

Ranges: composite measure phrases, modified numerals, and choice functions*

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Abstract Range expressions such as *between 3 and 8*, *from 3 to 8*, and *3 through 8* resemble modified numerals such as *at least 3* and have sometimes been mentioned under that rubric. This paper shows that they are crucially different in their distribution, the readings available to them, and their behavior with respect to quantifiers, and more generally that they have an intricate grammar of their own. We distinguish three classes of readings they can receive: singleton punctual readings, on which they often give rise to ignorance inferences; set punctual readings, which arise chiefly in the scope of quantifiers; and interval readings, where the range is interpreted exhaustively. We propose an analysis on which range expressions denote choice functions over degrees, which can in the right circumstances be parameterized.

Keywords: ranges, modified numerals, choice functions, measure phrases, indefinites, ignorance

1 Introduction

Our focus here will be range expressions such as those in (1):

- (1) a. Floyd saw **between 10 and 15** ferrets.
- b. The kids' ages ranged **from 10 to 15**.
- c. This volume covers $\left\{ \begin{array}{l} \text{from Lincoln to Taft} \\ \text{Lincoln through Taft} \end{array} \right\}$.

These expressions are akin to modified numerals such as *at least 3*, but as we'll show, they have an interesting and subtle grammar of their own. That's especially clear in (1c), of course, which shows that a range needn't even be numeric. Any set with an appropriate ordering—such as the alphabet or American presidents—will

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suffice.¹

We take range expressions to be, structurally, a special case of a more general phenomenon that in other work (Gobeski & Morzycki to appear) we've termed COMPOSITE MEASURE PHRASES:

- (2) a. The room is 10 feet by 15 feet.
b. Her odds of winning are $\left\{ \begin{array}{l} 1 \text{ in } 4 \\ 1 \text{ to } 3 \\ 1 \text{ out of } 4 \end{array} \right\}$.
c. The Padres beat the Red Sox by 3 to 2.

The distinguishing characteristic of these is that they're measure phrases with multiple measure phrases as subconstituents. The resemblance is chiefly syntactic, and we won't linger on it further here, but it's relevant to the larger point that these more complicated structural arrangements have subtle and intriguing semantic consequences.

Range expressions have sometimes been brought up as a form of modified numeral (Nouwen 2010; Rett 2014; Buccola & Spector 2016; Schwarz, Buccola & Hamilton 2012), but there is a considerably larger story to be told. That's reflected most clearly in non-numeric examples like (1c), but also in the selectional restrictions of predicates, among other things.

Section 2 lays out the crucial empirical facts, identifying a diagnostic for range expressions that distinguishes them from ordinary modified numerals. It also divides the readings available to them into three classes, and provides a diagnostic that adjudicates among them. Section 3 takes up the problem of ignorance inferences, which play a major part in the interpretation of modified numerals. Range expressions provide a different way of looking at this domain, and pose challenges to at least one line of thinking about how ignorance inferences arise. That challenge is connected to the question of whether certain range expressions are lexically indeterminate about whether they should get inclusive or exclusive readings. Section 4 proposes an analysis built on the idea that range expressions invoke choice functions over degrees and that provides a way of approaching the facts around indeterminacy, ignorance, and reading types. Section 5 raises two additional puzzles. One of these is about certain modifiers that can occur inside range expressions. The other is about what an analysis of this sort predicts about the availability of intermediate scope readings. Section 6 concludes.

¹ We won't pursue here the question of what properties this order must have, but it apparently needn't actually be a true partial order at all. (American presidents have served non-concurrent terms, but it's natural to build ranges on which they are arranged according to temporal precedence.)

2 The data

2.1 How range expressions are built

There are three basic varieties of range expressions in English, each introduced by distinct morphemes:²

- (3) a. between 3 and 8
 b. (from) 3 to 8
 c. 3 through 8

Predicates that select ranges First, as noted in section 1, there is a class of nouns and verbs that select range complements exclusively. This includes both the nominal and verbal forms of *range* and *span*, as well as verbs *cover*:

- (4) a. the $\left\{ \begin{array}{l} \text{range} \\ \text{span} \end{array} \right\} \left\{ \begin{array}{l} \text{between 2 and 6 / from 2 to 6 / 2 through 6 / up to 6} \\ \text{\#at least 2 / \#more than 2 / \#around 4 / \#under 6} \end{array} \right\}$
 b. This volume covers $\left\{ \begin{array}{l} \text{between G and L / from G to L / G through L / up to L} \\ \text{\#at least G / \#more than G / \#around G / \#under L} \end{array} \right\}$.

For this reason, this is a natural diagnostic for range expressions, especially in contradistinction to (other) modified numerals.

Distinctness and non-zero requirements Range expressions also require that the two constituents be distinct from each other:

- (5) a. #Sam saw between 3 and 3 monkeys.
 b. #Floyd ate a dozen to 12 cookies.

This is unsurprising, of course, but neither is it trivial. The requirement holds even when the numbers are introduced under different descriptions (*between 3 and the number on this playing card*, which is odd if both are 3). Nor, under typical circumstances, can ranges include zero except on a jokey reading:

² There may be a fourth type, which expresses ranges bounded on only one end:

- (i) a. from 3 up
 b. up to 8

In both the cases in (2), *up* is crucial to building a range expression. (Using the diagnostic immediately below: *the range from 3 up*, but not **the range from 3*.) However, the fact that these can be combined (*from 3 up to 8*) and that this looks similar to (3b) suggests that the *up* version may be a variant of (3b) rather than a distinct form. *Up to* is discussed extensively in Schwarz et al. (2012).

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- (6) Floyd is $\left\{ \begin{array}{l} \#0 \\ 1 \end{array} \right\}$ to 6 feet tall.

This restriction is present only where *0* is either independently infelicitous (*0 feet tall*) or a receives a reading similar to “no” (*Sam saw 0 monkeys*; Bylinina & Nouwen 2018). Places where *0* is felicitous also permit ranges that contain *0*:

- (7) a. This car goes from 0 to 60 (miles per hour) in 8 seconds.
b. This toy is for ages 0 through 3.

These facts in conjunction are likely the same phenomenon as Schwarz et al. (2012)’s bottom-of-scale effects. They observe that an expression such as *up to* doesn’t normally include 0 in the range:

- (8) #Up to 1 person died in the crash.

The measurement in (8) starts at 1—and since the two range expressions must be distinct, (8) is infelicitous. The infelicity is improved where 0 is an independently plausible value (*a toy for children aged up to 1*).³

Linear order restriction The linear order of the constituents of a range expression is required to accord with the order on the underlying scale:

- (9) a. Sam saw $\left\{ \begin{array}{l} \text{between 3 and 6} \\ \# \text{between 6 and 3} \end{array} \right\}$ monkeys.
b. We expect $\left\{ \begin{array}{l} \text{(from) 3 to 6} \\ \# \text{(from) 6 to 3} \end{array} \right\}$ people.
c. This class is for ages $\left\{ \begin{array}{l} 3 \text{ through } 6 \\ \#6 \text{ through } 3 \end{array} \right\}$.

This restriction is present for non-numeric ranges, as well as for negative adjectives such as *short* (where, depending on one’s assumptions about the negative adjectives, we might expect the scale to invert; see, among others, Kennedy 2001; Heim 2006):

³ This presupposes that the scale of ages includes a 0-age, which is the age of children younger than 1. More generally, the issues we’re examining are often complicated by assumptions about the internal structure of scales. For example, an exhaustive range expression like *houses 10 through 15* may pick out a range of houses that excludes 13 if that number has been skipped for superstitious reasons, and of course it would normally (but not necessarily!) skip fractional house numbers.

- (10) a. Floyd is $\left\{ \begin{array}{l} 4 \text{ to } 7 \\ \#7 \text{ to } 4 \end{array} \right\}$ inches shorter than Clyde.
 b. #This class is about the presidents $\left\{ \begin{array}{l} \text{between Lincoln and Washington} \\ \text{from Lincoln to Washington} \end{array} \right\}$.

Constituency facts One might in principle be skeptical that range expressions are actually constituents with a specialized structure. That skepticism is most plausible for *from/to* ranges, which one might suspect of being simply adjacent locative PP adjuncts. But the order of true locative PPs can be inverted:

- (11) Floyd drove $\left\{ \begin{array}{l} \text{from Boston to Cleveland} \\ \text{from Cleveland to Boston} \\ \text{between Boston and Cleveland} \\ \text{between Cleveland and Boston} \end{array} \right\}$. (locative)

That is not the case for range expressions:

- (12) We expect $\left\{ \begin{array}{l} \text{from 3 to 6} \\ *to 6 from 3 \end{array} \right\}$ people. (range)

Similarly, locative PPs cleft, but components of ranges don't:

- (13) a. It was *to Cleveland* that we drove *from Boston* __. (locative)
 b. *It was *to 6* that we expect *from 3* __ people. (range)

2.2 Three kinds of readings

We'll distinguish three kinds of readings that range expressions can receive, depending on their environment. We dub these *singleton punctual readings*, *interval readings*, and *set punctual readings*.⁴

Singleton punctual readings The reading that comes most readily to mind is the singleton punctual reading, on which there is a single crucial value within the range, which the speaker may or may not know:

⁴ The use of 'punctual' in this empirical neighborhood is inspired by Rett (2014).

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- (14) a. Floyd is $\left\{ \begin{array}{l} \text{between 5 and 6} \\ 5 \text{ to } 6 \end{array} \right\}$ feet tall.
- b. Floyd weighs $\left\{ \begin{array}{l} \text{between 150 and 200} \\ \text{from 150 to 200} \end{array} \right\}$ pounds.
- c. We expect $\left\{ \begin{array}{l} \text{between 10 and 15} \\ \text{from 10 to 15} \end{array} \right\}$ people.

In (14a), for example, the value of interest is normally Floyd's (maximum) degree of height. The range itself is present only as an epistemic fig leaf covering the speaker's uncertainty.

Interval readings By contrast, on interval readings the whole of the range is crucial:

- (15) a. Water is fluid $\left\{ \begin{array}{l} \text{between 32 and 212} \\ \text{from 32 to 212} \\ \text{at 32 through 212} \end{array} \right\}$ degrees Fahrenheit.
- b. This volume covers American presidents $\left\{ \begin{array}{l} \text{between Lincoln and Taft} \\ \text{from Lincoln to Taft} \\ \text{Lincoln through Taft} \end{array} \right\}$.

In (15a), we're not interested in a single temperature at which water is fluid, but rather the full interval of temperatures. Importantly, *through* seems to receive only interval readings. There is a helpful diagnostic for these readings—they alone allow exceptives:

- (16) a. Water is fluid $\left\{ \begin{array}{l} \text{between 32 and 212} \\ \text{from 32 to 212} \\ \text{at 32 through 212} \end{array} \right\}$ degrees Fahrenheit,
except at 100 degrees, for some reason.
- b. This volume covers American presidents $\left\{ \begin{array}{l} \text{between Lincoln and Taft} \\ \text{from Lincoln to Taft} \\ \text{Lincoln through Taft} \end{array} \right\}$
except (for) Garfield.

Of course, (16a) is false, but it is perfectly well-formed, as is (16b). Range expressions that receive a singleton punctual reading don't support exceptives:

- (17) a. Floyd is $\left\{ \begin{array}{l} \text{between 5 and 6} \\ 5 \text{ to 6} \end{array} \right\}$ feet tall **#except 5 foot 8.**
- b. Floyd weighs $\left\{ \begin{array}{l} \text{between 150 and 200} \\ \text{from 150 to 200} \end{array} \right\}$ pounds **#except 175.**
- c. We expect $\left\{ \begin{array}{l} \text{between 10 and 15} \\ \text{from 10 to 15} \end{array} \right\}$ people **#except 12.**

Set punctual readings On these readings, there are potentially multiple values in the range that are crucial—though unlike on interval readings, these crucial values needn't exhaust the full range. Set punctual readings arise mainly in plural and quantified contexts (including modals):

- (18) a. The children were $\left\{ \begin{array}{l} \text{(from) 5 to 6} \\ \text{between 5 and 6} \end{array} \right\}$ feet tall. *(plural)*
- b. Every student was $\left\{ \begin{array}{l} 5 \text{ to 6} \\ \text{between 5 and 6} \\ \text{?from 5 to 6} \end{array} \right\}$ feet tall. *(quantificational)*
- c. To ride this roller coaster, you must be $\left\{ \begin{array}{l} \text{between 5 and 6} \\ 5 \text{ to 6} \\ \text{?from 5 to 6} \end{array} \right\}$ feet tall. *(modal)*

They are also possible in temporal examples such as (19):

- (19) Floyd's weight fluctuated $\left\{ \begin{array}{l} \text{between 175 and 200} \\ \text{from 175 to 200} \end{array} \right\}$ pounds. *(temporal)*

These readings resemble interval readings in that they involve more than one crucial value, but the exceptive diagnostic demonstrates that they are in fact distinct—exceptives are very odd with set punctual range expressions:

- (20) a. The children were $\left\{ \begin{array}{l} \text{between 5 and 6} \\ \text{(from) 5 to 6} \end{array} \right\}$ feet tall, **??except (for) 5 foot 3.**
- b. Every student was $\left\{ \begin{array}{l} \text{between 5 and 6} \\ 5 \text{ to 6} \end{array} \right\}$ feet tall, **??except (for) 5 foot 3.**
- c. To ride this roller coaster, you must be $\left\{ \begin{array}{l} \text{between 5 and 6} \\ 5 \text{ to 6} \end{array} \right\}$ feet tall, **??except (for) 5 foot 3.**

The quantificational and plural forms in (20a) and (20b), respectively, are odd at best, while the modal form in (20c) is truly strange.

So as these facts illustrate, there is an intricate pattern of interactions between a number of factors regarding range expressions. We omit others for brevity, but the essential factors include the variety of the range (*between*, *from/to*, and *through*), the syntactic context of the range (measure phrase, adjoined, direct object, etc.), and the reading type of the range (singleton punctual, interval, and set punctual). There is also another important factor that influences the reading type: whether the range conveys speaker ignorance, an issue to which we now turn.

3 Uncertainty, ignorance, and indeterminacy

One of the major research questions in the modified numeral literature is what are called IGNORANCE INFERENCES. Geurts & Nouwen (2007) and Nouwen (2010) note that there seem to be two kinds of modified numeral, which Nouwen calls Class A and Class B. Class A modifiers simply express a particular cardinality, while Class B modifiers express a bound. For example, in (21), the modifier expresses a particular cardinality, even if it's not explicit, and the sentence is well-formed. In (21a), the modifier expresses a lower bound and thereby gives rise to the implication there are potentially multiple possible values, and thus the sentence is—for many speakers, to varying extents—odd:

- (21) a. CLASS A: A triangle has **more than** 2 sides.
 b. CLASS B: #A triangle has **at least 3** sides.

This distinction has been analyzed in multiple ways. Nouwen (2010), for instance, treats it as an issue of hidden modal force. Another influential strategy connects the issue to ignorance. We'll sketch the idea via Kennedy (2015), but there is an extensive literature on ignorance inferences with which we can't engage more fully here. It includes Büring (2007); Ciardelli, Coppock & Roelofsen (2018); Coppock & Brochhagen (2013); Coppock (2016); Cremers, Coppock, Dotlačil & Roelofsen (2021); Mayr (2013); Rett (2014); Schwarz (2016a,b); Westera & Brasoveanu (2014).

Kennedy (2015) argues that ignorance inferences arise systematically with non-strict orderings (such as \geq rather than $>$) via implicatures. For Kennedy, these inferences are calculated by comparing a (potentially) modified numeral to alternative modified forms of the same numeral. (Thus *three*, *at least three*, and *at most three* are competing alternatives, but not *three* and *four*.) These alternatives can be ranked according to truth-conditional strength, so they form Horn scales. A non-strict-ordering modified numeral (such as *at least three*) always has a strict-ordering alternative (in this case *more than three*) and consequently gives rise to the implicature that the speaker doesn't know whether the strict-ordering alternative is true (in this case, the speaker doesn't know whether using *more than three* would also be truthful).

Ranges complicate this picture. As we previously noted, they're an only intermittent topic of attention in the modified numeral literature, but two papers in which they are discussed—(Nouwen 2010) and (Rett 2014)—agree in classifying *between* ranges as strict/class A and *from/to* ranges as non-strict/class B.⁵ The sentences in (22) appear to bear this out:

- (22) a. A triangle has between 2 and 6 sides.
b. #A triangle has (from) 2 to 6 sides.

Between is said to be exclusive and therefore strict, so *between 2 and 6* excludes 2 and 6 and refers to the range $\{n \mid 2 < n < 6\}$. *From/to* is inclusive and therefore non-strict, so that *from 2 to 6* includes 2 and 6 and refers to the range $\{n \mid 2 \leq n \leq 6\}$.

But there's an additional wrinkle. As Buccola & Spector 2016 note, *between* can also be inclusive:

- (23) a. We expect between 5 and 6 people to show up.
b. Participants answered on a Likert scale between 1 and 5.
c. This volume covers between Lincoln and Taft.

If *between* were a class B modifier, we would expect (23a) to be necessarily false (or else necessarily about the grisly arrival of fractional humans). For (23b), it would be surprising if 1 and 5 were not included as values, and similarly for (23c), where we would likely conclude Lincoln and Taft are included.

One possible explanation could be that *between* is ambiguous between an exclusive version and an inclusive version, though it's not obvious that this would be analytically helpful, including in reconciling the *between* facts to the approach to ignorance inferences sketched above.⁶ But there is an even clearer challenge to such an approach—the intended reading of *between* can be indicated with endpoint modifiers:

- (24) The tourists saw $\left\{ \begin{array}{l} \text{between 3 and 6} \\ \text{from 3 to 6} \end{array} \right\}$ marmosets $\left\{ \begin{array}{l} \text{inclusive(ly)} \\ \text{exclusive(ly)} \end{array} \right\}$.

These modifiers seem to 'retroactively' change the interpretation of an earlier range expression, and thereby move it between the strict and non-strict classes. Yet

⁵ *Through* ranges seem not to have been previously recognized.

⁶ Presumably, *between* and *from/to* range expressions are each other's Horn-scale alternatives. If *between* is exclusive, *from/to* scales should give rise to the inference that the speaker doesn't know whether *between* would also yield a true sentence. But if it's ambiguous between inclusive and exclusive, should the inclusive and exclusive variants of *between* count as distinct alternatives? If so, how is an addressee to know which *between* has been used? The most epistemically conservative move would be to suppose it was the inclusive variant, but that would predict *between* should be like *from/to* in giving rise to ignorance implicatures.

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these don't change the facts surrounding ignorance implicatures and acceptability judgments in the predicted way:

- (25) a. ?A triangle has between 2 and 6 sides inclusive(ly).
b. ?A triangle has (from) 2 to 6 sides exclusive(ly).

It's unclear what the status of these sentences is, beyond simply annoying.

It seems, then, that *between* is indeterminate between inclusive and exclusive readings—and this makes the analysis of ignorance readings rather foggy. Our approach will be to develop a general analysis of ranges, one that is compatible with the indeterminacy of *between* and the existence of endpoint modifiers, in the hope that this will ultimately shed light on the source of ignorance readings too.

4 Range expressions via degree choice functions

4.1 The analytical direction

Our basic proposal is that range expressions are interpreted via quantification over degree choice functions. Before articulating it further, it may be helpful to suggest why such an approach might be appropriate. First, it resonates with analyses of indefinites, where choice functions have been argued to be crucial [Winter \(1997\)](#); [Reinhart \(1997\)](#); [Matthewson \(1998\)](#) and [Kratzer \(1998\)](#). That's especially clear in [Abels & Martí \(2010\)](#), which argues in part that cardinal numerals are interpreted as choice function indefinites. Second, using degree choice functions will make possible a semantics for *inclusive(ly)/exclusive(ly)*, and provide a means to express the indeterminacy of ranges—especially with *between*—without positing lexical ambiguity. Third, it will provide a means to explain contrasts among the types of readings range expressions receive, ultimately perhaps including the exhaustiveness requirement of *through*. Fourth, we have the vague hope that such an approach may shed light on ignorance readings. Epistemic effects of various sorts—including inferences of ignorance and indifference—are a component of the semantics of many determiners (see [Alonso-Ovalle & Ménéndez-Benito 2015](#)), so an analysis built from the building blocks of determiners might stand a chance of uncovering relevant connections.

To briefly gloss choice functions as a concept, they are functions that pick exactly one element out of a set. Choice functions over individuals are therefore of type $\langle et, e \rangle$, which is a natural type for a determiner. A specific indefinite such as *that one guy* might involve a specific-indefinite determiner *that one* choosing a particular guy from the set of guys. Of course, to know which individual a speaker intends, it's necessary to know what choice function they have in mind. One way to represent the denotation of *that one guy* would be $f(\llbracket \text{guy} \rrbracket)$, where f is a variable

over choice functions. How this variable is resolved differs from one analysis to another, but it is either closed existentially or else its value is provided contextually.

4.2 ‘Between’ and indeterminacy

What we will need here are degree choice functions, type $\langle ed, d \rangle$. The denotation of *between* is simply a variable over such choice functions, and it applies to a interval of degrees and picks a particular degree from that interval (we suppress the assignment function for simplicity):

- (26) a. $\llbracket \textit{between}_f \rrbracket = f$
 b. $\llbracket \textit{between}_f \textit{ 2 and 6} \rrbracket = f([2, 6])$

One additional tool will be necessary to show how this would work in a sentence. The standard analysis of cardinal expressions like *three ferrets* takes the numeral to be a measure-phrase specifier of an implicit cardinality adjective MANY ((Bresnan 1973) and many since). This adjective is standardly taken to have an ‘at least’ semantics, but following Nouwen (2010), we will use a variation that has an ‘exactly’ semantics, MANY_{MAX} . Nouwen uses this alongside a uniqueness quantifier $\exists!$ to represent the denotation of e.g. *three ferrets* as requiring that there is exactly one ferret plurality with a cardinality of three:⁷

- (27) a. $\llbracket \text{MANY}_{\text{MAX}} \rrbracket = \lambda d \lambda P_{\langle e, t \rangle} \lambda Q_{\langle e, t \rangle} . \exists! x [P(x) \wedge Q(x) \wedge |x| = d]$
 b. $\llbracket 3 \text{ MANY}_{\text{MAX}} \textit{ ferrets} \rrbracket = \lambda Q_{\langle e, t \rangle} . \exists! x [\textit{ferrets}(x) \wedge Q(x) \wedge |x| = 3]$

We follow similar lines for *between*:

- (28) $\llbracket \textit{between}_f \textit{ 2 and 6 MANY}_{\text{MAX}} \textit{ ferrets} \rrbracket$
 $= \lambda Q_{\langle e, t \rangle} . \exists! x [\textit{ferrets}(x) \wedge Q(x) \wedge |x| = f([2, 6])]$

The choice function variable is then existentially closed:⁸

- (29) Floyd saw *between 2 and 6 ferrets*.
 a. $\exists f [\textit{between}_f \textit{ 2 and 6 MANY}_{\text{MAX}} \textit{ ferrets}] \lambda x \text{ Floyd saw } t_x$
 b. $\exists f \exists! x [\textit{ferrets}(x) \wedge \textit{saw}(\text{Floyd}, x) \wedge |x| = f([2, 6])]$

This would be true iff there’s a way of choosing a degree from the interval $[2, 6]$ that is the total number of ferrets Floyd saw.

⁷ We’ve taken some small liberties.

⁸ In principle, there should be an explicit requirement here that f is a choice function, and it would probably be appropriate to include a few other general cognitive constraints on it as well.

One advantage of this approach is that it doesn't require us to resolve in the lexical entry whether *between* is exclusive or inclusive/strict or non-strict. A choice function can be either. On the exclusive reading, the intended choice function is one that picks a value other than either endpoint of the interval. Endpoint modifiers like *inclusively* simply indicate that an inclusive or exclusive choice function is intended:

- (30) Floyd saw between 2 and 6 ferrets, inclusively.
 a. $\exists f$ [between_f 2 and 6 MANY_{MAX} ferrets] λx Floyd saw t_x inclusively_f
 b. $\exists f \exists !x$ [ferrets(x) \wedge saw(Floyd, x) \wedge $|x| = f([2, 6]) \wedge$ inclusive(f)]

There are some subtleties in spelling out what this means precisely. When a speaker uses *inclusively*, are they signaling only that f is a function that yields endpoints for some interval in its range? Or is even this very weak semantics too strong, so that an inclusive choice function should be construed instead as one that *can* yield endpoints, in some possible world? The latter approach would interestingly echo Geurts & Nouwen (2007)'s proposal that some modified numerals lexicalize a modal component.

It's less clear to us what to do with the intuition that *between* is exclusive 'by default'. In formal semantics, one doesn't normally think in these terms, but various other frameworks have notions of inheritance that might be able to capture this property.⁹ But the deeper issue is what this intuition actually amounts to empirically—and what sort of evidence might be sufficient to falsify it. We leave this to future research.

4.3 'From-to' ranges, parameterization, and ignorance

What about *from/to*? There are two types of readings for a *from/to* range, depending (more or less) on whether the sentence is quantified:¹⁰

- (31) a. **No quantifier, so singleton punctual only:**
 Floyd saw from 2 to 6 ferrets. (ignorance implicature)
 b. **Quantified, so set punctual:**
 Every tourist saw from 2 to 6 ferrets. (not necessarily)

The challenge is to reflect how some range expressions are sensitive to quantification in a way that yields set punctual readings. We suggest that *from/to* introduces

⁹ Construction Grammar (Goldberg 1997), for example, assumes that constructions inherit properties from their parent constructions. Something similar may be possible in a Pustejovsky-style framework (Pustejovsky 1995) or in frame semantics (Fillmore 1975).

¹⁰ The plural cases may be reducible to quantification with a distributive quantifier. That would predict a correlation between distributive readings and set punctual readings.

parameterized choice functions, in the spirit of Kratzer (1998) and Abels & Martí (2010). A parameterized choice function is a choice function with an additional argument, or, to put it differently, it is a function that yields different choice functions depending on the value of its parameter. Kratzer proposed this as a means of analyzing specific indefinites like *a certain date* in e.g. *Every man forgot a certain date, namely, his wife's birthday*. The sentence involves a particular date, but a different particular date depending on the man. *From/to* ranges, we suggest, manifest such a sensitivity. We'll represent the parameter in the syntax as a subscript:

$$(32) \quad \llbracket \text{from}_{f,x} 2 \text{ to } 6 \rrbracket = f_x([2, 6])$$

This parameter yields set punctual readings when it's bound by the quantifier:

- (33) Every tourist saw from 2 to 6 ferrets.
- a. $\exists f$ [every tourist] λx [$\text{from}_{f,x}$ 2 to 6 MANY_{MAX} ferrets] λy x saw y
 - b. $\exists f \forall x [\text{tourist}(x) \rightarrow \exists! y [\text{ferrets}(y) \wedge \text{saw}(x, y) \wedge |y| = f_x([2, 6])]]$

The sentence is true iff there's a way of choosing a degree from $[2, 6]$ that, for every tourist, identifies the cardinality of the (maximal) ferret plurality that the tourist saw.

But what happens when there is no quantifier to bind the parameter, as in (31a)? This is where ignorance implicatures arise. In order to see why, we will first entertain a brief digression about trying to find the solution to a difficult problem. Someone who thinks they have found it might utter something like (34):

- (34) There's a way of getting to the right answer. It's exactly as follows ...

But things go slightly awry if we try to express this as in (35):

- (35) There's a way of getting *me* to the right answer. ??It's exactly as follows ...

Unlike (34), the first sentence of (35) conveys that the speaker doesn't know the right answer already, which is inconsistent with the next sentence. This shows that there seems to be a conceptual connection between making a choice function sensitive to the speaker and conveying speaker ignorance.

That's what leads to an ignorance implicature in (31a). In the absence of a quantifier, the choice function is parameterized with the speaker, which suggests that the speaker's knowledge state is somehow relevant to the choice of a value from the range. *From/to* range expressions obligatorily give rise to ignorance implicatures in the absence of quantifiers because it is in exactly those cases where the parameter is the speaker. (36) illustrates how this works:

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- (36) Floyd saw from 2 to 6 ferrets.
 a. $\exists f$ [from_{f, the-speaker} 2 to 6 MANY_{MAX} ferrets] λy Floyd saw y
 b. $\exists f \exists !y$ [**ferrets**(y) \wedge **saw**(**Floyd**, y) \wedge $|y| = f_{\text{the-speaker}}([2, 6])$]

This amounts to saying that there's a way of choosing a degree from [2, 6] that gets the speaker to the exact number of ferrets Floyd saw. The ignorance implicature arises via the Maxim of Quantity. The speaker would not say that there's a way to get *them* to the number if they already knew it.

Between, by contrast, doesn't give rise to ignorance implicatures, even on singleton punctual readings, because there's no parameter that can be relativized to the speaker:

- (37) a. Floyd saw between 2 and 6 ferrets.
 b. $\exists f \exists !x$ [**ferrets**(x) \wedge **saw**(**Floyd**, x) \wedge $|x| = f([2, 6])$]

This is the reason for the contrasts in (38):

- (38) a. This triangle has $\left\{ \begin{array}{l} \text{between 2 and 6} \\ \# \text{from 2 to 6} \end{array} \right\}$ sides.
 b. 3 is $\left\{ \begin{array}{l} \text{between 2 and 6} \\ \# \text{from 2 to 6} \end{array} \right\}$.

The explanation of the oddness of *from/to* ranges here is as before: that the ill-formed cases convey that the speaker is ignorant about something they could not be ignorant of if they know what a triangle is. What's different is the path to the implicature—not whether an ordering relation is strict or non-strict, but rather whether a choice functional range expression is relativized to the speaker.

4.4 Brief remark on ‘through’

The most striking property of *through* that, as noted in section 2.2, it gets interval readings only. Accordingly, (39a) entails that all the volumes in the range have been read, and (39b) is odd because interval readings are incompatible with cardinal uses (irrespective of whether *through* occurs in the scope of a quantifier):¹¹

- (39) a. We read volumes 2 through 4.
 b. $\left\{ \begin{array}{l} \# \text{Floyd} \\ \# \text{Every tourist} \end{array} \right\}$ saw 2 through 4 ferrets.

¹¹ It bears pointing out that the impossibility of interval readings in cardinal uses isn't self-evident. If, as on standard assumptions, numerals get ‘at least’ readings, we might have expected (39b) to be well-formed and true, because anyone who has seen 4 ferrets has also seen 2.

Our diagnostic for interval readings was the ability to license exceptives. This fact points toward an analysis. An essential generalization about exceptives is that they are licensed by universal quantification (von Stechow 1991, 1993). Thus it seems reasonable to suppose that interval readings can arise from universal quantification too—more precisely, that *through* induces universal quantification over choice functions.

The interpretation is in other respects similar to how *between* works:

- (40) Floyd read volumes 2 through 4.
 a. $\forall f$ [volumes 2 through_f 4] λx Floyd read t_x .
 b. $\forall f \exists x \left[\begin{array}{l} \mathbf{read}(\mathbf{Floyd}, x) \wedge \mathbf{volume}(x) \wedge \\ \mathbf{volume-number}(x) = f([2, 4]) \end{array} \right]$

The resulting interpretation is that any way of choosing from the range from 2 to 4 yields the number of a volume that Floyd read.

This accomplishes the required task, but the precise mechanism by which *through* ensures that the choice function it introduces is universally quantified isn't apparent. It's simply a stipulation. That's not unreasonable, but neither is it ideal.

It also doesn't suffice to explain interval readings in general, of course. Other range expressions also support interval readings, and in all those cases exceptives are also licensed. What we've proposed here is not a general theory of interval readings that connects them to universal quantification, but rather a means of ensuring that a particular word that supports only interval readings will give rise to them. The challenge of explaining interval readings across range expressions awaits future work.

5 Two unresolved issues and puzzles

5.1 Explicit ignorance (or set punctual?) modifiers

It's possible to explicitly mark a range expression as receiving an ignorance reading with the use of *somewhere* or *anywhere*:

- (41) Every tourist saw $\left\{ \begin{array}{l} \text{somewhere} \\ \text{anywhere} \end{array} \right\} \left\{ \begin{array}{l} \text{between 5 and 10} \\ \text{from 5 to 10} \end{array} \right\}$ capybaras.

These modifiers don't work with run-of-the-mill modified numerals:

- (42) #Floyd saw $\left\{ \begin{array}{l} \text{somewhere} \\ \text{anywhere} \end{array} \right\} \left\{ \begin{array}{l} \text{more than 5} \\ \text{at most 10} \\ \text{minimally 5} \end{array} \right\}$ capybaras.

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They do work with certain other modified numerals, however:

- (43) Floyd saw $\left\{ \begin{array}{l} \text{somewhere} \\ \text{?anywhere} \end{array} \right\} \left\{ \begin{array}{l} \text{around} \\ \text{over} \\ \text{under} \end{array} \right\} 8 \text{ capybaras.}$

Perhaps it's crucial that these involve modification via prepositions, ones that in other contexts are locative. One shouldn't overlook that *somewhere* and *anywhere* do have a *-where* in them, and this doesn't seem likely to be an accident. That locative flavor remains sufficiently that, for some speakers, there is a contrast between *somewhere under 6 people* and *?somewhere before 6pm*, where in the temporal uses *sometime* would be preferred.

It's not the case, however, that these modifiers can only mark ignorance. In contexts in which set punctual readings are possible as well, these modifiers fail to disambiguate in favor of ignorance:

- (44) a. A marmoset can eat anywhere from 2 to 6 dried cranberries.
b. The tourists saw anywhere from 2 to 6 marmosets each.

The acceptability of *somewhere* here is diminished. That accords with the intuition that *anywhere* might at heart be about domain-widening (Kadmon & Landman (1993)), with ignorance a side effect.

5.2 Intermediate scope

A major part of the motivation for using choice functions for specific indefinites in Kratzer (1998) is an analysis of apparent intermediate scope in sentences like (45):

- (45) Every professor_x rewarded every student who read some book they_x had reviewed for the *New York Times*.

This sentence has a reading under which *some book* seems to scope under *every professor* but above *every student*. On this reading, for every professor there's a particular book the professor reviewed and rewarded students for reading. Deriving this reading via Quantifier Raising alone is problematic because *some book* would have to move outside its clause. Kratzer's innovation was to show that if we take *some book* to be a choice function indefinite parameterized with an argument for the professor, the apparent intermediate scope reading can be derived without actually scoping *some book* over *every student* (we have the choice function existentially quantified here, but Kratzer's are supplied contextually):

- (46) a. $\exists f$ [every professor] λx [every student who read some _{f,x} book they _{x} reviewed] λy t_x rewarded t_y
 b. $\exists f \forall x: \mathbf{professor}(x) \forall y: \mathbf{student}(y)$
 $[\mathbf{read}(y, f_x(\mathbf{reviewed-book})) \rightarrow \mathbf{rewarded}(x, y)]$

This is what she calls pseudoscope.

It's natural to ask whether such readings arise with ranges.¹² However, the answer isn't clear. Examples that simply introduce a cardinality component seem to work, but that may reveal only a parameterized choice function reading of an implicit indefinite determiner associated with the numeral:

- (47) a. Every professor _{x} rewarded every student who read (from _{g,x}) three to six books they _{x} reviewed.
 b. $\exists f \exists g \forall x: \mathbf{professor}(x) \forall y: \mathbf{student}(y)$
 $[\mathbf{read}(y, f_x(\mathbf{reviewed-book})) \wedge |f_x(\mathbf{reviewed-book})| = g_x([3, 6]) \rightarrow \mathbf{rewarded}(x, y)]$

If we control for that implicit indefinite determiner, intermediate scope readings don't seem to be available:

- (48) a. Every professor rewarded every student whose shoe size was from _{f} 5 to 12.
 b. $\exists f \forall x: \mathbf{professor}(x) \forall y: \mathbf{student}(y)$
 $[\mathbf{shoe-size}(y) = f_x([5, 12]) \rightarrow \mathbf{rewarded}(x, y)]$

The scenario necessary to support an intermediate scope/pseudoscope reading is odd. Professors vary in their weird preference for shoe sizes. One professor likes only 5–6, another only 6–8, another 10–12. This reading doesn't seem to be possible. This seems to be a major challenge to the parameterized choice function analysis.

But it turns out things are more complicated still. For instance, (49a) does have an intermediate scope reading, but (49b) doesn't easily do so:

- (49) a. Every professor rewarded every student who read some number of books they reviewed.
 b. Every professor rewarded every student who read books they reviewed.

One might interpret this contrast as reflecting that English bare plurals normally take narrow scope (Chierchia 1998, among others), and—on a Kratzer-style analysis—therefore can't be choice-functional. But in the presence of a numeral, the intermediate reading returns:

¹² Thanks to Zhuo Chen for highlighting this issue for us.

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(50) Every professor rewarded every student who read two books they reviewed.

So any implicit indefinite determiner associated with bare plurals must differ from the implicit indefinite determiner associated with numerals. But this is of course the syntactic context that gave rise to an intermediate scope reading in (47). What this shows might be not that range expressions aren't choice functional but that their associated implicit determiner is, but rather that numerals *in general* get choice functional readings. The reason it's hard to disentangle the contribution of the implicit indefinite definite for numerals from the contribution of the range expression may be that they are one and the same.

Complicating matters further, intermediate scope is also possible even in the shoe size scenario—where there is no implicit independent determiner—with *somewhere*:

(51) Every professor rewarded every student whose shoe size was somewhere from 5 to 10.

This also has a wide scope ignorance reading and a narrow scope reading. However, *anywhere* only has the narrow scope reading:

(52) Every professor rewarded every student whose shoe size was anywhere from 5 to 10.

Clearly, then, there must be *some* way to get intermediate readings for ranges. The intricacy of these facts across different environments shows that there's still much more analytical work to do.

6 Conclusion

We've shown that range expressions constitute a natural class of expressions with its own complex grammar, only partly overlapping with modified numerals. They license three types of readings—singleton punctual, set punctual, and interval—that are available with different range expressions and in different structural circumstances. The analysis we've provided pursues the hypothesis that range expressions resemble choice-function indefinites, and that sensitivity to quantified contexts is evidence that for some range expressions, the choice function is a parameterized one. We've also taken a few steps toward an analysis of *through*, which seems not to have drawn attention previously. Range expressions also constitute a challenge for theories of ignorance inferences, which has led us to entertain a rather different kind of explanation for how such inferences arise. We've also embraced the indeterminacy of range expression with respect to whether they are inclusive or exclusive by making them lexically neutral on this point.

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