a \oplus c)] \land \neg R(a \oplus b \oplus c)
\end{array}
\end{aligned}
\]

b. $R(a) \land R(b) \land R(c)$

Because the predicate is distributive and cumulative, the first conjunct in (39a) is equivalent to (39b). With a distributive predicate, replacing the numeral in (39a) always yields an existential component and domain implicature equivalent to (39b) (and this meaning component is always inconsistent with the corresponding scalar
implicatures). The same is true in (37a), repeated in (40a), where numeral *any* combines with a necessity modal: the first conjunct in (40a) is equivalent to (40b), and other numerals would have also yielded an existential component and domain implicature equivalent to (40b). The situation is different with possibility modals: the first conjunct in (41a) entails (41b), but is not entailed by (41b).

(40) a. \[O_{\text{EXH-D}} O_\sigma \text{ any two books}_D \lambda_1 \text{ must}_C \text{ Bill read } t_1 \] = 
\[\Box_C R(a \oplus b) \land \Box_C R(b \oplus c) \land \Box_C R(a \oplus c) \land \neg \Box_C R(a \oplus b \oplus c)\]
b. \[\Box_C R(a) \land \Box_C R(b) \land \Box_C R(c)\]

(41) a. \[O_{\text{EXH-D}} O_\sigma \text{ any two books}_D \lambda_1 \text{ can}_C \text{ Bill read } t_1 \] = 
\[\Diamond_C R(a \oplus b) \land \Diamond_C R(b \oplus c) \land \Diamond_C R(a \oplus c) \land \neg \Diamond_C R(a \oplus b \oplus c)\]
b. \[\Diamond_C R(a) \land \Diamond_C R(b) \land \Diamond_C R(c)\]

In a sense, then, the numeral is redundant in episodic sentences and where *any* combines with a necessity modal. This motivates the economy constraint in (42):

(42) \textit{The Scale Economy Constraint}

\(\star [O \text{ FCI}_i \ldots] \text{ if } \text{FCI}_i \in \langle \text{FCI}_1 \ldots \text{FCI}_n \rangle (n > 2) \text{ and } \forall j (1 \leq j \leq n) [O \text{ FCI}_i \ldots] = [O \text{ FCI}_j \ldots] \) (Chierchia 2013: 333)

In positive episodic sentences with distributive predicates, a violation of the Scale Economy Constraint is guaranteed (and so is the derivation of a pathological meaning). This corresponds to the deviance of numeral *any*. In sentences with a necessity modal, however, a violation of the Scale Economy Constraint (and the derivation of a contradiction) can be avoided by letting numeral *any* scope under the modal—at the cost of violating the Wide Scope Constraint. In this case, the resulting meaning is not contradictory, and it does not violate the Scale Economy Constraint. To illustrate, consider (43):

(43) \[O_\sigma \text{ must}_C \text{ any two books}_D \lambda_1 \text{ Bill read } t_1 \] =
\[\Box_C [R(a \oplus b) \lor R(b \oplus c) \lor R(a \oplus c)] \land \neg \Box_C R(a \oplus b \oplus c)\]

The set containing the negation of the pre-exhaustified domain alternatives to (43) is in (44) below. If (43) is true, all the antecedents in these conditionals must be false. The strengthened meaning, in (45), is not a contradiction. It entails that every group of two books is a permitted option for Bill, and is therefore true in the model.

6 We exclude from the set below \(\neg \Box_C (R(a \oplus b \oplus c))\), since it is entailed by the assertion.
7 Consider the first three antecedents. If any of these antecedents were true, given the consequents, the second conjunct in (43) would be false. Now consider any of the other conditionals, for instance (i) below. Given that the first three antecedents are false, if the antecedent of (i) is true, there will be accessible worlds of two types: worlds where \(R(a \oplus b)\) is true, and worlds where \(R(b \oplus c)\) are true. If the first disjunct in the consequent is true, any \(R(a \oplus b)\) world will have to be a world where
The proposition in (i) will convey that every group of three books is a permitted option. The antecedent containing a disjunction under the scope of the modal.

To summarize: the Wide Scope Constraint and the Scale Economy Constraint rule out numeral *any* in episodic sentences, but not in sentences containing a possibility or a necessity modal, as desired.

We turn next to the Viability Constraint Analysis.

### 3.2 The Viability Constraint Analysis (Dayal 2013)

Like the Wide Scope Constraint Analysis, the Viability Constraint Analysis assumes that *any* is an existential, and that it triggers and excludes pre-exhaustified domain alternatives.

The Wide Scope Constraint, Modal Containment, and Scale Economy Condition constraints are however replaced by the single constraint in (47) below:

\[
\begin{align*}
& \Box_c R(a \oplus b) \rightarrow [\Box_c (R(b \oplus c) \vee \Box_c (R(a \oplus c))], \\
& \Box_c R(a \oplus c) \rightarrow [\Box_c (R(b \oplus c) \vee \Box_c (R(a \oplus b))], \\
& \Box_c R(b \oplus c) \rightarrow [\Box_c (R(a \oplus c) \vee \Box_c (R(a \oplus b))], \\
& \Box_c [R(a \oplus b) \vee R(b \oplus c)] \rightarrow [\Box_c (R(b \oplus c) \vee R(a \oplus c)) \vee \Box_c (R(a \oplus b) \vee R(a \oplus c))], \\
& \Box_c [R(b \oplus c) \vee R(a \oplus c)] \rightarrow [\Box_c (R(a \oplus b) \vee R(b \oplus c)) \vee \Box_c (R(a \oplus b) \vee R(a \oplus c))], \\
& \Box_c [R(a \oplus b) \vee R(a \oplus c)] \rightarrow [\Box_c (R(a \oplus b) \vee R(b \oplus c)) \vee \Box_c (R(b \oplus c) \vee R(a \oplus c))]
\end{align*}
\]

(44)

\[
\begin{align*}
[O_{\text{EXH-D}} O_\sigma \text{ must any two books } \lambda_1 \text{ Bill read } t_1] = \\
[\Box_c (R(a \oplus b) \vee R(b \oplus c) \vee R(a \oplus c)) \wedge \neg \Box_c [R(a \oplus b) \vee R(b \oplus c)] \wedge \neg \Box_c [R(a \oplus b) \vee R(a \oplus c)] \wedge \neg \Box_c [R(b \oplus c) \vee R(b \oplus c)] \\
\wedge w_1 R(a \oplus b) \wedge \neg R(b \oplus c) \wedge \neg R(a \oplus c) \\
\wedge w_2 R(b \oplus c) \wedge \neg R(a \oplus b) \wedge \neg R(a \oplus c) \\
\wedge w_3 R(a \oplus c) \wedge \neg R(a \oplus b) \wedge \neg R(b \oplus c)
\end{align*}
\]

(45)

(46)

To summarize: the Wide Scope Constraint and the Scale Economy Constraint rule out numeral *any* in episodic sentences, but not in sentences containing a possibility or a necessity modal, as desired.

We turn next to the Viability Constraint Analysis.

\[
\begin{align*}
R(a \oplus b \oplus c) \text{ is true. But in that case it will be true that all worlds are worlds where } R(b \oplus c) \text{ are true, which contradicts the assumption that the first three antecedents are false. We can reason likewise for the second disjunct—and the same point can be made with any of the other two conditionals with an antecedent containing a disjunction under the scope of the modal.}
\end{align*}
\]

(i) \[\Box_c [R(a \oplus b) \vee R(b \oplus c)] \rightarrow [\Box_c (R(b \oplus c) \vee R(a \oplus c)) \vee \Box_c (R(a \oplus b) \vee R(a \oplus c))]]\]

8 The proposition in (i) will convey that every group of three books is a permitted option. The proposition in (ii) will be true in models where Bill is required not to read two or more books.

\[
\begin{align*}
(i) \ [O_{\text{EXH-D}} O_\sigma \text{ must any three books } \lambda_1 \text{ Bill read } t_1] \\
(ii) \ [O_{\text{EXH-D}} O_\sigma \text{ must any one book } \lambda_1 \text{ Bill read } t_1]
\end{align*}
\]

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The Viability Constraint:

i. when any does not outscope a modal, each pre-exhaustified domain alternative must be true in the world of evaluation,

ii. when any outscopes a modal with domain C, each pre-exhaustified domain alternative must be true in the world of evaluation when the domain of the modal is restricted to a subset of C.

The Viability Constraint cannot be satisfied in cases where any is in a positive episodic sentence, as in (48a), since, in those cases, the pre-exhaustified domain alternatives, in (48b), are mutually exclusive and, therefore, cannot all be true in any given world. The same is true when any scopes under a modal.

For the Viability Constraint to have a chance to be satisfied, any has to scope over a modal. When any scopes over a possibility modal, as in (49a), the Viability Constraint can be satisfied. Consider, for instance, the model in (28b), repeated in (50) below. When C = \{w_1, w_2\}, (49a) will be true in (28b). The same is true for (49c), the result of strengthening (49a) with the negation of the pre-exhaustified alternatives. At the same time, each pre-exhaustified domain alternative can be true when the domain of its modal is a subset of C, if we let g(C') = \{w_1\} and g(C'') = \{w_2\}, so the Viability Constraint is satisfied, predicting the acceptability of any with possibility modals.

The situation changes when any scopes over a necessity modal. When the assertion in (51a) is strengthened with the negation of each pre-exhaustified alternative in (51b), we end up with the conjunction in (51c). If (51c) is true, then, no pre-exhaustified domain alternative will be true when its modal domain is the value of C or a subset of it. The Viability Constraint necessarily fails.

The wording of the constraint is ours.
Numeral *any*: the view from Farsi

c. \( □_C R(a) ∧ □_C R(b) \)

To account for the difference between *any* and numeral *any*, the Viability Constraint Analysis—like the Wide Scope Constraint Analysis—assumes that the existential component of *any* scopes under the modal. Unlike the Wide Scope Constraint Analysis, the Viability Constraint Analysis takes numeral *any* to introduce two existentials: one corresponding to the numeral (a generalized quantifier ranging over degrees, in (52)) and the other to *any*. In (53), the numeral moves from a DP internal position (the sister of a covert MANY (54)) over the modal, creating a property of degrees (the property that is true of any \( d \) such that in all permitted worlds Bill reads at least one group consisting of exactly \( d \)-many books).

(52) \([\text{two}] = \lambda d. P(d) \cdot \exists d [d = 2 ∧ P(d)]\)

(53) \([\text{two} \lambda_2 \text{ must } [\text{any} t_2 \cdot \text{MANY books}] \lambda_1 \text{ Bill reads } t_1] = □_C [R(a ⊕ b) ∨ R(b ⊕ c) ∨ R(a ⊕ c)]\)

(54) \([\text{MANY}] = \lambda d. \lambda x. |x| = d\)

The truth-conditions for (53) correspond to the truth-conditions that we get for narrow scope numeral *any* under the Wide Scope Constraint analysis (excluding exhaustification.) We also get the same pre-exhaustified alternatives. The Viability Condition is checked at the smallest constituent containing every component of the free choice item, so, in this case, it is checked at the topmost node, because the numeral component takes maximal scope. When the assertion is strengthened with the exclusion of the pre-exhaustified domain alternatives, the Viability Constraint can be satisfied in models like (46), repeated in (55) below, accounting for the acceptability of numeral *any* with necessity modals.

| \( w_1 \) | \( R(a ⊕ b) ∧ ¬R(b ⊕ c) ∧ ¬R(a ⊕ c) \) |
| \( w_2 \) | \( R(b ⊕ c) ∧ ¬R(a ⊕ b) ∧ ¬R(a ⊕ c) \) |
| \( w_3 \) | \( R(a ⊕ c) ∧ ¬R(a ⊕ b) ∧ ¬R(b ⊕ c) \) |

(55)

This completes the overview of the Wide Scope Constraint Analysis and the Viability Constraint Analysis, which are tailored to account for the distribution of English *any* and numeral *any*. Next, we get back to the Farsi data. In the next section, we will see that either theory can accommodate the Farsi data under minimal modifications. Section 5 will show that the two theories make different predictions when we move from distributive to collective predicates. In that case, as anticipated above, the minimally modified Wide Scope Constraint correctly predicts the attested interpretations, but the Viability Constraint Analysis, as is, undergenerates.
4 Back to Farsi

Recall the Farsi data. We have seen that, as illustrated in (5), (7), and (9), repeated below in (56-58), Farsi har -i DPs mimic the behavior of English any: moving beyond downward entailing environment, they are deviant in positive episodic sentences, like (56), and in sentences containing necessity modals, like (57), but licensed by possibility modals, as in (58).

(56) * Roya har ketab-i xund.
    Roya HAR book-IND read-3.SG

(57) * Roya bayad har ketab-i bexun-e.
    Roya must HAR book-IND read-3.SG

(58) Roya mitun-e har ketab-i bexun-e.
    Roya can-3.SG HAR book-IND read-3.SG

‘Roya can read any book.’

As we saw before, English and Farsi differ, however, with respect to numeral any: unlike in English, Farsi har -i DPs containing numerals behave like their counterparts with no numeral: they are deviant in positive episodic sentences like (59) and with necessity modals, as in (60), but they are licensed by possibility modals, as seen in (61).

(59) * Roya har do ta doone ketab-i xund.
    Roya HAR two CL CL book-IND read-3.SG

(60) * Roya bayad har do ta doone ketab-i bexun-e.
    Roya must HAR two CL CL book-IND read-3.SG

(61) Roya mitun-e har do ta doone ketab-i bexun-e.
    Roya can-3.SG HAR two CL CL book-IND read-3.SG

‘Roya can read any two books.’

How can the Wide Scope Constraint Analysis and the Viability Constraint Analysis be extended to cover the Farsi data and the contrast with English? We will start by considering the Wide Scope Constraint Analysis.

When discussing the behavior of numeral any in sentences with necessity modals, we saw that the Wide Scope Constraint Analysis assumes that the Wide Scope Constraint can be violated. The Wide Scope Constraint is violated when not doing so would result in a violation of the Scale Economy Condition. This is what allows numeral any to take scope under a necessity modal. With this in mind, one natural way for the Wide Scope Constraint Analysis to derive the Farsi data would be to assume that (for reasons to be determined) the Wide Scope Constraint cannot be
violated in Farsi. Under this assumption, the har -i data would be analyzed as the basic English any data. These DPs obligatorily take scope over modals. As seen in Section 3.1, exhaustification derives a contradiction across the board (accounting for the deviance of any in positive episodic sentences and with necessity modals), but the Modal Containment Constraint can rescue this contradiction in sentences containing a possibility modal, deriving their acceptability.

As discussed in Section 3.1, numeral any derives a pathological meaning and violates the Scale Economy Constraint in positive episodic sentences. The same would be true for numeral har. When numeral any or numeral har scope over a necessity modal, they also derive a contradiction, which Modal Containment cannot rescue. The derived meaning violates the Scale Economy Constraint, too. The derivation of a pathological meaning and the violation of the Scale Economy Constraint would correlate with the unacceptability of numeral har in positive episodic sentences and in sentences containing a necessity modal. If the Wide Scope Constraint cannot be violated in Farsi, numeral har will not be able to scope under the necessity modal, and the deviance of this item with necessity modals, in contrast to English, would be expected.

With possibility modals, wide scope numeral har does not derive a pathological meaning, and, furthermore, the meaning that it yields does not violate the Scale Economy Constraint. We then expect numeral har to be fully acceptable in this environment, just like its English counterpart is.

Assuming that the Wide Scope Constraint cannot be violated in Farsi derives the basic pattern within the Wide Scope Constraint Analysis, then. We turn now to considering how to accommodate the Farsi data within the Viability Constraint Analysis. A natural possibility to consider would be to assume that the numeral and existential components in numeral har do not split. To illustrate, we will assume, as in Section 3.1, that Farsi numerals are given an ⟨e, t⟩ type, as in (29), repeated in (62) below, and are interpreted within the har DP that contains them.

(62) \[ \lambda x. |x| \geq 2 \]

Let us consider the LFs in (63) below:

(63) a. [ har two booksD ] \( \lambda_1 \) Roya read \( t_1 \)
    b. mustC / canC [ [ har two booksD ] \( \lambda_1 \) Roya read \( t_1 \) ]
    c. [ har two booksD ] \( \lambda_1 \) mustC / canC Roya read \( t_1 \)

The Viability Constraint fails in (63a), since the pre-exhaustified domain alternatives in (64) are mutually exclusive.

(64) \[ R(a \oplus b) \land \neg R(a \oplus c) \land \neg R(b \oplus c) \]
    \[ R(a \oplus c) \land \neg R(b \oplus c) \land \neg R(a \oplus b) \]
    \[ R(b \oplus c) \land \neg R(a \oplus c) \land \neg R(a \oplus b) \]
Because the Viability Constraint is checked at the smallest constituent containing the free choice item, it also fails when numeral *har* scopes under a modal, any modal, as in (63b).

When numeral *har* scopes over a possibility modal, this setup replicates the results of the Wide Scope Constraint Analysis: the strengthened meaning in (65), where we assume $g(C) = \{w_1, w_2, w_3\}$, is true in a model like (66), where the Viability Constraint can be satisfied (when $g(C') = \{w_1\}$, $g(C'') = \{w_2\}$, and $C''' = \{w_3\}$), given the pre-exhaustified domain alternatives in (67).

(65) $\diamond_c R(a \oplus b) \land \diamond_c R(b \oplus c) \land \diamond_c R(a \oplus c)$

<table>
<thead>
<tr>
<th>$w_1$</th>
<th>$R(a \oplus b) \land \neg R(b \oplus c) \land \neg R(a \oplus c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2$</td>
<td>$R(a \oplus c) \land \neg R(a \oplus b) \land \neg R(b \oplus c)$</td>
</tr>
<tr>
<td>$w_3$</td>
<td>$R(b \oplus c) \land \neg R(a \oplus b) \land \neg R(a \oplus c)$</td>
</tr>
</tbody>
</table>

(66) $\diamond_{c'} R(a \oplus b) \land \neg \diamond_{c'} R(a \oplus c) \land \neg \diamond_{c'} R(b \oplus c)$

(67) $\diamond_{c''} R(a \oplus c) \land \neg \diamond_{c''} R(b \oplus c) \land \neg \diamond_{c''} R(a \oplus b)$

When numeral *har* scopes over a necessity modal, as in (63c) above, the Viability Constraint fails, since the strengthened meaning in (68) entails that the pre-exhaustified domain alternatives in (69) are false, no matter which subset of C their variables denote.

(68) $\Box_c R(a \oplus b) \land \Box_c R(b \oplus c) \land \Box_c R(a \oplus c)$

| $c'$  | $\Box_c R(a \oplus b) \land \neg \Box_c R(a \oplus c) \land \neg \Box_c R(b \oplus c)$ |
|-------|==========================================================|
| $c''$ | $\Box_{c''} R(a \oplus c) \land \neg \Box_{c''} R(b \oplus c) \land \neg \Box_{c''} R(a \oplus b)$ |

While we have made two assumptions that would eventually need to be justified (that the Wide Scope Constraint cannot be violated in Farsi, or that the existential and numeral components of numeral *har* must go together) both analyses, under minimal modifications, can capture the Farsi data presented so far. The situation changes, however, when we consider what happens with collective predicates.

5 Collective predicates

Consider the sentence in (70) below:

(70) Bayad dore *har* do ta ketab-i ye tanab bekesh-i.

‘You must wrap a rope around each group of two books.’
As the translation shows, the sentence in (70) makes a universal claim: that for each group of two books \( x \), the addressee is required to wrap a rope around \( x \). The sentence is judged to be false in a situation where the requirement simply conveys that the addressee must wrap a rope around some group of two books or other—and it is up for him to decide. The reported intuition is that the addressee would be disobeying if there are some groups of two books that he doesn’t wrap a rope around.

From the perspective of the modified Wide Scope Constraint Analysis, this interpretation is predicted. The sentence in (70) has the LF in (71a) below:

\[
\begin{align*}
\text{(71) a.} & \quad [O_{\text{EXH-D}} \ O_\sigma \ \text{har two books}_D \ \lambda_1 \ \text{must}_C \ \text{you wrap a rope around} \ t_1] = \\
& \quad \Box W(a \oplus b) \land \Box W(b \oplus c) \land \Box W(a \oplus c) \ldots \land \neg \Box W(a \oplus b \oplus c) \ldots \\
& \quad \text{(assertion + domain implicature)} \quad \text{(scalar implicature)} \\
\text{b.} & \quad \Box W(a \oplus b \oplus c)
\end{align*}
\]

The LF in (71a) satisfies the Wide Scope Constraint. Because the predicate is collective, the proposition that it denotes is not a contradiction: the first conjunct does not entail (71b) and is compatible with the second. The proposition conveys universal quantification over groups of two books (the addressee is required to wrap a rope around each such group) and it conveys that the addressee is not required to wrap a rope around any larger group of books. Furthermore, the Scale Economy Constraint is not violated: again, because the predicate is collective, quantifying over groups of books containing more or less books would have yielded a different meaning.

The predictions of the modified Wide Scope Constraint Analysis contrast with those of the modified Viability Constraint Analysis. Consider the two possible LFs of the sentence in (70) above, in (72a) and (73a):

\[
\begin{align*}
\text{(72) a.} & \quad \text{must}_C \ [ \ \text{har two books}_D \ ] \ \lambda_1 \ \text{you wrap a rope around} \ t_1 \\
& \quad \text{b.} \quad \text{PDAs: } W(a \oplus b) \land \neg W(b \oplus c) \land \neg W(a \oplus c) \ldots \\
\text{(73) a.} & \quad [ \ \text{har two books}_D \ ] \ \lambda_1 \ \text{must}_C \ \text{you wrap a rope around} \ t_1 \\
& \quad \text{b.} \quad \text{PDAs: } \Box_C W(a \oplus b) \land \neg \Box_C W(b \oplus c) \land \neg \Box_C W(a \oplus c) \ldots \\
& \quad \text{c.} \quad \Box_C W(a \oplus b) \land \Box_C W(b \oplus c) \land \Box_C W(a \oplus c) \ldots
\end{align*}
\]

In the LF in (72a), the Viability Constraint is checked at the IP below the modal. The Viability Constraint requires all pre-exhaustified domain alternatives, in (72b), to be true at the world of evaluation. Since these alternatives are mutually exclusive, the Viability Constraint fails to be satisfied.

The wide scope construal of numeral \textit{har}, in (73a), does not fare much better. The assertion, together with negation of the pre-exhaustified domain alternatives derives the conjunction in (73c). If (73c) is true, then the addressee wraps a rope
around each group of two books in all permitted worlds, and, so no pre-exhaustified domain alternative will be true with respect to any subdomain of permitted worlds.

The Viability Constraint fails whether numeral *har* is given wide or narrow scope with respect to the modal. The sentence in (70) is then ruled out. In contrast with the Wide Scope Constraint, the Viability Constraint Analysis undergenerates.

6 To conclude

We have seen that the contrast in distribution between *any* and numeral *any* that the Wide Scope Constraint and the Viability Constraint analyses aim to capture is not universal: in Farsi, the distribution of the counterparts of *any* and numeral *any* mirror each other.

Because the contrast between *any* and numeral *any* is not universal, the issue of how the Wide Scope Constraint and the Viability Constraint analyses might be extended to capture the observed cross-linguistic variation arises.

In this paper, we have entertained two natural ways in which these analyses could be extended to capture the Farsi data: the Wide Scope Constraint Analysis can assume that the Wide Scope Constraint cannot be violated in Farsi, and the Viability Constraint Analysis can assume that the numeral is interpreted within the DP.

While both extended analyses would capture the basic data, the extended Viability Constraint Analysis is too restricted, as it fails to predict the acceptability of sentences with numeral *har* containing collective predicates.

Some questions are left open for further research. One concerns the parallelism between numeral *any* and numeral *har*. Numerals *any* and numeral *har* differ in one respect that we have not discussed: while numeral *any* has been described not to be sensitive to subtrigging (Dayal 2005), numeral *har* (like English *any*) is, as the counterparts of the sentences in (12) and (13) with clausal modifiers—in (74) and (75) below—show.

(74) Roya har do ta doone ketab-i ke roo miz boode bashe xund-e.
    Roya HAR two CL CL book-IND that on table was SUBJ read-3.SG
    ‘Roya read any two books that were on the desk.’
(75) Roya bayad har do ta doone ketab-i ke peyda mikon-e bexun-e.
    Roya must HAR two CL CL book-IND that find does-3.SG read-3.SG
    ‘Roya must read any two books that she finds.’

More importantly, a really pressing question for the Wide Scope Constraint Analysis remains: What determines whether or not the Wide Scope Constraint can be violated in a given language? Where does the difference between Farsi and English ultimately derive from? We hope to address this question in future work.
Numeral any: the view from Farsi

References


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