# Conjunction is parallel computation\*

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**Abstract** This paper proposes a new, game theoretical, analysis of conjunction which provides a single logical translation of *and* in its sentential, predicate, and NP uses, including both Boolean and non-Boolean cases. In essence it analyzes conjunction as parallel composition, based on game-theoretic semantics and logical syntax by Abramsky (2007).

**Keywords:** conjunction, quantifier independence, game theoretic semantics

#### 1 Introduction

An adequate analysis of conjunction *and* should provide a uniform analysis of the meaning of *and* across its various uses. It should apply to various instances of coordinate structures in a compositional fashion, and capture their interpretational properties. In this paper I develop a proposal that unifies Boolean conjunction of sentences with collective (plural), branching, as well as *respectively* readings of coordinate NPs.

A well-known problem for the Boolean approach to conjunction (Gazdar 1980; Keenan & Faltz 1985; Rooth & Partee 1983) are plural readings of coordinate NPs, as in *John and Mary are a nice couple*, which occur in the context of collective (group) predicates. While in many contexts *John and Mary* can be interpreted as a generalized quantifier  $I_j \wedge I_m$ , predicates like *meet*, *be a nice couple*, *be the only survivors* etc. force a group construal of coordinate referential NPs.

And applied to properly quantified NPs can produce a quantifier branching reading (Barwise 1979). Sum formation can be seen as a special case of branching: essentially, plurality formation is branching of Montagovian individuals. I will focus here on NPs with distributive universal quantifiers, such as:

(1) Every man and every woman kissed (each other).
'For every man x and for every woman y, x and y kissed (each other)', or 'every man-woman pair kissed.'

<sup>\*</sup> This paper owes much to the author's discussions of the topic with Chris Barker, Adrian Brasoveanu, Sam Cumming, Jakub Dotlacil, Edward Keenan, Jessica Rett, Barry Schein, and Edward Stabler. All the errors are my own.

In a model with k men and m women, the truth of (1) (in this particular reading) requires  $k \times m$  kissing events. (Reciprocal *each other* is used in this example solely to force the reading of *kissed* as a predicate on pluralities.) Branching quantification applies not only to universals but also to other quantifiers, which do not have to be identical like in (1):

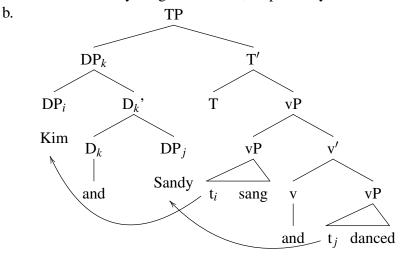
(2) Quite a few boys in my class and most girls in your class have all dated each other. (Barwise 1979)

It is the distributive quantifiers (such as *every man*) that make the branching reading most readily available when coordinated. Quantified NPs that naturally receive collective interpretation (e.g. *all students, five boys*), get a group construal:

(3) Five boys and three girls stay in two different rooms.
'There is a group of five boys staying in one room and a group of three girls staying in another room.'

A peculiar reading of coordination in English is forced with the adverb *respectively*. Although *respectively* constructions can involve various syntactic categories, I consider here only those with coordinate NPs and respective coordinate predicates. Zhang (2007) analyzes examples with *respectively* through sideward movement of two NPs from coordinate sentences into two coodinate positions (Zhang 2007: 51c):

(4) a. Kim and Sandy sang and danced, respectively.



Note that under Zhang's proposal the surface subject  $DP_k$ , contrary to familiar conventions in generative grammar, does not bind a trace or occur as an argument of a predicate. If Zhang's analysis of *respectively* statements is correct, it challenges

standard assumptions about the semantic composition of quantifiers because  $DP_i$  and  $DP_j$  in (4) do not c-command their traces. I address this challenge below.

Meanings of different coordination constructions have been formalized in many ways: Boolean greatest lower bound (Keenan & Faltz 1985); mereological sums (Link 1983); quantifier branching (Barwise 1979); quantifier resumption (Paperno 2010); cross-product (Heycock & Zamparelli 1999); "simultaneous partial interpretation" (Moltmann 1992). The variety of proposals suggests that we are missing a generalization.

## 2 Informal proposal

I propose that to unify coordination of sentences and NPs, we need a dynamic approach to quantification and coordination. One can treat all NPs as quantifiers, and this is in fact a standard move (Barwise & Cooper 1981). But to fully unify sentential and quantifier conjunction, we should treat quantifiers dynamically, on par with sentences, indeed *as* sentences — in a way common for dynamic semantic theories where both quantifiers and sentences are interpreted as context updates. NPs (like *some man*) have the potential to introduce a new referent to the context. Sentences can introduce multiple referents and facts. If quantifier denotations are a variety of sentential denotations, quantifiers can be combined via sentential *and*. Below, I will provide a simple implementation of this basic idea in the framework of Game Theoretic Semantics (GTS).

The interpretation of *and* that I will rely upon is parallel combination of games (Abramsky 2007); the underlying idea belongs to van Benthem (2003), who proposed interpreting branching quantification through concurrent games.

Game semantics is accepted here because of its relative simplicity; game semantics has sufficient expressive power without reference to higher order entities (such as sets or functions) in the object language. But the same proposal translates into other semantic theories as long as they treat quantifiers dynamically, for example, Plural Compositional DRT (Brasoveanu 2007) or Plural Dynamic Logic (Van den Berg 1996).

Informally, I propose to interpret sentences like (a) as paraphrases like (b):

- (5) a. Every man and every woman kissed each other.
  - b. Take an arbitrary man x and take an arbitrary woman y; they (x,y) kissed.

Similar paraphrases can be construed for existentially quantified or referential phrases; note that these paraphrases translate NP coordination by sentential *and*:

(6) a. Some man and some woman kissed each other.

'There is a man x and there is a woman y; they (x,y) kissed each other.'

b. John and Mary are afraid. 'Take John, and take Mary; they are afraid.'

'Take an arbitrary x' is an informal description of the game theoretic semantics for the universal quantifier  $\forall x$ , 'there is an x' is an informal description of the game theoretic semantics for the existential quantifier  $\exists x$ . Respectively-coordination is semantically fully analogous to other coordinate NPs:

- (7) a. John<sub>i</sub> and Bill<sub>k</sub>  $t_i$  sing and  $t_k$  dance, respectively.
  - b. 'Take  $x_i$ =John and take  $x_k$ =Bill,  $x_i$  sings and  $x_k$  dances.'

As the paraphrases suggest, we need a quite particular notion of quantifier. 'Take an arbitrary x' (the paraphrase for  $\forall x$ ) is an instruction; so we need a theory of meaning where quantifiers are treated as instructions — a dynamic semantic theory. I propose a precise formalization below.

#### 3 Insufficiency of Existing Accounts

But before turning to details of my proposal, let us review theories of *and* and the problems they encounter with (1) which motivate the move to a different (game theoretic) semantic framework. The standard crosscategorial theory of conjunction (Rooth & Partee 1983; Keenan & Faltz 1985) predicts the sentence *Every man and every woman kissed (each other)* to be equivalent to \*every man kissed (each other) and every woman kissed (each other). The event-based theory of generalized conjunction by Lasersohn (1995) makes an analogous (wrong) prediction.

Translation of *and* as mereological sums is not applicable to quantified NPs because of a type mismatch: mereology is defined on entities but not on quantifiers (see however Krifka (1990); Gawron & Kehler (2004); Chaves (2007) for attempts to extend mereology to other types).

Hoeksema (1988) proposed a way to save the mereological formalization of and. In his analysis, and is interpreted as Linkian sums, but the two NPs scope out of the conjoined structure, predicting correct truth conditions for (7). Hoeksema's solution, however, is problematic for two reasons. First, it runs contrary to independent evidence that quantifiers normally don't scope out of a conjoined structure. Second, Hoeksema's technique falsely predicts a scope dependency between the two quantifiers. But in fact conjoined quantifiers tend to be scope-independent:

- (8) a. Three boys kissed three girls
  b. Three boys and three girls kissed (each other).
  conjunction
- (7) but not (8) admits a **scope dependent reading**: 'There are three boys such that

each of them kissed three girls' (triples of girls kissed may vary with the boy). The same contrast in scopal dependency arises with distributive quantifiers:

- (9) a. Every man kissed almost every woman.

  Scopal dependency: each man kissed a vast majority of the women; the set of women kissed may vary arbitrarily with the man, to the degree that there might be few or no women that all men kissed.
  - Every man and almost every woman kissed each other.
     No scopal dependency: all men kissed a fixed majority of women.

Consider one model that highlights the semantic contrast between the two sentences. Let there be a large number of men and a large number of women. Assume that every woman is some man's first love. Now assume that every man kissed, and was kissed by, every woman except his first love. In this setting, no woman was kissed by all men: every woman is some man's first love, and every man kissed every woman except his first love, therefore for each woman there was a man who didn't kiss her. Under these circumstances, *Every man kissed almost every woman* can be judged as true; indeed, every man kissed all but one woman. On the other hand, *Every man and almost every woman kissed each other* could be judged false in this scenario; there's no majority of women (in fact, no women) that every man kissed.

Winter (2001) relies on the Boolean theory of conjunction as primary, but adds a shifting operator to account for plurality readings. Winter's proposal, while making correct predictions for the most basic cases, makes wrong predictions for some quantified NPs — in particular, downward monotone ones like *few*, *fewer than six* etc. When it comes to the kinds of examples we consider here, Winter's operator  $\mathbb{C}$  gives as an outcome a single plurality of all men and women (or, more precisely, a Montagovian individual based on that plurality):

(10) 
$$\mathbb{C}(\text{every.man} \land \text{every.woman}) = \lambda P.P(\text{man} \cup \text{woman})$$

So Every man and every woman kissed each other should be paraphrased as 'The group of all men and all women kissed each other.' Although the exact truth conditions of this paraphrase depend on the interpretation of the reciprocal, which can vary (Dalrymple, Kanazawa, Kim, McHombo & Peters 1998), it is clearly different from the reading we aim to capture, and it does not amount to  $k \times n$  kissing events for k men and n women. On a stronger interpretation, the paraphrase would say that everyone who is a man or a woman kissed everyone else who is a man or a woman, implying  $(k+n-1) \times (k+n-1)$  individual kissing events. On a weaker interpretation, it would mean that everyone who is a man or a woman kissed

<sup>1</sup> Both sentences have a scope-independent group reading 'a group of three boys was engaged in kissing with a group of three girls'.

someone else who is a man or a woman; this reciprocal meaning would imply just n + k kissings. A prediction similar to Winter's is made by the event-based approach to NP conjunction by Schein (1993), who relies on a radical neo-davidsonian theory of argument structure in the spirit of Parsons (1990).

Barwise (1979) proposed a simple formula capturing branching:

(11) 
$$\exists P, P'. Q(P) \land Q'(P') \land P \times P' \subseteq R$$
,

and observed that it only applies to MON↑ quantifiers: non-upward monotone quantifiers are either degraded in branching contexts or lead to a different interpretation. In particular, for downward monotone quantifiers (MON↓) Barwise proposed a formula for "branching" that essentially expresses cumulative quantification. Branching combination of quantified NPs is an observationally adequate translation for *and* in sentences like *Every man and every woman kissed (each other)*, but it does not generalize beyond quantified phrases. Moreover, Barwise's formula is an *ad hoc* characterization of branching examples, not related to other uses of *and*. This makes branching combination of quantifiers an unlikely candidate for an independent meaning of *and*.

Krifka (1990) proposes a general way to extend non-Boolean meaning of 'and' to arbitrary semantic types. In particular, when applied to generalized quantifiers, Krifka's semantics of conjunction derives the branching combination of upward monotone quantifiers as introduced by Barwise (1979). However, despite its advantages, Krifka's proposal on crosscategorial non-Boolean and did not catch on. One of the reasons is that Krifka's theory is fairly complex, more so than the standard order-theoretic Boolean semantics of Keenan & Faltz (1985) or mereological semantics for entities (Link 1983). More seriously, in contrast to other accounts Krifka's method gives only an "approximation" for the meaning of and for non-basic types, leaving a significant amount of work to maximalization operators. Most importantly for us, Krifka's proposal is not general enough because it assumes both mereological and Boolean notions for 'and' to be basic for types e and t, respectively.

## 4 Dynamic Semantics and Game Semantics

## 4.1 First approximation: Dynamic Predicate Logic

In dynamic approaches to meaning logical formulas relate to programs (instructions). For example,  $\exists x.P(x)$  reads as an instruction to do two things: assign a value to x ( $\exists x$ ), and then check if P(x) is true. Dynamic Predicate Logic (Groenendijk & Stokhoff 1991) is a paradigm example of a dynamic semantic theory. In DPL, the denotation of a formula is the set of ways of following its instructions (e.g.  $\exists x.P(x)$  denotes the set of ways of updating the value of x so that P(x) is true).

One of our examples is immediately formalizable in DPL. DPL has the very useful feature that quantifiers do not have to c-command variables they bind, so for instance  $\exists x.\phi(x)$  in DPL is equivalent to  $(\exists x) \land \phi(x)$ . Then the sentence *Some man and some woman met* can be straightforwardly interpreted as  $(\exists x \land \exists y) \land \mathbf{met}'(x,y)$ , where  $(\exists x \land \exists y)$  is an instance of dynamic conjunction. In DPL,  $(\exists x \land \exists y) \land \mathbf{met}'(x,y)$  is equivalent to the dynamically interpreted  $\exists x.\exists y.\mathbf{met}'(x,y)$ , with exactly the same truth conditions. This logical translation is compositional: the coordinate NP is translated as a subformula  $\exists x \land \exists y$ , which consists of two quantifiers connected with a conjunction.

But this is as far as DPL can get us. In DPL,  $\exists$  is dynamic:  $\exists x.\phi$  changes the value of x, and the value of x can be invoked outside of the scope of  $\exists x$ . But the universal quantifier  $\forall$  is (externally) static.  $\forall x.\phi$  in DPL is defined as a test (roughly, check whether  $\forall x.\phi$  is true in the classical sense) and the value of the variable x can not be invoked outside of the syntactic scope of  $\forall x$ . So one can not eliminate the scope of a universal quantifier the way one can do away with the existential quantifiers;  $(\forall x.1) \land P(x)$  is equivalent in DPL to P(x), not to  $\forall x.P(x)$  because  $\forall x$  fails to bind x in P(x). So the equivalence which guaranteed compositional logical translation of a conjunction of two existential quantifiers does not hold for universal quantifiers. To model compositional interpretation of quantifier conjunction, we need a theory that treats both existential and universal quantifiers dynamically.

### 4.2 Adding another agent: Game Theoretic Semantics (GTS)

Groenendijk & Stokhoff (1991) propose a computational interpretation of DPL that uses a single **agent** that follows programs — in particular, updates variable values (when instructed by quantifiers) and does tests (e.g. for atomic formulae). The main differentiating trait of Game Theoretic Semantics (GTS) is that GTS uses two agents, so both  $\exists$  and  $\forall$  are dynamic, being instructions for the two agents.

In game semantics, any formula denotes a debate about its truth between the two agents. One of them, called variously "Me", "Verifier", or "Eloise" ( $\exists$ ), tries to prove the formula true, and the other, "Nature", "Falsifier", or "Abelard" ( $\forall$ ) aims to refute it. In an existentially quantified statement  $\exists x.\phi$ , Verifier provides a verifying example (updates the value of x), then the debate proceeds for  $\phi$  for the given x. When one debates the truth of a universally quantified formula  $\forall x.\phi$ , Falsifier proposes a potential counterexample for the universal claim (i.e. Falsifier updates the value of x), then the debate continues for  $\phi$  for the given x.

In frameworks that don't use multiple agents but treat universal quantification dynamically, Falsifier's role is mimicked by adding *all* possible values for *x* to the representation of the context (Van den Berg 1996; Brasoveanu 2007).

The debate between the Verifier and the Falsifier is a zero-sum game, where just

one of the two players wins in the end, and the other loses. In a game denoted by an atomic formula (e.g.  $[P(x)]^{M,g}$ ) Verifier wins iff the formula is true in the classical sense, i.e if P(g(x)) holds. In the game  $[\exists x.P(x)]^{M,g}$  Verifier wins iff she wins in the subgame  $[P(x)]^{M,g}$ , i.e. iff she updates x so that P(x) holds.

A semantic game  $G_A(M, \phi_0, g_0)$  for a formula  $\phi_0$  and a (partial) assignment function  $g_0$  includes a set of actions (positions in the game)  $A = \{(\psi, g)\}$ . Each action is a pair consisting of a subformula  $\psi$  of  $\phi$  and an assignment g produced by extending the initial assignment  $g_0$ . Each move in the game is a shifting from one position to another. Sequences of actions form possible game *histories*, or *plays*.

The set of possible histories  $H_{(\phi_0,g_0)}$  is defined recursively.

- (12) a.  $H_{(\phi_0,g_0)}$  includes the starting action  $(\phi_0,g_0)$ ;
  - b. if  $((\phi_0, g_0), ..., (\phi_{n-1}, g_{n-1})) \in H_{(\phi_0, g_0)}$  and  $\phi_{n-1} = \psi \land \xi$  or  $\phi_{n-1} = \psi \lor \xi$  then  $((\phi_0, g_0), ..., (\phi_{n-1}, g_{n-1}), (\psi, g_{n-1})) \in H_{(\phi_0, g_0)}$  and  $((\phi_0, g_0), ..., (\phi_{n-1}, g_{n-1}), (\xi, g_{n-1})) \in H_{(\phi_0, g_0)}$ ;
  - c. finally, if  $((\phi_0, g_0), ..., (\phi_{n-1}, g_{n-1})) \in H_{(\phi_0, g_0)}^{(v,v)}$  and  $\phi_{n-1} = \exists v. \psi$  or  $\forall v. \psi$  then for all h that differ from g only with respect to v,  $((\phi_0, g_0), ..., (\phi_{n-1}, g_{n-1}), (\psi, h)) \in H_{(\phi_0, g_0)}$ .

 $Z_{(\phi_0,g_0)}=\{h\in H_{(\phi_0,g_0)}\mid \forall h'\in H_{(\phi_0,g_0)}h \text{ is a prefix of }h' \text{ iff }h=h'\}$ , the set of all *terminal histories*, corresponds to complete games. The utility functions  $u_V,u_F$  determine the payoffs for the players in each complete game, so for all terminal histories  $h=((\phi_0,g_0),\ldots,(\phi_n,g_n))\in Z_{(\phi_0,g_0)},u_V(h)=1$  iff  $[\![\phi_n]\!]^{M,g_n}=\top$  in the classical sense, and  $u_V(h)=-1$  iff  $[\![\phi_n]\!]^{M,g_n}=\bot$  in the classical sense;  $u_F(h)=-u_V(h)$  (whenever defined) and  $u_F(h)$  is defined just in case  $u_V(h)$  is defined.

At each point in the game, the structure of the current subformula determines who chooses the next position. The function from (non-terminal) histories to players responsible for the next move P is defined as

$$P(((\phi_0,g_0),\ldots,(\phi_n,g_n))) = egin{cases} V & ext{if} \phi_n = \psi \lor \xi \ F & ext{if} \phi_n = \psi \land \xi \ V & ext{if} \phi_n = \exists v. \psi \ F & ext{if} \phi_n = \forall v. \psi \end{cases}$$

In Game Theoretic Semantics as in Dynamic Predicate Logic the notion of truth is secondary. A formula is true iff there's a way for Verifier to win the game it denotes no matter what his opponent, Falsifier, does. A way to win is called a winning strategy. Formally, a strategy  $\sigma$  for player p is a function on the set of game histories determining p's choice of next position whenever p is responsible for this choice.  $\sigma$  is a winning strategy for player p iff any terminal history that follows  $\sigma$  in moves for which p is responsible is won by p.

By definition, a formula  $\phi$  is true for M, g iff Verifier has a winning strategy in the game  $[\![\phi]\!]^{M,g}$ . In a deep sense the notion of truth is the same in DPL and in game theoretic semantics: a formula is true iff one can successfully fulfill the variable assignment update that the formula requires. The main difference is that in DPL there is no Falsifier whose potential interference has to be taken into account.

*Example*. In the game denoted by  $\forall x. \exists y. P(x,y)$ , Falsifier updates the value for variable x; then Verifier updates y; after that, we check if P(x,y) is true. If it is, Verifier wins, otherwise Falsifier wins.  $\forall x. \exists y. P(x,y)$  is *true* iff Verifier has a winning strategy, i.e. iff no matter how Falsifier updates x, Verifier can choose a y making P(x,y) true.

Not just quantifiers but all operators of first order logic have a game theoretic interpretation. Moreover, for formulas of predicate logic, game-theoretic truth is equivalent to classical truth. The formal metalanguage that many semanticians of natural language use is usually based on predicate logic. So we lose nothing by switching to a game-theoretic interpretation of semantic formulae for the semantic analysis of natural language. Moreover, we can actually gain something if we add new, specifically game theoretic connectives.

### 4.3 Conjunction in GTS

In classical GTS, a conjunction  $\phi \land \psi$  denotes a game in which Falsifier chooses one of the subgames  $\xi = \phi$  or  $\xi = \psi$ . Then  $\xi$  is played, and whoever wins  $\xi$ , wins  $\phi \land \psi$ . Truth conditionally, game theoretic conjunction defined this way is equivalent to conjunction in classical logic, defined by a truth table, even though the game semantics of propositional operators  $\land$  and  $\lor$  is not truth functional. Recall that truth is defined not as a characteristic of a particular game history, but as the existence of a winning strategy. It is possible to construct a winning strategy  $\sigma''$  for Verifier in  $\phi \land \psi$  from winning strategies  $\sigma$  for  $\phi$  and  $\sigma'$  for  $\psi$ , and vice versa; informally, if Verifier can win in  $\phi$  or  $\psi$  not matter which is played, she can win in both. So by the game theoretic definition of truth,  $\phi \land \psi$  is true (for the given M, g) iff both  $\phi$  and  $\psi$  are true for those M, g.

This notion of game theoretic conjunction is not the only logical possibility. Abramsky (2007) proposes other operators for multiagent interactions inspired by linear logic: parallel composition of games ( $\phi \mid \psi$ ) and sequential composition of games ( $\phi \cdot \psi$ ). Under sequential composition  $\phi \cdot \psi$ ,  $\phi$  is played followed by  $\psi$ . Under parallel composition  $\phi \mid | \psi$ ,  $\phi$  and  $\psi$  are played in parallel. Verifier wins a composition of  $\phi$  and  $\psi$  iff she wins both subgames  $\phi$  and  $\psi$ . For any game history  $h \in H_{\phi_0,\psi_0,g_0}$  for a sequential composition  $\phi_0 \cdot \psi_0$ , either  $h \in H_{\phi_0,g_0}$ , or

$$h = ((\phi_0, g_0), \dots (\phi_{k-1}, g_{k-1}), (\psi_0, g_{k-1}), \dots (\psi_{n-1}, g_{n-1}))$$

where 
$$((\phi_0, g_0), \dots (\phi_{k-1}, g_{k-1})) \in Z_{\phi_0, g_0}$$
 and  $u_V(((\phi_0, g_0), \dots (\phi_{k-1}, g_{k-1}))) = 1$ , and  $((\psi_0, g_{k-1}), \dots (\psi_{n-1}, g_{n-1})) \in H_{(\psi_0, g_{k-1})}$ .

For closed formulae, both parallel and sequential composition are truth conditionally equivalent to classical  $\wedge$ . The following equivalence holds for both quantifiers:

(13) a. 
$$\exists x.P(x) \equiv \exists x \cdot P(x)$$
  
b.  $\forall x.P(x) \equiv \forall x \cdot P(x)$ 

Technically,  $\exists x$  and  $\forall x$  may be taken to represent  $\exists x. \top$  and  $\forall x. \top$  where  $\top$  stands for any tautology. This observation sets the stage for coordinating quantifiers in the same way as sentences.

Parallel composition of games can be formalized via informational independence, whereby the two subgames might be played in a particular order but the players have to forget how exactly the first game was played when they play the second one. Technically, this involves counting only those strategies as legitimate winning strategies which do not differentiate game histories of the first subgame when applied to the parallel subgame. Implementing the second approach, we can define parallel composition as follows. A history  $h \in H_{\phi_0||\psi_0,g_0}$  for a parallel composition  $\phi_0 \mid |\psi_0$  of  $\phi_0$  and  $\psi_0$  is either in  $H_{\phi_0,g_0}$  or  $h = ((\phi_0,g_0),\dots(\phi_{k-1},g_{k-1}),(\psi_0,g_{k-1}),\dots(\psi_{n-1},g_{n-1}))$  where  $((\phi_0,g_0),\dots(\phi_{k-1},g_{k-1})) \in Z_{\phi_0,g_0}$  and  $u_V(((\phi_0,g_0),\dots(\phi_{k-1},g_{k-1}))) = 1$ , and  $((\psi_0,g_{k-1}),\dots(\psi_{n-1},g_{n-1})) \in H_{(\psi_0,g_{k-1})}$ , or

$$h = ((\phi_0, g_0), \dots (\phi_{k-1}, g_{k-1}), (\psi_0, g_0), \dots (\psi_{n-1}, g_{n-1}), (\top, g_{k-1} \cup g_{n-1}))$$

where 
$$((\phi_0, g_0), \dots (\phi_{k-1}, g_{k-1})) \in Z_{\phi_0, g_0}$$
 and  $u_V(((\phi_0, g_0), \dots (\phi_{k-1}, g_{k-1}))) = 1$ , and  $((\psi_0, g_0), \dots (\psi_{n-1}, g_{n-1})) \in Z_{\psi_0, g_0}$  and  $u_V(((\psi_0, g_0), \dots (\psi_{n-1}, g_{n-1}))) = 1$ .

Parallel games introduce *partial information* into the game semantics. To relate informational independence to truth, we may refine the notion of strategy  $\sigma$  so that a player in a subgame can not use information on the other parallel subgame to make a move. Since under our definition of parallel composition all the relevant information is encoded in the history but not in the current assignment function, we can require that for all  $h = ((\phi_0, g_0), \dots, (\phi_{n-1}, g_{n-1}), (\phi_n, g_n))$  and  $h' = ((\phi_0, g_0), \dots, (\phi'_{n-1}, g'_{n-1}), (\phi_n, g_n))$ ,  $\sigma(h) = \sigma(h')$  if  $((\phi_0, g_0), \dots, (\phi'_{n-1}, g'_{n-1})) = ((\phi_0, g_0), \dots, (\phi_{n-1}, g_{n-1}))$ .

## 5 'And' in natural language as parallel composition of games

I propose to represent the denotation of *and* in game theoretic terms as parallel composition. This applies to sentential and NP coordination alike. The case of sentential conjunction is straightforward because  $\phi \mid \mid \psi$  is truth conditionally equivalent to

 $\phi \wedge \psi$ . Indeed, Verifier has a winning strategy in  $\phi$  and  $\psi$  played in parallel iff she has a winning strategy for each of them. So leaving the rest of the semantics unchanged, we lose nothing by switching to parallel composition || as the logical counterpart of conjunction in natural language.

If we treat NPs as quantifiers, NP coordination can also be represented via parallel composition, in the same way as sentential coordination. Let us adopt the notation for quantifier restriction  $Q^{[A]}$  from Peters & Westerståhl (2006: 87) where Q is a quantifier and A is a restriction set. In the game theoretic setting quantifier restriction constrains the choice of referent. For example,  $\exists^{[A]}x$  is a move by the Verifier who updates the value of variable x with a model element  $a \in A$ . Likewise,  $\forall^{[A]}x$  is a move by the Falsifier who updates the value of variable x with a model element  $a \in A$ . If so, we can encode the meanings of quantified phrases like *every man* and *some man* as syntactic units of our logical language ( $\exists^{[man]}$  and  $\forall^{[man]}$ ). Building a restriction set into the game semantics of a quantifier follows van Benthem's (2003) approach to modality in game theoretic semantics.

Then the coordinate NP *every man and every woman* receives a logical translation as follows:

(14) 
$$\forall^{[\mathbf{man}]} x \mid \forall^{[\mathbf{woman}]} y$$

Note that this is a combination of semantic values for coordinated NPs *every man*  $(\forall^{[\mathbf{man}]}x)$  and *every woman*  $(\forall^{[\mathbf{woman}]}x)$ . In game semantics this means that Falsifier updates the value of x with a male (discourse) referent, and parallel to that Falsifier updates the value of y with a female referent. In other words, x and y simultaneously receive arbitrary values from the sets of men and women, respectively.

We can attribute the sentence *Every man and every woman kissed (each other)* a logical form like the following:

(15) 
$$[\forall^{[\mathbf{man}]}x \mid | \forall^{[\mathbf{woman}]}y] \cdot \mathbf{kissed\text{-}each\text{-}other}(x,y)$$

where **kissed-each-other**(x,y) stands for  $\exists e(\mathbf{kiss}(e,x,y)||\mathbf{kiss}(e,y,x))$  or whatever the proper denotation of *kissed each other* is (Heim, Lasnik & May 1991; Dalrymple et al. 1998).

So a sentence with conjoined NPs Every man and every woman kissed (each other), represented with a formula  $[\forall^{[\mathbf{man}]}x \mid |\forall^{[\mathbf{woman}]}y] \cdot \mathbf{kissed}(x,y)$ , denotes a game in which Falsifier updates the value of x with a male referent, and parallel to that updates the value of y with a female referent. The outcome of the game is determined by whether x and y kissed each other. This game semantics formalizes the informal description we started with, 'Take an arbitrary man x and take an arbitrary woman y; they kissed each other'.

But universal and existential quantifiers, even relativized to a restriction set,

do not exhaust the range of quantifiers expressible in natural language (Barwise & Cooper 1981; Keenan & Moss 1985). Moreover, the range of natural language quantifiers goes beyond first order definable ones. So it is justified to use second order quantification (quantification over sets); in a different approach to implementing generalized quantifiers in game theoretic semantics, Pietarinen (2007) proposes to use sequences instead of sets. I will use capital letters as variables over sets/predicates; restrictions of second-order quantifiers (still marked as superscripts) are now not sets but sets of sets. For any type  $\langle 1,1 \rangle$  quantifier like *most*, *two*, or *infinitely many*, call it Q, and set A, define Q(A) as  $\{A' \subseteq A \mid Q(A,A')\}$ . Then we can introduce Q into the logic by translating a quantificational statement "QA are B" as

$$(16) \qquad \exists^{[Q(A)]} A'. \forall^{[A']} x. B(x)$$

(16) is equivalent to Q(A,B) for conservative upward monotone quantifiers; note that monotone quantifiers enjoy some nice logical properties (Makowsky & Tulipani 1977) and can be considered basic in natural language quantification (Barwise & Cooper 1981). The translation in (16) allows us again to dissociate the quantifier from its scope:

(17) 
$$Q^{[A]}x.B(x) = Q^{[A]}x \cdot B(x)$$

A conjunction of two such quantifiers  $Q_1, Q_2$  produces the following formula

(18) 
$$(\exists^{[Q_1(A)]}A'.\forall^{[A']}x) \mid (\exists^{[Q_2(B)]}B'.\forall^{[B']}y) \cdot R(x,y)$$

which turns out to be truth conditionally equivalent to Barwise's (1979) branching combination for right increasing quantifiers:

$$(19) \qquad \exists A' \exists B'. F(A') \land G(B') \land A' \times B' \subseteq R$$

where  $F = Q_1(A)$  and  $G = Q_2(B)$ . In other words, we derive quantifier branching as a compositional combination of distributively interpreted generalized quantifiers. The derivation is valid only for monotone increasing quantifiers, exactly the class of quantifiers for which Barwise proposed (19). Compare the following example and its logical translation:

(20) Many a man and every other woman know each other.

(21) 
$$(\exists^{[\mathbf{many}(\mathbf{man})]} A'. \forall^{[A']} x) \mid | (\exists^{[\mathbf{half}(\mathbf{woman})]} B'. \forall^{[B']} y) \cdot \mathbf{know} \text{ each other}(x, y)$$

where the Verifier chooses sets A' of many men and B' that contains half the women, and then for arbitrarily and independently chosen  $x \in A'$  and  $y \in B'$  one checks if x and y know each other. That branching interpretation is not always available for coordinated quantified phrases (even with reciprocal predicates) must be due to the

fact that most quantifiers can be interpreted collectively rather than distributively. If we take a collective interpretation of a quantifier Q to be  $\exists^{[Q(A)]}A'$  instead of the distributive  $\exists^{[Q(A)]}A'. \forall^{[A']}x$ ,

(22) Two boys and three girls like each other.

could be expressed as

(23)  $(\exists^{[\mathbf{two(boy)}]}A \mid | \exists^{[\mathbf{three(girl)}]}B) \cdot \mathbf{like \ each \ other}(A,B)$  'There is a group of two boys A, and there is a group of three girls B, and groups A and B like each other.'

This seems to correctly represent the truth conditions of (22), which are weaker than the branching combination of two quantifiers.

The operator of parallel processing || is designed to handle quantifier independence; indeed, expressing scope independence was the main stimulus for developing game semantics for predicate logic. Take the example of quantifier independence *Every man and almost every woman kissed each other* (9b). The vague quantifier  $[almost\ every\ woman]^{M,g}$  can be formalized as a game where Verifier picks a sufficiently big subset W of women, and the Falsifier picks an arbitrary  $x \in W$ . So (9b) can be given a logical translation of

(24) 
$$[\forall^{[\mathbf{man}]}x \mid ] \exists^{[\mathbf{almost.every(woman)}]}W\forall^{[W]}y] \cdot \mathbf{kissed}(x,y)$$

This denotes a game in which Falsifier picks an arbitrary man x, and parallel to that Verifier restricts the set of women to W, throwing away a few negligible exceptions. Falsifier chooses a woman  $y \in W$ . Any time the outcome of the game is determined by the truth of 'x and y kissed each other', y is arbitrarily chosen from W, which in turn is independent of the choice of x. In order for Verifier to have a winning strategy in this game, she must be able to pick a set W of almost all women that all men kissed.

The discussion so far has been driven by universally quantified NPs, but the approach to conjunction proposed here extends to other NPs as well. A natural extension is to indefinite NPs, which can be treated as existential quantifiers. So  $[someman] = \exists^{[man]} x$ , 'pick a man x'. Coordinated indefinite NPs as in *Some man and some woman kissed each other* can be translated compositionally as a game where Verifier picks a man x, and parallel to that picks a woman y, as expressed by the formula  $[\exists^{[man]} x \mid \exists^{[woman]} y] \cdot kissed(x, y)$ .

Extension to referential NPs is just as straightforward. It is a standard technique to present names as a special case of quantifiers. Among other ways to accomplish this, one can treat referential NPs as a trivial instance of existential quantifiers, so that Mary is interpreted as  $\exists x = \mathbf{m}$ . This applies to all proper names:  $Brezhnev_i$ 

translates as 'update the value of variable i with the referent Brezhnev',  $Honekker_j$  into 'update the value of variable j with the referent Honekker', so the sentence Brezhnev and Honekker kissed translates into  $[\exists^{[\{\mathbf{b}\}]}x \mid | \exists^{[\{\mathbf{h}\}]}y] \cdot \mathbf{kissed}(x,y)$ , with appropriate truth conditions.

More has to be said on negative quantifiers, as in

- (25) a. No man and no woman kissed each other.
  - b. Not every man and not every woman kissed each other.

A careful discussion of negation in game theoretic semantics and a proper implementation of non-upward monotone quantifiers in this framework would go beyond the limits of this paper; for a discussion of properties of game-theoretic negation, see Hintikka (2002, 2006); Caicedo, Dechesne & Janssen (2009). Let me just note that negation translates into role permutation: Verifier takes the role of Falsifier and vice versa. Changing roles and then playing parallel games is equivalent to changing roles in each of the parallel subgames. So in particular — formalizing (25)—  $(\neg \exists x) \mid (\neg \exists y)$  is equivalent in the game logic to  $\neg(\exists x \mid | \exists y)$ , and likewise  $(\neg \forall x) \mid (\neg \forall y)$  is equivalent to  $\neg(\forall x \mid | \forall y)$  (Abramsky 2007), so game semantics has the potential to explain the lack of double negation readings with coordinate negative quantifiers.

#### **6** Syntax Semantics Interface

### 6.1 Plural predicates

The semantics of conjunction outlined above invites a reanalysis of collective predicates. In examples with collective predicates, conjuncts correspond to parts of one plural argument. A collective predicate can combine with a single plural DP, or with a conjunction of arbitrarily many DPs. The usual assumption is that conjoined DPs translate into a single argument of the collective predicate. But one can assume, alternatively, that collective predicates have flexible argument structure, and can take arbitrarily many arguments. Let *lift-the-piano-together* be the linkian one-place predicate that takes one plurality as an argument. Then we can take the denotation of the VP *lift the piano together* to be *lift-the-piano-together\**, a predicate with flexible arity, such that *lift-the-piano-together\**( $x_1, x_2, ..., x_k$ )=*lift-the-piano-together*( $x_1 \oplus x_2 \oplus ... \oplus x_k$ ). This move builds Link's sum operator  $\oplus$  into the predicate. So we can treat predicates over pluralities as having variable arity, taking one or more arguments per thematic role.

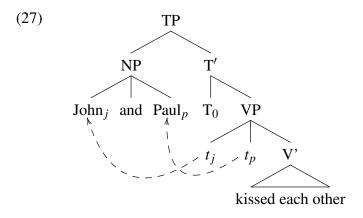
The idea of plural predicates as polymorphic is not new, and was entertained (though not accepted) by various students of plurality, e.g. Landman (1989). Motivation for this comes from examples like (26a) and (26b) which turn out synonymous

under the standard mereological approach to NP conjunction:

- (26) a. The cards below seven and the cards from seven up are separated. (Landman 1989: 574: ex. 27)
  - b. The cards below ten and the cards from ten up are separated.

The coordinate NPs in these two sentences seem to refer to the same set of cards and are combined with the same predicate, yet, intuitively, the meanings of the two sentences are different. Unless the predicate *be separated* is treated as binary, the meaning contrast in (26) either has to be dismissed as in Schwarzschild 1996 or explained through a more complex plural ontology which effectively amounts to representing plural predicates as non-unary. The main argument against modeling plural predicates as polyadic has been that of compositionality: if a coordinate NP is a syntactic unit, it should denote a semantic unit rather than two distinct ones. This becomes a non-issue if we allow coordinate phrases function as polyadic quantifiers. In this case a coordinate phrase is still a semantic unit but it saturates multiple valencies of a predicate it combines with. Now I will present one implementation of this idea that bridges the Chomskian syntactic framework with GTS.

The multiple arguments of collective predicates could be represented as multiple syntactic arguments that merge in sister positions and receive the same thematic role, then are moved into coordinate positions, and check case or other features as a single coordinate constituent:



The idea of movement into coordinate positions follows the proposal by Niina Zhang for *respectively*-statements, which posits sideward movement into coordinate positions (Zhang 2007: (51c)). The current proposal extends Zhang's analysis to all NP coordination (at least the non-Boolean cases). But I do not merely extend Zhang's syntactic proposal to new empirical domains, I also complement it with a compositional semantics.

### 6.2 Compositionality

Now let us define a game theoretic interpretation for syntactic structures of natural language, in particular ones with coordination. Assume the following

### (28) Principles of compositionality

i.each verb is interpreted as a corresponding predicate;

ii.each trace  $t_i$  as a variable i;

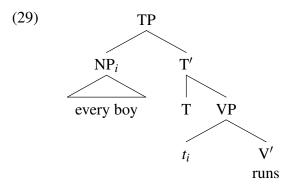
iii.each quantified noun phrase  $NP_i$  as a quantifier binding the variable i,

iv.phrase [A B] is interpreted via function application if  $[A]^{M,g}$  and  $[B]^{M,g}$  are of appropriate semantic types

v.[A B] is interpreted as sequential composition  $[\![A]\!]^{M,g} \cdot [\![B]\!]^{M,g}$  if both A and B denote formulae.

vi.finally, coordinate structures of the form [A and B] can be interpreted as parallel composition  $(\phi \mid \mid \psi)$  of  $[A]^{M,g}$  and  $[B]^{M,g}$ .

These principles of compositionality are standard, with the exception of adding game theoretic operators: sequential composition and parallel composition. Parallel composition is simply the denotation of *and*-coordinated structures (whether quantifiers or sentences). Sequential composition helps connect quantifiers with their scope; given that quantifiers are now formulae on their own, they are combined via the sentential connective (·). Semantic compositionality in action is best understood by way of example. Take a simple sentence *Every boy runs*, represented as



Elements of this structure are mapped into semantic units:  $runs \longrightarrow predicate runs$ ; trace  $t_i \longrightarrow variable i$ ;  $every boy_i \longrightarrow \forall^{[boy]}i$  (ignoring the internal structure and semantic composition of the noun phrase). The predicate run combines with the variable i via function argument application giving an atomic formula run(i). The quantifier  $\forall^{[boy]}i$ , itself a formula, has a sister node denoting run(i), so by principle v above they are combined via sequential composition, giving

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(30) 
$$\forall^{[\mathbf{boy}]} i \cdot \mathbf{run}(i)$$

### **6.3** Interpreting Coordinate Structures

Now let us see how the compositionality principles described above apply to various coordination patterns. Sentential coordination is the most obvious case. The meanings of coordinate clauses are simply combined via parallel composition:

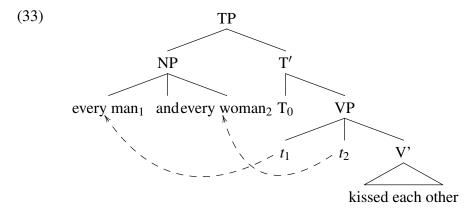
- (31) a. It rains and it snows.
  - b. rain||snow

Predicate coordination is fully analogous if treated as coordination of formulae with an open variable in each formula, where the variables could be the semantic correlates of traces left by ATB movement, compare the English sentence and its logical translation in (32):

- (32) a. Some  $man_i$  ( $t_i$  dances and  $t_i$  sings).
  - b.  $\exists^{[\mathbf{man}]} i \cdot (\mathbf{dance}(i) \mid\mid \mathbf{sing}(i))$

paraphrased in English as 'Pick a man i ( $\exists^{[man]}i$ ), and then (·) check if i dances, and at the same time (||) check if i sings'.

The compositionality principles proposed above also apply to NP conjunction. The syntactic structure of conjunction in our branching example *Every man and every woman kissed (each other)* is as follows:



The rules of compositional interpretation translate (33) into the very formula I proposed above as its semantic representation, namely

(34) 
$$[\forall^{[\mathbf{man}]}x \mid | \forall^{[\mathbf{man}]}y] \cdot \mathbf{kissed}(x,y)$$

The compositionality rules apply to sentences with *respectively*, allowing a compositional semantic treatment without recourse to special interpretational devices such as the pair building denotation of *and* or the plural predicate building meaning of *and* as in Chaves 2012. In fact, all instances of *and* in *respectively* sentences translate into parallel composition, provided that we assume the syntactic derivation of (4) as proposed by Zhang (2007).

In Zhang's account, *respectively* is a semantically vacuous<sup>2</sup> marker of structures with parallel extraction, which guarantees proper coindexing of quantifiers and traces. Given that names are treated as trivial quantifiers ( $[Kim_j]^M = \exists^{[\{Kim\}]} j$ ), (34) translates into the formula

(35) 
$$[\exists^{[\{\mathbf{Kim}\}]} x \mid \exists^{[\{\mathbf{Sandy}\}]} y] \cdot (\mathbf{sang}(x) \mid | \mathbf{danced}(y))$$

interpreted, informally, as 'take x to be Kim, and parallel to that take y to be Sandy, and then check that x sang and y danced', and equivalent to

(36) 
$$\exists x = \mathbf{Kim}. \exists y = \mathbf{Sandy}. \mathbf{sang}(x) \land \mathbf{danced}(y)$$

#### 7 Summary

We saw that the instances of coordination patterns — sentential conjunction, branching readings, group denoting coordination, *respectively* readings, — are served by one simple compositional mechanism that relies on game theoretic semantics. All the differences between these types of coordination constructions, however dramatic, are merely syntactic.

The choice of *parallel*, as opposed to sequential composition, as the denotation of *and*, is intended to capture quantifier independence in coordinate structures. The proposal, presented here in game theoretic terms, can be translated into other theories that treat both existential and universal quantifiers dynamically. Under certain assumptions about dynamic rendering of generalized quantifiers we derive Barwise's generalization that (a particular notion of) branching quantification is restricted to right upward monotone quantifiers. Special "maximality" readings reported for nonmonotone quantifiers by Sher (1990) can be derived via independently motivated maximality operators (Robaldo 2011).

<sup>2</sup> The semantic vacuousness of *respectively* reflects the observation, acknowledged by other scholars (Chaves 2012), that the relevant reading can be observed in sentences without the adverb, but is forced by the overt *respectively* or *correspondingly*.

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