A Framework for Focus-Sensitive Quantification

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1. Focus as a Source of Semantic Partition

Recent work on the semantics of natural language has shown that instances of quantification can be analyzed in terms of relations between two predicate meanings. That is, quantifications involve so-called TRIPARTITE STRUCTURES, consisting of a QUANTIFIER that identifies the relation, a RESTRICTOR as its first argument, and a MATRIX as its second argument. The prototypical case are quantificational NPs like (1), where the determiner, here *most*, is the quantifier, the noun, here *frogs*, is the restrictor, and the verbal predicate, here *croaked*, is the matrix:

(1) Most frogs croaked
    \[ \text{MOST}((\forall x \text{name}(x)))(\forall x \text{croak}(x)), \]  
    with \( \text{MOST} = \lambda X \lambda Y [\#(X \cap Y) > \frac{1}{2} \#(X)] \)

Tripartite structures can also be identified with adverbial quantification:

(2) Mostly / Most of the time, if a frog is happy, it croaks
    \[ \text{MOST}((\langle s, x \rangle \text{name}(x) \& \text{happy}(x, s)))(\langle s, x \rangle \text{croak}(x, s)) \]

We have to assume that (2) contains both a quantification over objects and over situations; this is implemented as a quantification over PAIRS of entities \( x \) and situations \( s \). Quantifications over more than one entity are called quantification over CASES, following Lewis (1975). In the example at hand, the quantifier is an adverbial, the restrictor is supplied by the if-clause, and the matrix is given by the matrix clause.

Other cases of quantification, for example by verbal affixes, have been identified in the Amherst project on quantification (Bach, Kratzer, Partee 1989; Partee 1991). I should mention that the quantifier may be implicit, as in *If a frog is happy, it croaks*. In such cases, the inherent quantifier is the generic operator (cf. Krifka e.a., to appear), which can be interpreted as a quantifier with a modal component. In this article I will concentrate on non-generic cases in order to avoid additional complications involving quantification over possible worlds.

An obvious question at this point is how the mapping of semantic material of a quantificational expression to the restrictor and the matrix, respectively, is grammatically determined. This mapping, which has come to be called SEMANTIC PARTITION (cf. Diesing 1990), may depend on a range of different factors. For example, one obvious source is PHRASE STRUCTURE. In the case
of quantificational NPs like *most frogs*, the nominal predicate, here *frogs*, forms
the restrictor, and the matrix is determined by general scoping rules for quantified
NPs -- in the simple case of (1), it is the VP. Another source for semantic partition
are special SYNTACTIC OR MORPHOLOGICAL MARKERS, like the if of
conditional sentences: it marks the clause that it c-commands as restrictor (cf. 2).

Another way to mark semantic partition is FOCUS, which is typically
marked by sentence accent in languages like English, cf. Rooth (1985), also
Newton (1979) and Schubert & Pelletier (1989) for generic sentences. In general,
expressions that are in focus are mapped to the matrix. This effect shows up in the
following minimal pair discussed by Rooth:

(3) a. [In St. Petersburg], OFFICERS, always escorted ballerinas.
EVERY(∃x∃y×escorted(x,y,s) & ballerina(y)))
(∃x∃y×officer(x) & escorted(x,y,s) & ballerina(y)))
b. [In St. Petersburg], officers always escorted BALLERINAS.
EVERY(∃x∃y×officer(x) & escorted(x,y,s)))
(∃x∃y×officer(x) & escorted(x,y,s) & ballerina(y)))

In this article, I will concentrate on focus. See Krifka (1992) for a discussion of
other sources of semantic partition, like article choice, case marking, word order,
scrambling, and context).

2. Rooth’s Treatment of Focus-Sensitive Quantification

In this section, I will discuss Rooth’s theory of focus-sensitive quantification and
some of its problems.

We assume that focus is represented by a feature F that applies to syntactic
constituents and may be spelled out by sentence accent on certain syllables of
certain words of the constituent in focus. The constituent in focus may be
associated with a focusing operator such as only that c-commands its focus. Focus
marking by sentence accent is often ambiguous, as shown with the following
example, where the main accent is on Sue:

(4) John only introduced Bill to SUE.

a. John only introduced Bill to [SUE]
   "John introduced Bill to Sue and to no one else"
b. John only [introduced Bill to SUE]
   "John introduced Bill to Sue and did nothing else"

In Rooth’s theory, semantic representations consist of two parts, the usual
meaning, and a set of alternatives (therefore we may call it ALTERNATIVE
SEMANTICS, following von Stechow 1989). The set of alternatives is generated by the expression(s) in focus.

For example, in the (a) reading of (4) the alternatives to the item in focus, Sue, is a set ALT(s). This set of alternatives generates alternative sets for more comprehensive expressions in a compositional way. The alternative set for (4.a) without the operator only then is the set of propositions given in (5.a). In a similar way, the alternative set for (4.b) without only can be derived as (5.b):

(5)  
   a. John introduced Bill to [SUE]_P  
   Meaning: introduce(j,s,b).  
      Alternatives: {p | x [x ∈ ALT(s) & p=introduced(j,x,b)]}  
   b. John [introduced Bill to SUE]_P  
   Meaning: same as in (a)  
      Alternatives: {p | x | P ∈ ALT(λx.introduced(x,s,b)) & p=P(j)}

The operator ONLY simply states that the meaning itself is the only element in the set of alternatives that is true. (Actually, Rooth allows for focusing operators to be combined with VP-meanings. As in the preceding section, I will restrict the discussion here to focusing operators that take sentential scope; the generalization to other types is straightforward.)

(6)  
   ONLY(M,A) iff true(M) & ∀p [p ∈ A & true(p) → p=M]

Let us now turn to Rooth’s treatment of focus-sensitive quantification. In Rooth (1985), he only treats cases that imply quantifications over situations (which he captures by quantifications over times, following Stump 1981). He assumes that episodic sentences are true of situations. Then the meaning of a sentence like (7.a) can be described as the set of situations in which Mary took John to the movies (7.b). Applied to a specific situation s, it is expressed that s is a situation in which Mary took John to the movies (7.c).

(7)  
   a. Mary took John to the movies.  
   b. \{sitook(m,j,s)\}  
   c. s ∈ \{sitook(m,j,s)\}

Focus on John will create the following representation:

8. Mary took [JOHN]_P to the movies.  
   Meaning: As above, (b)  
   Alternatives: {s | x [x ∈ ALT(j) & S=sitook(m,x,s)]}

Focus-sensitive quantifiers relate the set of alternatives to the meaning. More precisely, they can be spelled out as quantifiers with the union of alternatives in
the restrictor, and the meaning in the matrix. For example:

(9) *most of the time*, applied to meaning M and alternative set A:
    \[ \text{MOST}(UA)(M). \]

Let us look at one example:

(10) Most of the time, Mary took [JOHN]_j to the movies.
    \[
    \text{MOST}(U\{\exists x[x \in \text{ALT}(j) \& S=\{\text{took}(m,x,s)\}]\})((\text{took}(m,j,s)))
    = \text{MOST}(\{\exists x[x \in \text{ALT}(j) \& \text{took}(m,x,s)]\})((\text{took}(m,j,s)))
    \]

(10) can be paraphrased as: "In most situations in which Mary took an alternative to John to the movies, she took John to the movies". The context may provide a set of alternatives. If it does not, we can assume that the alternatives are the set of all suitable entities of the type of the expression in focus. For example, the alternatives of j will be the set of all individuals. In this case, the meaning of our example reduces to:

(11) \[ \text{MOST}(\{\exists x.\text{took}(m,x,s)\})((\text{took}(m,j,s))) \]

Rooth's reconstruction of adverbial quantification seems to be a good starting point. However, there are several problems with it.

One problem is that the generated readings often seem to be too liberal, which Rooth himself sees as a "possible point of dispute" (p. 173). There is an interpretation of (10) where this sentence is true if, and only if, in most cases in which Mary took someone to the movies she took John and no one else. The phenomenon is obviously related to the exhaustive interpretation we often find with sentences containing a focus.

One way to handle this problem is to treat exhaustivity by assuming pragmatic interpretation rules that can be spelled out by ONLY. In the case of sentences with the adverbial quantifier *most of the time*, we would like to get something like the following interpretation instead of (9):

(12) \[ \text{MOST}(UA)(\text{ONLY}(M,A)) \]

That is, most situations that are in the union of the alternatives are such that the meaning is the only one among the alternatives that holds for them. To get the types right, \text{ONLY}(M,A) must be interpreted as a set of situations. The definition that comes to mind is the following one:

(13) \[ \text{ONLY}(M,A) = \{ s \in M \& \forall S[S \in A \& s \in S \rightarrow S=M] \} \]

That is, \text{ONLY}(M,A) holds for situations s that satisfy the meaning M, but no
proper alternative to it. It seems that (13) is a straightforward reformulation of (6) in the new, situation-based framework. However, it does not capture the meaning of only or the exhaustive interpretation. This shows up in examples like the following one:

(14) Mary took JOHN to the movies.

\[ \{\text{stook}(m,j,s) \land \forall s[\exists x[x \in \text{ALT}(j) \land S = \{\text{stook}(m,x,s)\}] \rightarrow S = \{\text{stook}(m,j,s)\} \}] \]

\[ = \{\text{stook}(m,j,s) \land \forall s'[\exists x[x \in \text{ALT}(j) \land \text{took}(m,x,s') \land \text{took}(m,j,s') \land \neg \text{took}(m,j,s')] \}] \]

This applies to situations \( s \) in which Mary took John to the movies, and for which it holds that for every situation \( s' \) where Mary took some alternative to John to the movies, she took John to the movies. Now, the most natural interpretation of the conditions for \( s' \) is that Mary can take more than one person to the movies at the same occasion, that is, it is possible that \( \text{took}(m,j,s') \) and \( \text{took}(m,b,s') \) for the same situation \( s' \). But then (13), and consequently (12), cannot give us the required exhaustive interpretation. In order to arrive at a more adequate representation we would need to refer to the content of the item in focus and say that it is the only one among the alternatives that satisfies the proposition. But this is not possible in Rooth’s framework, where we cannot refer to the meaning contribution of the focus directly.

Another problem is that we may have anaphoric bindings between the restrictor and the matrix:

(15) Most of the time, a frog that sees a fly tries to CATCH it.

\[ \text{MOST}((\exists x,y[\text{frog}(x) \land \text{fly}(y) \land \text{see}(x,y,s)]) \land \{\text{try-to-catch}(x,y,s)\}) \]

In the most straightforward representation given in (15), the variables \( x \) and \( y \) in the matrix remain unbound, hence the indicated formula is not an acceptable representation.

In particular, we find adverbial quantifications also in sentences that arguably have no situation argument to quantify over, as in the following example, which expresses a quantification over three-coloured cats instead of situations:

(16) Most of the time, a three-coloured cat is INFERTILE.

Obviously, examples like (15) and (16) are donkey sentences, and we should expect that a combination of focus representation with a framework like Discourse Representation, File Change Semantics or another dynamic semantic representation is called for.
3. The Structured Meaning Theory of Focus

I have suggested that one shortcoming of Alternative Semantics is that we cannot refer directly to the meaning contribution of an item in focus. There is another framework for the semantic representation of focus, Structured Meanings, developed by von Stechow and Jacobs, whose basic assumptions can be traced back to Jackendoff (1972, Ch. 6). In this framework, focus induces a partition into a BACKGROUND part and a FOCUS part, which is commonly represented by a pair of semantic representations <B,F>, where B can be applied to F, and B(F) is the standard interpretation. Focus operators take such focus-background structures as arguments. The examples (4.a,b) given above would be treated as follows:

(17) a. ONLY(<λx.introduced(j.x,b),s>)
    b. ONLY(<λP.P(j), λx.introduced(x,s,b)>

Assuming the following meaning postulate for ONLY

(18) ONLY(<B,F>) :→ B(F) & ∀X[X ∈ ALT(F) & B(X) → X=F],
    where X is a variable of the type of F
    and ALT(F) is the set of alternatives to F.

we get the following representations:

(19) a. introduced(j.s,b) & ∀x[x ∈ ALT(s) & introduced(j.x,b) → x=s]
    b. introduced(j.s,b) & ∀P[P ∈ ALT(λx.introduced(x,s,b)) & P(j) →
       P=λx.introduced(x,s,b)]

The Structured Meaning framework can capture complex foci (20.a, by list representations) and multiple foci (20.b, by recursive focus-background structures).

(20) a. John only<sub>1</sub> introduced BILL<sub>y1</sub> to SUE<sub>y1</sub>,
    ONLY(<λx•y.introduced(j.x,y), s•b>)
    b. Even<sub>1</sub> JOHN<sub>x1</sub> met only<sub>2</sub> SUE<sub>x2</sub>,
    EVEN(<λx.ONLY(<λy.met(x,y), s>), j>)

Krifka (1992) has developed a framework in which examples of these types are analyzed in a compositional way. In this framework, the focus on a constituent with the semantic representation A introduces a focus-background structure with "empty" background, <λX.X, A>, where X is of the type of A. This focus-background structure is projected through semantic compositions. For example, if the original semantic composition rule called for application of B to A, then application of B to a structured meaning <λX.C,D> will yield <λX[B(C)],D>. If the original rule called for application of A to B, then application of the structured
meaning $\lambda X.C,D$ to $B$ will yield $\lambda X[C(B)],D$. Thus, information about the focus and the place in the background where it has to be interpreted is projected through semantic composition. Finally, focus-sensitive operators are applied to such background-focus structures.

The Structured Meaning framework provides us with a more articulate representation of expressions with focus than Alternative Semantics, insofar as we can access the meaning of an item in focus directly. In general, Alternative Semantics representations can be derived from Structured Meaning representations, but not vice versa. And it seems that we will need this additional information provided by Structured Meanings in order to cover the exhaustive interpretations discussed in the last section.

4. A Framework for Dynamic Interpretation

In this section, I will develop a framework for dynamic interpretation to capture anaphoric bindings in quantificational structures (cf. section 2). It will be related to Rooth (1987), mainly because I feel that its representations render the underlying ideas most perspicuously. The main differences to Rooth (1987) are that I will work with partial assignment functions (cf. Heim 1983), and that I will assume indices for possible worlds to capture modal quantifications and, in general, the increase of propositional information. Furthermore, I will use some abbreviatory conventions that hopefully improve the readability of the formulas.

Let us assume a countable infinite set of DISCOURSE REFERENTS (or INDICES) DR, for which I use natural numbers 1, 2, 3 etc. Let us call the domain of entities D, and let G be the set of ASSIGNMENT FUNCTIONS, that is, the set of partial functions from DR to D: $G = U\{G'|\exists X[X \subseteq DR \& G' = D^X]\}$. If $g$ is an assignment function and $d$ is an index in its domain, then I will write $g_d$ instead of $g(d)$. Two assignment functions $g,k$ are said to be COMPATIBLE, $g=k$, iff they are identical for their shared domain: $g=k \iff \forall d \in \text{DOM}(g) \& d \in \text{DOM}(k) \rightarrow g_d=k_d$. The AUGMENTATION of $g$ with $k$, $g+k$, is defined as $g \cup k$, if $\text{DOM}(g) \cap \text{DOM}(k) = \emptyset$, and undefined otherwise.

I will use the following notations for VARIANTS of assignment functions; contrary to usual conventions, they will denote sets of assignment functions. First, $g[d]$ should be the set of assignment functions that is like $g$ with the addition that they map the index $d$ to some entity in $D$, that is, $g[d] = \{k \exists x [x \in D \& k=g+\{<d,x>\}]\}$. Second, $g[d/a]$ should be the set of assignment functions that are like $g$ with the addition that they map the index $d$ to the entity $a$, that is, $g[d/a] = \{k \exists x [x \in D \& k=g+\{<d,a>\}]\}$; note that this will be a singleton set. Be aware that these notations are defined only if $d \notin \text{DOM}(g)$. The two notations can be combined; for example, $g[1/a,2,3/b]$ stands for $\{k \exists x [x \in D \& k=g+\{<1,a>,<2,x>,<3,b>\}]\}$.

The interpretation of natural-language expressions will, in general, be with respect to an INPUT ASSIGNMENT, an OUTPUT ASSIGNMENT, and a
POSSIBLE WORLD. NPs are related to discourse referents; I assume that their syntactic indices are interpreted as semantic indices. Indefinite NPs bear indices that are new with respect to the input assignment, definite NPs have old indices, and quantificational NPs have new indices that are "active" only within the scope of quantification. Also, episodic verbs introduce discourse referents for situations; they are, in general, new.

For objects I will use variables x, y, ..., for situations s, s', ..., for possible worlds w. I assume a relation then for situations; s-then-s' means that the situation s is followed by the situation s', and that both situations together form a larger, coherent situation. Worlds determine the meanings of constants; I assume that constants, in general, have a world argument, which will be written as a subscript. I use v as a meta-variable over vectors of individual terms of length ≥ 0. I use Q, Q' etc. as variables for entities of type \(<g,k,w,v>l...\), T, T' etc. as a variable for entities of type λQ.\(<g,k,w,v>l...\), and X, Y for variables of any type. For assignments I use variables g,h,k,i,j,f. Semantic combinations are typically by functional application. To save space, I will write tuples without commas; for example, instead of \(<g,k,w,y,k,s>l\) I will write \(<gkwyks>l\).

Indices of NPs are introduced by determiners, the functional heads of NPs. Indices of indefinite determiners are new, whereas indices of definite NPs and pronouns are old. NPs are of a type that maps tuples \(<gkwxv>l\) to tuples \(<gkvwx>l\), that is, they reduce the arguments of the verbal predicate, xv, by one to v. In general, I assume that the first available entity variable is bound, which implies that grammatical functions are encoded sequentially.

The situation variable of an episodic verb is bound by an operator that introduces a new index for that situation. This operator may be associated with the syntactic position of INFL as the functional head of a sentence, and therefore I will attach the corresponding syntactic index to the finite verb (cf. Kratzer 1989, who suggests that tense, a feature of INFL, specifies and binds the Davidsonian argument). INFL can be applied at different stages of the syntactic derivation. In particular, it might be applied as the last operator, or it might be applied before the subject. In this way, internal subjects and external subjects in the sense of Kratzer (1989) and Diesing (1990) can be modelled. Tense will be disregarded throughout.

The following example shows the treatment of indefinite NPs and sentences with transitive verbs. I will use capital letters in brackets, like [A], as abbreviation.
(21) A frog saw a fly.

$$\text{see}, \{<\text{ggwyxs}>\text{see}_w(x,y,s)\}, = [A]$$

$$\text{fly}, \{<\text{ggwx}>\text{fly}(x)\}, = [B]$$

$$\lambda Q.\exists h \exists j[<\text{ghwh}_3> \in Q' \land j \in h[1] \land <\text{jkwh}_3> \in Q], = [C]$$

$$\lambda Q.\exists h \in g[3] \land \text{fly}_w(h) \land <\text{hkwh}_3> \in Q], = [D]$$

$$\text{see}_w(x,k_3), = [E]$$

$$\lambda Q.\exists h \in g[2] \land \text{PAST}(g_2) \land <\text{hkwh}_2> \in Q], = [F]$$

$$\lambda Q.\exists h \in g[2,3] \land \text{fly}_w(k_3) \land \text{see}_w(x,k_3,k_2)$$

$$\lambda Q.\exists h \in g[1] \land \text{frog}_w(h) \land <\text{hkwh}_1> \in Q], = [G]$$

The next example shows the treatment of anaphoric reference. As mentioned above, definite NPs and pronouns presuppose that their index is already in the domain of the input assignment. Similarly, the situation index of an episodic verb might indirectly refer to some situation index introduced before, insofar as its situation index is located after that previous index (see Partee 1984 for temporal anaphora in narrative discourses). I assume here that the INFL operator may have two indices, one referring to an antecedent situation index, and the other representing its own situation. Assuming that in the following sentence, which continues example (21), it refers to the frog, the fly refers to the fly, and INFL refers to a situation that follows the seeing situation, we get the following interpretation:
(22) It$_1$ caught$_{2,4}$ the$_3$ fly.

catch, \{$<gkwxyss>\text{catch}_w(x,y,s)\}$

\[
\text{the}_3 \text{fly}, \lambda Q. \{<gkwv>\text{fly}_u(g_3) \& <gkwg_yv>\in Q\}
\]

\[
/\]

\[
\text{catch the}_3 \text{fly,} \ {<gkwxyss>\text{fly}_u(g_3) \& \text{catch}_w(x,g_3,s)}
\]

\[
/\]

\[
\text{INFL}_{2,4}, \lambda Q. \{<gkwv>\exists h[g \in [4] \& g \_ \text{-then-} g_4 \& <hkwv_h>\in Q]\}
\]

\[
/\]

\[
\text{caught}_{2,4} \text{the}_3 \text{fly,}
\]

\[
\{<gkwxyss>\exists k[g \in [4] \& \text{fly}_u(g_3) \& k_2 \_ \text{-then-} k_4 \& \text{catch}_u(x,k_3,k_4)\}
\]

\[
/\]

\[
\text{it}_1, \lambda Q. \{<gkwv>\langle gkwg_yv>\in Q\}
\]

\[
/\]

\[
\text{it}_1 \text{caught}_{2,4} \text{the}_3 \text{fly,}
\]

\[
\{<gkwxyss>\exists k[g \in [4] \& k_2 \_ \text{-then-} k_4 \& \text{fly}_u(k_3) \& \text{catch}_u(k_1,k_3,k_4)\}, = \{H\}
\]

We can combine the first sentence with the second one by dynamic conjunction, for which I will use the semicolon.

(23)

\[
\text{A}_1 \text{frog saw}_{2} a_3 \text{fly,} \ [G]
\]

\[
/\]

\[
\text{It}_1 \text{caught}_{2,4} \text{the}_3 \text{fly,} \ [H]
\]

\[
/\]

\[
\text{A}_1 \text{frog saw}_{2} a_3 \text{fly. It}_1 \text{caught}_{2,4} \text{the}_3 \text{fly.} \ [G]; [H]
\]

\[
\{<gkwv>\exists h[g \in [4] \& <hkwv>\in [G] \& <hkwv>\in [H]\} = \{<gkwxyss>\exists k[g \in [1,2,3,4] \& \text{frog}_w(k_1) \& \text{fly}_w(k_3) \& \text{saw}_w(k_1,k_3,k_2)
\]

\[
& \& \text{catch}_u(k_1,k_3,k_4) \& k_2 \_ \text{-then-} k_4\}
\]

Quantified NPs do not introduce any anaphoric possibilities beyond their scope, that is, their input assignment and output assignment are the same. They are "tests", according to Groenendijk & Stokhof (1991). For example, the meaning of the determiner most$_4$ can be given as follows:

(24) most$_4$:

\[
\lambda Q' \lambda Q. \{<gkwv>\text{MOST}((x \exists h.k[h \in g[d/x] \& <hkwv>\in Q'])
\]

\[
((x \exists h.k,j[h \in g[d/x] \& <hjwv>\in Q' \& <jkwv>\in Q]))
\]

See Chierchia (1990) for the main lines of this reconstruction of quantification with "built-in" conservativeness. It represents the "weak" reading, as identified by
Rooth (1987), Kadmon (1987), and Schubert & Pelletier (1989). In terms of the standard example *Every farmer who owns a donkey beats it* we get a reading where it is sufficient that every farmer who owns a donkey beats at least one donkey that he owns. The STRONG INTERPRETATION -- that every farmer who owns a donkey must beat every donkey he owns -- can be generated with a slightly different scheme for quantifier meanings:

\[(25)\]  
\[\lambda Q\lambda R.(<g g w v> \downarrow \text{MOST}((x \exists h, k[h \in g(d/x) \& <h k w x> \in Q])) \rightarrow \exists k.<j k w x> \in Q))\)

So far, we have construed the dynamic meaning of discourses. The truth conditions for discourses are given by existential closure over the assignments and the world arguments with respect to the "actual" world: A text A is true with respect to the world w iff there are assignments g, k such that <g k w> ∈ A. And A is true w.r.t. an input assignment g and a world w iff there is an output assignment k such that <g k w> ∈ A.

5. Structured Meanings in the Dynamic Framework: The Case of "Only"

Let us now enrich the framework of the last section with structured meanings. This is fairly straightforward -- we might assume pairs of meanings <B,F>, where B and F are dynamic. However, we must reconsider the notion of alternatives to the focus meaning.

In a dynamic framework, the meaning of a focus constituent will naturally be dynamic. We indeed need dynamic foci, as they may exhibit anaphoric bindings:

\[(26)\] - Did John introduce every lady to her partner at left and her partner at right?  
- John only introduced every, lady to [her, partner at LEFT].

In the given context, the alternatives are anaphorically related to *every lady*. Furthermore, the choice of alternatives itself is dependent on the context in which the expression in focus is evaluated, as it will vary for different contemplated ladies. I will capture this dependency of the alternative sets to a focus F on an input assignment g by the notation \(\text{ALT}_g(F)\).

Since the elements of alternative sets are dynamic, we must take care that they do not introduce their own binding possibilities and lead to an unwelcome inflation of alternative sets. For example, assume that that *Mary, and the, woman with a hat* refer to the same person (but, of course, with different anaphoric potential). Obviously, we must exclude that the dynamic meaning of both NPs are
in the same alternative set. In general, we want that all proper alternatives to a focus meaning refer, in the given input context, to an entity that is different from the entity to which the focus meaning refers, with respect to the input context.

The analysis for only that is closest to the non-dynamic counterpart (cf. 27) is the following (again, I assume for simplicity that only is a sentence operator):

\[ \text{ONLY}(B,F) = \{ <gkw>! <gkw>! \in B(F) \land \forall X, h[X \in \text{ALT}_g(F) \land <ghw>! \in B(X) \rightarrow X=F] \} \]

The problem with this formulation, however, is that we take the alternatives with respect to the global input assignment g. What we would like to have is alternatives with respect to the local input assignment at which the focus is interpreted, as the discussion of sentences like (26) shows. A treatment of only that works with local alternative selection is the following,

\[ \text{ONLY}(B,F) = \]

\[ \text{(i) } B(F \land \{ <gkw>! \in Q \forall k[Q \in \text{ALT}_g(F) \land <ghw>! \in Q \rightarrow Q=F] \}) \]

\[ \text{if F is of type } \{ <gkw>! \ldots \} \]

\[ \text{(ii) } B(F \land \lambda Q. \{ <gkw>! \in Q \exists k[T \in \text{ALT}_g(F) \land <ghw>! \in T(Q) \rightarrow T=F] \}) \]

\[ \text{if F is of type } \lambda Q. \{ <gkw>! \ldots \}. \]

in which the intersection of functional expressions is the type-lifted version of standard intersection: \( A \cap B = \lambda X[A(X) \cap B(X)] \). For simplicity, I assume again that only is a sentential operator; the treatment as a VP operator is quite straightforward. (See Krifka (1992) for further discussion.)

6. Focus-Sensitive Quantification

Let us return to focus-sensitive adverbial quantification. As a meaning rule for most of the time, I would like to propose the following:

\[ \text{MOSTLY}(B,F) = \]

\[ \{ <gkw>! \in \text{MOST}([\forall h[f=g+h \land <gfw>! \in \text{B}([\{ <gkw>! \exists Q \exists k[Q \in \text{ALT}_g(F) \land <ghw>! \in Q] \}]])]) \}

\[ \text{if F is of a type } \{ <gkw>! \ldots \} \]

That is, most of the time expresses a quantification over augmentations h of the input assignment g. As restrictor we take all the cases in which the input g and the output f, \( f=g+h \), satisfy the background applied to some alternative of F, where the set of alternatives is again taken with respect to that input assignment at which the focus constituent is interpreted. We prevent the alternatives from introducing their
own binding possibilities by binding the assignment \(j\) existentially -- in a sense, we skip over the indices introduced within the focus. As matrix we take the cases in which \(g\) and \(g+h\) satisfy the background applied to the focus directly, and in which the focus is the only item among the alternatives that yields the required result. Actually, we have to introduce an assignment \(i\) that is compatible with \(g+h\), as the focus might introduce its own binding possibilities that are not captured yet by \(h\).

The meaning rule in (29) gives the exhaustive interpretation. The non-exhaustive interpretation can be specified by changing the second argument of MOST to the somewhat simpler \(\langle h \forall i (i = g+h \& <giv> \in B(F)) \rangle\).

Let us look at some examples to see this meaning rule at work. I will compute the exhaustive interpretation.

(30) Most of the time, \(a_1\) frog that sees \(a_3\) fly [climbs \(a_5\) \(REED\)]

\(a_1\) frog that sees \(a_3\) fly, \(\lambda Q.\{<gkwv>\exists h [h \in \{1,2,3\} \& frog_1(h_1) \& fly_2(h_3) \& see_4(h_1,h_2) \& <hkwh_4> \in Q]\}, = [I]

climb \(a_5\) reed, \(\{<gkwxs>\exists k \in \{5\} \& reed_4(k_4) \& climb_2(x,k_5,s)\}, = [J]

[climb \(a_5\) \(REED\)] \(\langle \lambda Q, [J]\rangle\)

INFL \(\langle I\rangle\), \(\lambda Q.\{<gkwv>\exists h [h \in \{4\} \& g_1 \text{ then-} g_4 \& <hkwh_4> \in Q]\}, = [K]

[climbs \(a_5\) \(REED\)] \(\langle [K],[J]\rangle\)

\(a_1\) frog that sees \(a_3\) fly [climbs \(a_5\) \(REED\)] \(\langle \lambda Q, [K]([K](Q)), [J]\rangle\)

most of the time, \(\lambda <B,F>\).MOSTLY(\(<B,F>\))

most of the time, \(a_1\) frog that sees \(a_3\) fly [climbs \(a_5\) \(REED\)]

\(<gkw>\lambda \exists! f (f = g+h \& \langle gfw > \in [I][K][\{<gkwxs> \exists Q \exists j (Q \in ALT_f ([J]) \& <gjwxs> \in Q\}]])\)

\(<gkw>\lambda \exists! f (f = g+h \& \langle gfw > \in ONLY(\langle \lambda Q, [K]([K](Q)), [J]\rangle))\)

Where the first argument of MOST reduces to:

\(\{h \exists f (f = g+h \& f \in \{1,2,3,4\} \& frog_1(f_1) \& fly_2(f_3) \& see_4(f_1,f_3,f_2) \& f_1 \text{ then-} f_4 \& \exists Q \exists j (Q \in ALT_f ([J]) \& <fjwfw> \in Q]\}\)

and the second argument of MOST reduces to:
We get a dynamic meaning that accepts those input functions g, without changing them, and worlds w such that:
- most augmentations h of g such that f=g+h and f₁ is a frog, f₂ is a fly, f₃ sees f₄, in a situation f₅, and there is a situation f₆ occurring after f₇ such that f₈ does something that is an alternative to climbing a reed in f₉,
- are such that they can be extended to i, where i contains a j such that j₁ (=f₁) is a frog, j₂ (=f₂) is a fly, j₃ sees j₄ in j₅ (=f₅). j₆ (=f₆) is a situation following j₇, j₈ climbs a reed in j₉, and climbing a reed is the only thing j₁ does in j₄ among the alternatives, in the given context j.

In this formalization, then, one problem we found with the treatment in Rooth (1985) is solved: We can express quantifications over cases, not only quantifications over situations. Note that any bindings between elements in the background and elements in the focus are only expressed within the second argument of MOST.

What about the problem of exhaustivity? This is taken care of by the operator ONLY. To see how things work, let us have a look at the treatment of example (10). Here, the item in focus is a term, John, which is not of a type for which the meaning rule for most of the time was defined in (29). Terms are of a type represented by λQ.\{<gkwv>\...Q...\}, where Q stands for the verbal predicate to which the term is applied. As in (48), we have to introduce in the restrictor some existentially bound assignment j that allows us to skip over the indices introduced by the item in focus. But in this case, we must make sure that we do not skip over the indices introduced by the verbal predicate for which Q stands for -- that is, we have to exempt indices that are introduced within Q. A meaning rule for most of the time that does that is the following one. The relevant part is the formula ∃v[<gkwv>ε Q], which guarantees that indices introduced within Q are not affected.

(31) \{<gkwv>\}
    MOST((h∈f=g+h & gfwvε B(λQ.\{<gkwv>ε Q\} &
        ∃T\{j=f & Tε ALT,\{F\} & <gkwv>ε T(Q)\}))))
    ((h∈f=g+h & gfwvε ONLY(\{B,F\}))))
    if F is of a type λQ.\{<gkwv>\...\}

Let us now have a look at our example. I change it slightly to one that contains an indefinite NP in focus instead of a name, in order to show the point of the above
definition. Imagine that little Mary has many stuffed animals, among them several teddy bears. She likes to take one of them to bed with her. Let us look at the following sentence in this context:

(32) Most of the time, Mary takes [a TEDDY bear] to bed with her

\[ a_3 \text{ teddy bear}, \lambda Q. \{ \langle gkwv \rangle \exists h [h \in g[3] \& \text{teddy}_w(h) \& <hkwv> \in Q] \}, = [L] \]

\[ [a_3 \text{ TEDDY bear}], <\lambda T. [L]> \]

take to bed, \[ \langle ggwyxs \rangle \text{take}_w(x, y, s), = [M] \]

\[ \text{take} [a_3 \text{ TEDDY bear}] \text{ to bed}, <\lambda T.((M)), [L]> \]

\[ \text{INFL}_2, \lambda Q. \{ \langle gkwv \rangle \exists h [h \in g[2] \& <hkwvh>_2 \in Q] \} \]

\[ \text{takes}_2 [a_3 \text{ TEDDY bear}] \text{ to bed}, \]

\[ <\lambda T. \{ \langle gkwv \rangle \exists h [h \in g[2] \& <hkwvh>_2 \in (T([M]))) \}, [L]> \]

\[ \text{Mary}_1, \lambda Q. \{ \langle gkwv \rangle \exists g_1 = m_w \& <gkwg'_1v> \in Q], = [N] \]

\[ \text{Mary}, \text{ takes}_2 [a_3 \text{ TEDDY bear}] \text{ to bed}, \]

\[ <\lambda T. [N] \{ (\langle gkwv \rangle \exists h [h \in g[2] \& <hkwvh>_2 \in (T([M]))) \}, [L]> \]

most of the time, \[ \lambda <B, F>. \text{MOSTLY}(<B, F>) \]

\[ \text{most of the time, Mary}, \text{ takes}_2 [a_3 \text{ TEDDY bear}] \text{ to bed}, \]

\[ \text{MOSTLY}(<\lambda T. [N] \{ (\langle gkwv \rangle \exists h [h \in g[2] \& <hkwvh>_2 \in (T([M]))) \}, [L]>), \]

\[ = \{ \langle gkwv \rangle \}

\[ \text{MOST}(\{ \langle h \exists f = g + h \& <gfw> \in [N] \{(\langle gkwv \rangle \exists h [h \in g[2] \& <hkwvh>_2 \in (\langle gkwvx \rangle \exists y [y \in g[2] \& <gkwv'y^2> \in [M] \& \exists T_3 [j = f \& T \in ALT_L([L]) \& <gkwv> \in T([M]))]))})

\[ ((\langle h \exists i = g + h \& <giw> \in \text{ONLY}(\langle \lambda T. [N] \{ (\langle gkwv \rangle \exists h [h \in g[2] \& <hkwvh>_2 \in (T([M]))) \}}, [L]>)) \}) \}

The first argument of MOST reduces to:

\[ \{ \langle h \exists f = g + h \& g_1 = m_w \& f \in g[2] \& \exists y, x, s [\text{take}_w(x, y, s)] \& \exists T_3 [j = f \& T \in ALT_L([L]) \& <fiw, k_2> \in T([M]))] \} \]

The second argument of MOST reduces to:
\{ h: \exists i = g + h \land g_1 = m_i \land \exists f \in g[2] \land i \in h[3] \land \text{teddy}_v(i) \land \text{take}_v(g_1, i_2, f_2) \land \\
\forall T \forall j \forall \ell \forall \mathcal{L} \in \text{ALT}_v(T(\mathcal{L})) \land \langle f j w g, f_2 \rangle \in T(\mathcal{L} \rightarrow T(\mathcal{L}))) \}

This accepts input assignments \( g \), without changing them, and worlds \( w \) such that:
- most augmentations \( h \) of \( g \) with \( \text{DOM}(h) = \{2\} \) and \( f = g + h \) such that \( f_1 (= g_1) \) is Mary, \( f_2 \) is a situation where \( f_1 \) takes something to bed, and \( f_1 \) takes some alternative to a teddy bear to bed
- are such that they can be extended to \( i \), where \( g_1 (= i) \) is Mary, \( i_2 \) is a teddy bear, \( g_1 \) takes \( i_2 \) to bed in situation \( i_2 (= f_2) \), and \( g_1 \) doesn't take any alternative to a teddy bear to bed in \( i_2 \). The alternatives here are with respect to an input assignment \( f \) that contains reference to the situation.

This gives us the right reading. We effectively quantify only over situations in which Mary takes something to bed with her, as the augmentations \( h \) just capture the situation variable. A simpler paraphrase would be: Most of the time when Mary took something to bed with her, she took a teddy bear and only a teddy bear with her. The crucial difference to the extension of Rooth's treatment developed in section (3) is that we do not express uniqueness through the situation variable, but more directly by referring to the constituent in focus. In order to do so, we have to IDENTIFY THE CONTENT OF THE ITEM IN FOCUS. Hence, we make use of the additional information that Structured Meaning representations provide us, compared to Alternative Semantics.

7. **Conditionals**

In this section, I will discuss certain effects of focus in conditional sentences. In conditionals the antecedent clause should be part of the restrictor of a quantifier, and hence be part of the background. Now, this clause can have its own focus-background structure, which has an interesting effect on quantification. Kadmon (1987) observed that different accents within the antecedent clause lead to different types of asymmetric quantification: It seems that the quantification is not over all the indices provided by the antecedent clause (cf. also Kratzer 1989, Heim 1990). If we paraphrase usually by most, then we get contrasts like the following one:

(33) a. If a painter lives in a VILLAGE, it is usually nice.
    "Most painters that live in a village live in a nice one"

b. If a PAINTER lives in a village, it is usually nice.
    "Most villages in which there lives a painter are nice"

Recently two theories have been put forward to explain these differences, namely Kratzer (1989) and Chierchia (1990); see also de Swart (1991) for a comparison
of those theories. According to Kratzer, we have existential closure over the VP of the if-clause, which prevents the indices of NPs within the VP to be quantified over. This principle must be supplemented by assumptions that the subject can be interpreted VP-internally, and that constituents that are originally VP-internal may be scrambled outside of the VP, thus escaping existential closure. In Krifka (1992), I discuss some of the problems of this approach. According to Chierchia (1990), quantification is only over topical constituents, which typically are unaccented. One problem with this explanation is that in languages that have an explicit topic marker, like Japanese or Korean, NPs marked as topics do not occur within antecedent clauses.

I think that what matters is not topicality, but being part of the background of a background-focus structure. The right generalization seems to be that indices introduced by expressions in the BACKGROUND of a conditional clause are BOUND BY THE QUANTIFIER, whereas the indices introduced by expressions in the FOCUS are subjected to existential closure and thus are PREVENTED FROM BEING QUANTIFIED OVER. Given that analysis, we would get the right readings if, in (33.a), a painter is in the background, and in (33.b), a village is in the background. Actually, (33.b) would have at least two different interpretations: either the meaning of a village is the only item in the restrictor, or both lives and a village are in the restrictor.

We can express the influence of focus-background articulation in conditional clauses with the following meaning rule:

(34) MOSTLY(if <B,F> then C):
    \{<gkw>!MOST(\{h\exists f=g+h & <gfw>\in B(\{<gjwv>\exists j[<gjwv>\in F])\})
    \{h\exists k[k=g+h & <gkw>\in B(F);C]\}),
    if F is of a type \{<gkw>!...\}.

That is, MOSTLY(if <B,F> then C) is true of input assignments g and worlds w iff most ways h to augment g such that the input g, the output g+h and the world w satisfy the background applied to the focus (where indices introduced within the focus are existentially bound) are such that g+h can be extended to k such that the input g, the output k and the world w satisfy the background applied to the focus, composed with the consequent C. The essential part of this meaning rule is that indices that are introduced within the focus are existentially bound with narrow scope in the semantic representation of the antecedent, hence they are not accessible to the main quantifier. Let us discuss an example:
(35) Most of the time, if a frog sees a fly, it catches it.

\[
\begin{align*}
\text{[see a frog [sees a fly] \& it catches it]} & \\
= \lambda Q. Q. \{<\text{gkwv}x> | x \in [3] \& \text{see}_w(x, k_2, s) \& \text{fly}_w(k_3) \},
\end{align*}
\]

\[
\begin{align*}
\text{INFL}_2, \lambda Q. \{<\text{gkwv}> \exists h \in [2] \& <\text{hkvh}_2> \in Q \},
\end{align*}
\]

/ \\

\[
\begin{align*}
\text{[sees a fly \& it catches it]} & \\
= \lambda Q. \{<\text{gkwv}> \exists h \in [2] \& <\text{hkvh}_2> \in Q \}, \{O\}
\end{align*}
\]

\[
\begin{align*}
\text{a frog,} \lambda Q. \{<\text{gkwv}> \exists h \in [1] \& \text{frog}_w(h_1) \& <\text{hkvh}_2> \in Q \}
\end{align*}
\]

/ \\

\[
\begin{align*}
\text{it catches it,}
\end{align*}
\]

\[
\begin{align*}
\{<\text{gkwv}> | k \in [4] \& k_2 \text{-then-} k_4 \& \text{catch}_w(k_1, k_3, k_4) \}, = [Q]
\end{align*}
\]

\[
\begin{align*}
\text{most of the time}
\end{align*}
\]

/ \\

most of the time, if a frog sees a fly, it catches it.

\[
\begin{align*}
\{<\text{gkwv}> \text{MOST}((\exists f \exists h \in [1,2] \& \text{frog}_w(f_1) \& \exists j \in [3] \& \text{see}_w(f_1, j_3, f_3) \& \text{fly}_w(j_3)))
\end{align*}
\]

\[
\begin{align*}
= \{<\text{gkwv}> \text{MOST}((\exists f \exists h \in [1,2] \& \text{frog}_w(f_1) \& \exists j \in [3] \& \text{see}_w(f_1, j_3, f_3) \& \text{fly}_w(j_3)))
\end{align*}
\]

This accepts input assignments g, without changing them, and worlds w such that most extensions h of g, where the domain of h is [1,2] and h_1 is a frog that can be extended to j, where j_3 is a fly and h_2 is a seeing of j_3 by h_1, are such that they can be extended to k, where k_3 = h_1 is a frog, k_3 is a fly, k_4 = h_2 is a seeing of k_3 by k_2, and k_4 is a situation following k_2 in which k_4 catches k_3. This gives us the intuitively correct interpretation of the most prominent reading: we quantify over frogs h_1 and situations h_2 in which h_1 sees a fly.

The semantic rule (34) is restricted to foci of non-functional types. How can we extend it to cover cases where, e.g., an NP is in focus, that is, an expression of a type \( \lambda Q. \{<\text{gkwv}> \} \)? The extension is relatively straightforward. However, we have to make sure that indices that are introduced within Q are accessible for the quantifier, that is, we have to exempt them from existential quantification. This is done by the following meaning rule:
(36) \( \{ <\text{gkw}>!\text{MOST}(\{h; f=g+h & <\text{gfw}>\in B(\lambda Q.\{ <\text{gkw}>\in Q \& \exists j(j=k & <\text{gjw}>\in F(Q))\})\})) \)
\( \{ <h; k=k' & <\text{gkw}>\in [B(F); C] \}) \)

if \( F \) is of a type \( \lambda Q.\{ <\text{gkw}>!..Q.. \} \)

See Krifka (1992) for further examples and for a discussion of some observations by Kratzer (1989) and de Swart (1991) that quantificational adverbials need a variable to quantify over.

8. Final Remarks

In this paper, I tried to give a formal account of the influence of focus on quantification, in particular on the semantic partition into restrictor and matrix. This was carried out in a framework that combined the Structured Meaning representation of focus with a version of Dynamic Semantics to capture anaphoric bindings. In developing it, we had to pay attention to the notion of focus alternatives within a dynamic setting.

There are several areas that need elaboration. One is that I assumed that focus-sensitive operators apply to sentences. This is not true in general: Particles like only and quantifiers like always clearly can be VP operators. It is relatively straightforward to generalize the semantic types of these operators such that they can be applied to VP meanings of the type \( \{ <\text{gkw}>!.. \} \).

One point which I have suppressed in this paper is that focus can have different sources -- it might be focus associated with an overt operator, or it might be focus associated with the illocutionary operator, so-called "free" focus. We can assume that it is always the focus associated to the highest operator that is spelled out by sentence accent (cf. Jacobs 1991, Krifka 1991). This can easily lead to confusion. In the following example, focus on SUE does not indicate that this phrase is interpreted in the matrix; John is interpreted in the matrix, as focus on SUE is licensed by the illocutionary operator.

(37) [Did Mary always take JOHN to the movies?]

No, SUE always took John to the movies.

Finally, we might question whether the restrictor of an adverbial quantifier is always given by focus-background structures (if not provided by the context). There is an interesting case involving relative clauses which Anna Szabolcsi brought to my attention with examples like (38.a):

(38) a. We should thank the man whom Mary always took to the movies.
b. We should thank the man whom Mary only took to the movies.
(38.a) can be interpreted as: "We should thank the man such that, if Mary took someone to the movies, it was him". But note that the representative of the man within the relative clause, whom (or the empty element coindexed with whom) is not stressed. We might say that relative pronouns, let alone empty elements, cannot bear stress, but still may be in focus. However, if this is so, then only in (38.b) should be associable with the object NP, yielding the reading "We should thank the man such that Mary took only HIM to the movies", which is not available. Szabolcsi suggests that the creation of an empty element by WH-movement is crucial for the construction of the restrictor, which in our example yields the semantic representation $\lambda x \exists s[\text{took}(m,x,s)]$. Szabolcsi (1985), who discusses the focus-sensitivity of superlatives, takes the creation of empty elements as the crucial property even in the cases with focus, following the focus theory of Chomsky (1977), according to which focus implies Wh-movement. However, assuming movement is problematic, as it would not abide by syntactic island constraints (cf. Jackendoff 1972, Krifka 1991).

There is another type of case where we might question how predictive focus is in determining semantic partition of adverbial quantifiers. Schubert & Pelletier (1989), in their discussion of "reference ensembles" (roughly, restrictors), give a number of examples for which they do not claim that focus plays a role. One of their examples is

(39) Cats usually land on their feet.

Note that we could explain this example in terms of background-focus structure: The main accent probably is on feet, hence we have Cats usually land [on their FEET], as a plausible analysis, which would generate the reading: Usually, when cats land on something (a body part of them), then they land on their feet. However, it seems that (39) has a very similar interpretation with the whole VP land on their feet in focus. In this case, Schubert & Pelletier's suggestion that PRESUPPOSITIONS may furnish the reference ensembles (i.e., the restrictor of the quantification) seems to be on the right track, as every case of landing on one's feet presupposes that one is coming down in the first place.

I don't see Schubert & Pelletier's presuppositional theory and the focus theory proposed here as necessarily being in conflict with each other. It seems plausible to assume that the background of a focus-background structure provides or identifies the presuppositions of an expression. If this is so, the role of focus-background structures in semantic partitions could ultimately be subsumed under a general theory of the role of presupposition in quantification.
References


