Ordinal numbers: Not superlatives, but modifiers of superlatives*

Johanna Alstott
Massachusetts Institute of Technology

Abstract  The few existing accounts of the semantics of ordinal numbers attribute to them all or almost all of the semantic properties of superlatives. This work discusses a construction problematic for these existing theories: the ordinal superlative construction (John climbed the third highest mountain). Existing theories give ordinals and superlatives such similar semantics that they struggle to explain how an ordinal and a superlative could join together and form a complex modifier. As an alternative, I propose a semantics according to which ordinals are exceptive modifiers of superlatives. For example, the n-th highest mountain is the mountain that, with the exception of n - 1 others, is the highest. When an ordinal does not co-occur with an overt superlative (e.g. the second train), I posit a covert superlative adjective that represents the contextual ordering. Not only does this approach account for the ordinal superlative construction, but it lends itself to a principled explanation of differences between ordinals and superlatives with respect to plurality.

Keywords: ordinals, superlatives, degree semantics, plurality

1 Introduction

The semantics of ordinal numbers has seen little attention in the literature. The few existing accounts attribute to ordinals all or almost all of the semantic properties of superlatives (Bhatt 2006; Herdan & Sharvit 2006; Sharvit 2010; Yee 2010; Yee 2011; Bylinina, Ivlieva, Podobryaev & Sudo 2014). ¹ Indeed, ordinals and superlatives have striking similarities that seem to support a twin semantics. Ordinals and superlatives both exhibit Szabolcsi’s (1986) relative-absolute ambiguity, license NPIs and Bhatt’s (2006) non-modal subject infinitival clauses, and are similarly focus-sensitive.

This paper discusses a construction problematic for these existing theories of ordinals: the ordinal superlative construction (1). To my knowledge, the only previous work on ordinals that includes an analysis of ordinal superlatives comes from Yee (2010, 2011).

* I thank Amir Anvari, Martin Hackl, Elizabeth Coppock, Kai von Fintel, and audiences at MIT LFRG and SALT 33 for their insights, feedback, and discussion. All remaining errors are my own.
1 Even Bylinina et al.’s (2014) “non-superlative” semantics for ordinals treats them similarly to superlatives, differentiating them mainly in their scope possibilities. I discuss Bylinina et al.’s (2014) approach in section 2 and their data concerning scope possibilities in section 6.

©2023 Alstott
Existing theories give ordinals and superlatives such similar semantics that they struggle to explain how an ordinal and a superlative could join together to form a complex modifier like *third highest*, as acknowledged by Bylinina et al. (2014: 36) and Yee (2011: 274) for their analyses. Yee (2010, 2011) attempts to sweep this problem under the rug by tacitly giving *-est* in *third highest mountain* no semantic contribution. But treating *-est* in (1) as vacuous is not empirically tenable (at least for English), as indicated by semantic contrasts like *the fourth longest home run of Judge’s career vs. the fourth long home run of Judge’s career.*

As an alternative, I propose a new semantics according to which ordinal numbers are exceptive modifiers of overt or covert superlatives. My approach, according to which ordinal numbers are functions that take the superlative morpheme *-est* as an argument, formalizes the following sort of intuition about ordinal superlatives: *the n-th highest mountain is the mountain that, with n - 1 exceptions, is the highest.* The resulting proposal is compatible with a broad range of assumptions about superlatives, including *in situ* theories (e.g. Heim 1999; Sharvit & Stateva 2002) and movement theories (e.g. Heim 1999; Hackl 2009; Bumford 2018).

Since this approach requires something of *-est*’s type as an ordinal’s argument, the question arises of what happens when an ordinal appears in the absence of an overt superlative (e.g. *the fourth train*). In this paper, I pursue a hypothesis according to which *the fourth train* contains a covert superlative adjective that represents the contextual ordering of trains. The idea is that in one context, *the fourth train* might be shorthand for *the fourth earliest train*. But in a different context (say, one where the speaker is counting a line of trains starting from the left), *the fourth train* is likely shorthand for *the fourth leftmost train*.

Armed with this baseline account for ordinals with and without co-occurring superlatives, I turn to a prediction made by this account. The proposal advanced here draws no distinction at LF between ordinal superlatives (*second R-est*) and bare ordinals (*second*): the only difference concerns the overt vs. covert status of the superlative adjective. As such, if we can find a way in which bare ordinals semantically behave differently from bare superlatives, my account predicts that ordinal superlatives should pattern with bare ordinals. I argue that this prediction holds in the realm of plurality. For example, consider (2). For (2a) to be true, A and B need not have arrived at the same time; all that matters is that A and B are the only trains that meet or exceed the contextual threshold of earliness. But (2b) is felicitous and true only if A and B arrived at the same time, after ten other trains. In this respect, ordinal superlatives pattern with bare ordinals and unlike bare superlatives, as predicted: (2c) leads to the inference that A and B arrived at the same time.

(2) a. A and B were the earliest trains to arrive.

(1) Joel climbed the third highest mountain.
Ordinals as modifiers of superlatives

b. A and B were the eleventh trains to arrive.

c. A and B were the eleventh earliest trains to arrive.

Since (2c) patterns with (2b) and unlike (2a) in this regard, an approach that gives (2b) and (2c) identical LFs, such as my own, is well-equipped to account for these data. Building on previous approaches to plural superlatives, I derive the contrast between (2a) vs. (2b-c) within my system via a presupposition held by ordinals but not by \(-est\).

The rest of this paper is structured as follows. Section 2 provides background on superlatives and ordinals, explaining why existing proposals for ordinals have trouble with ordinal superlatives. Section 3 proposes a new semantics where ordinals are exceptive modifiers of superlatives. Section 4 discusses a core prediction of this account, while section 5 argues that the prediction holds in the realm of plurality and sketches an analysis of plural ordinals and ordinal superlatives. Section 6 concludes.

2 The puzzle

This section explains the puzzle that ordinal superlatives pose for existing theories of ordinals. I first outline some standard assumptions about the semantics of superlatives and introduce previous theories of ordinals, which are all fairly similar (section 2.1). In section 2.2, I show that pairing previous proposals about ordinals with my standard assumptions about superlatives leads to an inability to derive the correct meaning for ordinal superlatives. Section 2.3 critiques the only previous account of ordinal superlatives known to me (Yee 2010, 2011), arguing that Yee’s treatment of \(-est\) in ordinal superlatives as vacuous is a non-solution to section 2.2’s puzzle.

2.1 Background: superlatives and ordinals

In this paper, I adopt a standard set of semantic assumptions about superlatives that stems from Heim 1999, the starting point for the contemporary semantic literature on the topic. Heim (1999) foregrounds a systematic ambiguity in sentences like (3) known as the relative-absolute ambiguity, first identified by Szabolcsi (1986).

(3) John climbed the highest mountain.

Suppose that (3) is uttered as a report of what John did on his mountain-climbing club’s trip throughout the Andes. In this context, (3) has two readings. On its absolute reading, (3) asserts that John climbed the mountain higher than all other mountains in the Andes, namely Aconcagua. On its relative reading, (3) asserts that John climbed a higher mountain than anyone else in his club. On this reading, (3) can be true even if John only climbed a mountain a fraction of the size of Aconcagua.
Heim (1999) introduces two approaches to the ambiguity of examples like (3), one of which derives the two readings via domain restriction and the other of which derives them via movement. The two approaches share common assumptions about the lexical entries for gradable adjectives and *-est. Heim (1999) adopts a standard “at least” semantics for gradable adjectives and uses the entry for *-est shown in (4b).

\[
\begin{align*}
(4) & \quad a. \quad \text{[high]} = \lambda d. \lambda x. x' s\ \text{height} \geq d \\
& \quad b. \quad \text{[}-est\ C\text{]} = \lambda C(e,e_1).\lambda G(d,e_2).\lambda x. \forall y [y \in C \rightarrow \exists d [G(d)(y) = 1]]. \\
& \quad \quad \exists d [G(d)(x) = 1 \text{ and } \forall z ([z \in C \text{ and } z \neq x] \rightarrow \neg [z \text{ is a mountain and } z' s\ \text{height} \geq d])] 
\end{align*}
\]

The first argument of (4b) is a comparison class, a null variable denoting the relevant set of entities from which a superlative description picks out the one that ranks most highly on the specified dimension of comparison. For example, the comparison class for the absolute reading of (3) in our Andes scenario would be the set of Andes mountains, as that is the relevant set from which the superlative description *the highest mountain* picks out the member that ranks above all others in height. (4b) takes two arguments other than the comparison class (a degree predicate G and an individual x), and it returns true if and only if x has a higher degree of G than any other entity in the comparison class.²

Heim (1999)’s two approaches not only use the same lexical entries for gradable predicates and *-est but also both use LF (5a) for the absolute reading of (3).

\[
\begin{align*}
(5) & \quad a. \quad \text{John climbed [the [}-est\ C\text{]}[\lambda d \ [d\text{-high mountain}]]] \\
& \quad b. \quad \text{[}-est\ C\text{]}[\lambda d \ [d\text{-high mountain}]] = \lambda x. \exists d \ [x \text{ is a mountain and } x' s\ \text{height} \geq d \text{ and } \forall z ([z \in C \text{ and } z \neq x] \rightarrow \neg [z \text{ is a mountain and } z' s\ \text{height} \geq d])] 
\end{align*}
\]

In (5a), [}-est C] is base-generated as the sister of *high* and undergoes movement to the edge of DP. The LF constituent corresponding to *highest mountain* denotes the singleton consisting of the mountain that is higher than all others in C, as shown in (5b). (5a) as a whole is thus true if and only if John climbed the mountain that is higher than all other mountains in C, as desired.

Heim (1999)’s two approaches diverge in their treatment of relative readings. According to her “in situ” theory, (5a) is the only LF for sentence (3). This approach derives relative and absolute readings by varying the choice of comparison class. For example, an in situ theory would derive the relative reading for (3) in our Andes scenario by setting C to the set of Andes mountains climbed by a member of John’s club rather than the set of all Andes mountains (as in the absolute reading). With C

² (4b) also contains two presuppositions about C. First, the subject of the superlative predication must be in C; second, the members of the comparison class must be commensurable in the sense that they all have degrees of G and can thus be compared in terms of their G-degrees. Going forward, I often omit these presuppositions so the reader can focus on assertional content.
Ordinals as modifiers of superlatives

\(\{x: x \text{ is an Andes mountain climbed by a member of John’s club}\}\), (5a) asserts that the highest Andes mountain climbed by any member of John’s club was climbed by John. This is the relative reading.

Heim (1999)’s other approach derives relative readings via movement of -est beyond the DP boundary, as in the following LF for the relative reading of (3):

\[
(6) \quad \begin{align*}
&\text{a. } \text{John } [[-\text{est } C][\lambda d \text{ climbed A d-high mountain}]] \\
&\text{b. } [[-\text{est } C][\lambda d \text{ climbed A d-high mountain}]] = \lambda x. \exists d \text{ [x climbed a d-high mountain and } \forall z \text{ [z } \in \text{ C and } z \neq x \text{ ] } \rightarrow \text{ z did not climb a d-high mountain}]]
\end{align*}
\]

The derivation for LF (6a) involves not a comparison of relevant mountains in terms of their height but rather a comparison of relevant mountain-climbers in terms of how high a mountain they climbed. The sister of John (6b) denotes the singleton set consisting of the individual that climbed a higher mountain than any other individual in C did, and (6a) asserts that the member of this set is John.

Since Heim 1999, the choice between in situ and movement-based theories of relative readings has become the main locus of debate in the semantic literature on superlatives, and Heim (1999) herself argued in favor of movement. I remain neutral on that debate in this paper; as discussed in section 3, my proposal for ordinal superlatives is compatible with both kinds of theories.

Having established some standard assumptions about superlatives, I next discuss previous perspectives on the semantics of ordinals. These previous proposals take as a starting point striking semantic similarities between ordinals and superlatives. Both exhibit the relative-absolute ambiguity (7), license NPIs and Bhatt’s (2006) non-modal subject infinitival clauses (8), and exhibit similar focus-sensitivity (9).

\[
(7) \quad \text{Ellie caught the second train.}
\]

\[
(8) \quad \begin{align*}
&\text{a. } \text{Absolute: Of the relevant trains, Ellie caught the second earliest one.} \\
&\text{b. } \text{Relative: Only one person caught a train before Ellie did.}
\end{align*}
\]

\[
(9) \quad \begin{align*}
&\text{a. } \text{Chetney gave [Dorian]}_F \text{ the largest/first/second present.} \\
&\quad \leadsto \text{The largest/first/second present Chetney gave was given to Dorian.} \\
&\text{b. } \text{[Chetney]}_F \text{ gave Dorian the largest/first/second present.} \\
&\quad \leadsto \text{The largest/first/second present Dorian received was given by Chetney.}
\end{align*}
\]

3 To derive the relative reading correctly, unadorned movement analyses such as the one presented here must swap the definite determiner in (3) for an indefinite in the relative-reading LF (Szabolcsi 1986; Heim 1999). For recent discussion of this fairly ad hoc aspect of the movement theory and on definiteness in superlatives in general, see Krasikova 2012, Coppock & Beaver 2014, Bumford 2017.
On the basis of these observations, the previous literature has proposed lexical entries for ordinals that are similar to the lexical entry for -est shown in (4b). These lexical entries for ordinals differ in small details from proposal to proposal but express virtually the same semantics. As such, I present here an amalgamation of existing proposals, noting the differences between them where relevant.

Existing proposals give a DP like the second train an LF like (10). As with superlatives, the function of the comparison class C is to restrict the set of individuals being ordered to the relevant ones and, in some proposals, to explain the relative-absolute ambiguity via domain restriction (Sharvit 2010; Bylinina et al. 2014).

(10) \[ \text{the [[second C] train]] \]

According to previous proposals, second is a function that, in (10), takes a comparison class C and the denotation of train and returns the set consisting of the train in C that is number two on the contextually relevant ordering of trains in C. The simple lexical entry in (11a) formalizes this idea, where \( o \) is an ordering retrieved from context and \( a >_o b \) iff a outranks b in \( o \). As shown in (11b), this semantics returns that second train is a predicate that is true of a train in the comparison class that ranks below exactly one other train in the comparison class in the relevant order.

(11) a. \[ n\text{-th} = \lambda C_{\langle e, t \rangle}. \lambda P_{\langle e, t \rangle}. \lambda x: x \in C \text{ and } |C| \geq n. \]

\[ P(x) = 1 \text{ and } |\{ y: y \in C \text{ and } P(y) = 1 \text{ and } y >_o x \}| = n - 1 \]

b. \[ [[\text{second C] train}\]} = \lambda x: x \in C \text{ and } |C| \geq 2. \]

\[ x \text{ is a train and } |\{ y: y \in C \text{ and } y \text{ is a train and } y >_o x \}| = 1 \]

With the exception of Yee 2010, 2011, all previous proposals for ordinals known to me establish an ordering via a non-syntactically realized variable akin to \( o \) in (11). As discussed in section 2.3, Yee (2010, 2011) instead establishes an ordering in the second train via a covert gradable adjective rendered overt in ordinal superlatives.

While existing proposals all assign ordinals similar lexical entries, they derive the relative-absolute ambiguity in (7) differently. Some derive relative readings for ordinals via domain restriction (Sharvit 2010, Bylinina et al. 2014), while others posit that ordinals undergo DP-external movement (Bhatt 2006). As with superlatives, I remain neutral on the question of movement vs. domain restriction for ordinals here.

2.2 The ordinal superlative construction

Regardless of how they derive relative readings, existing proposals for ordinals struggle with ordinal superlatives like (12). Like ordinals and superlatives, ordinal superlatives have both an absolute and a relative reading; existing proposals struggle to derive the correct meaning for (12) even for the simpler case of the absolute reading, so that is that case I focus on in this subsection.
Ordinals as modifiers of superlatives

(12) John climbed the eighth highest mountain.

a. **Absolute**: John climbed the mountain that ranks eighth in height.

b. **Relative**: Seven people climbed a higher mountain than John did.

In a broad sense, existing proposals for ordinals struggle with ordinal superlatives because (11a), their proposed lexical entry for \textit{n-th}, is extremely similar to (4b), the entry for \textit{-est}. For one thing, (11a) and (4b) have nearly identical semantic types (\langle et,\langle et,et\rangle \rangle \text{ and } \langle et,\langle det,et\rangle \rangle , respectively). The two entries also make very similar semantic contributions: while (4b) takes two arguments (including a comparison class) and returns the singleton consisting of the individual that ranks most highly on a given dimension, (11a) takes two arguments (including a comparison class) and returns the singleton consisting of the individual that ranks n-th most highly on a given dimension. These similarities leave existing proposals unable to explain how \textit{n-th} and \textit{-est} can join together to form a complex modifier like \textit{eighth highest}.

More concretely, existing proposals’ entry for \textit{eighth} cannot combine directly with \textit{-est} in \textit{eighth highest mountain} due to type mismatch, and structures in which \textit{eighth} outscopes \textit{-est} or vice versa end up with ill-formed meanings:

(13) the eighth highest mountain

a. eighth > highest mountain
   ‘the element that is #8 in the order \textit{o} of elements in [highest mountain]’

b. -est > eighth d-high mountain
   ‘the highest member of \{x: x is a mountain that is #8 in the order \textit{o}\}’

In (13a), \textit{eighth} outscopes \textit{highest mountain} and takes it as an argument. \textit{Eighth} in (13a) will, according to the lexical entry in (11a), attempt to pick out the element in \textit{highest mountain} that, in the contextually-determined respect, ranks below exactly seven other elements that are in \textit{highest mountain}. Since \textit{highest mountain} denotes a singleton set (see 5b), there is by definition no such eighth element in \textit{highest mountain}. As such, the extension of \textit{eighth highest mountain} on the construal in (13a) will be empty, rendering the definite description (13) undefined. But of course, \textit{the eighth highest mountain} is perfectly felicitous in actuality.

In (13b), \textit{-est} moves above \textit{eighth} and, in essence, picks out the highest member of the set of mountains that are eighth in the contextually relevant order. One problem with (13b) is a threat of overgeneration related to the presence of the free variable \textit{o}. If we let \textit{o} denote any relevant order, we end up predicting clearly unavailable readings. For example, suppose that we’ve been ranking mountains by famousness for hours, making it so that famousness is the contextually relevant order. If \textit{o} is allowed to order mountains in terms of how famous they are, then \textit{the eighth highest mountain} on the construal in (13b) will need to be a mountain that ranks eighth in
famousness. But clearly, ranking eighth in famousness cannot be part of what it means to be the eighth highest mountain.

Even if we can somehow guarantee that *eighth* in (13b) orders the mountains in terms of their height, (13b) still runs into trouble with giving *-est* the right sort of input. To see what I mean, suppose (as is most natural) that there is only one mountain that is eighth in height. Then, \( \{ x : x \text{ is a mountain that is eighth in height} \} \) denotes a singleton, and *-est* in (13b) will redundantly pick out the highest member of this singleton set (i.e. its only member). Other cases in which superlatives are fed a singleton set in this way are infelicitous, as discussed in Hackl 2009 ("my friendliest biological mother").\(^4\) So a structure for the *eighth highest mountain* where *-est* is fed a singleton does not seem to be on the right track: we have reason to believe that such a structure is ill-formed, but *the eighth highest mountain* is felicitous.

As such, one can salvage (13b) only if *-est* is fed a set with more than one element whenever *the eighth highest mountain* is felicitous. There are some cases in which a hypothetical proponent of (13b) could, via a somewhat *ad hoc* ordering, get the right meaning for *the eighth highest mountain* without feeding *-est* a singleton. As an example, suppose that there are ten relevant mountains M1-M10, where M1 is the highest, M2 is second highest, etc. The lexical entry in (11a) does not, strictly speaking, rule out an ordering where M8, M9, and M10 all count as eighth: that is, they are unranked relative to each other but rank below exactly seven other mountains (M1-M7). If M8, M9, and M10 all count as #8 in the given ordering, *-est* in (13b) will pick out the highest mountain in the set \{M8,M9,M10\}, namely M8.

However, there is one case where *the eighth highest mountain* is acceptable but where a proponent of (13b) must feed *-est* a singleton: namely, the case where there are only eight relevant mountains M1-M8. In order for (13b) to correctly denote M8 in this scenario, *-est* in (13b) must be fed a subset of the mountains in which M8 is the highest. But the only subset of our eight mountains in which M8 is the highest is the singleton set \{M8\}. If referring to the shortest of eight mountains as *the eighth highest mountain* involves feeding *-est* a singleton, this use of *the eighth highest mountain* should be just as infelicitous as other cases in which *-est* is fed a singleton, like #my friendliest biological mother. But in actuality, #my friendliest biological mother is significantly more degraded.

2.3 Yee (2010, 2011)

In the previous subsection, I showed that pairing a standard entry for *-est* with (11a), the sort of lexical entry for ordinals used in previous proposals, leads to problems

---

\(^4\) Due the infelicity of such examples, some lexical entries for *-est* (including Hackl’s (2009)) include a presupposition that *-est* is not fed a singleton. But one can also rule out these cases by exploiting their redundancy (Marty 2017) rather than by stipulating a lexical presupposition.
with deriving the correct meaning for ordinal superlatives. In section 3, I will propose a solution to the problem of ordinal superlatives that involves a lexical entry for ordinals quite different from the one in (11a). But first, to lend credence to the idea that ordinal superlatives necessitate a significant departure from (11a), I critique the analysis of ordinal superlatives from Yee 2010, 2011, which keeps the lexical entry in (11a) fairly intact. As far as I know, this is the only previous theory of ordinals that includes an analysis of ordinal superlatives.\footnote{Yee (2010, 2011) adopts Discourse Representation Theory (DRT). Since I lack the space to provide background on DRT here, I translate Yee’s analysis into the Heim & Kratzer (1998)-style notation used so far. I believe that Yee’s semantics for ordinals does not rely on DRT-specific machinery.}

Yee (2010, 2011) proposes an entry for ordinals that differs from (11a) in only one respect. Instead of ranking individuals via an ordering variable \(o\), Yee’s ordinals take an overt or covert gradable adjective \(G\) as an additional argument and rank individuals in terms of their degree of \(G\). One can arrive at this Yee-style entry for \(n\)-th by slightly modifying (11a), adding a \(G\) argument and swapping \(>o\) for \(>_G\).

\begin{equation}
\begin{aligned}
&\left[\text{n-th}\right] = \lambda C(e,t) \cdot \lambda G(d,e,t) \cdot \lambda P(e,t) \cdot \lambda x: \\
&\quad x \in C \text{ and } \forall y \left[ y \in C \rightarrow \exists d \left[ G(d)(y) = 1 \right] \right]. \\
&\quad P(x) = 1 \text{ and } \left| \left\{ z : z \in C \text{ and } P(z) = 1 \text{ and } z >_G x \right\} \right| = n - 1
\end{aligned}
\end{equation}

Using the lexical entry in (14), a bare ordinal like the second train will denote the train in the comparison class with the second highest degree of \(G\), where \(G\) is a covert argument that represents the contextually-determined dimension along which the trains are ordered. Yee (2010, 2011) proposes that in ordinal superlatives, this \(G\) argument is given overtly by the gradable adjective stem of the superlative, e.g. high in third highest mountain. Under the assumption that high is the \(G\) argument of third, the third highest mountain will correctly denote the mountain in the comparison class with the third highest degree of height (see 15). Yee (2010, 2011) only gives a non-decomposed entry for \(n\)-th highest mountain of the sort shown in (15) rather than a full compositional derivation for it.

\begin{equation}
\begin{aligned}
&\left[\text{third C highest mountain} \right] = \lambda x : x \in C \text{ and } \forall y \left[ y \in C \rightarrow \exists d \left[ y\text{'s height} \geq d \right] \right]. \\
&\quad x \text{ is mountain and } \left| \left\{ z : z \in C \text{ and } z \text{ is a mountain and } z >_{\text{high}} x \right\} \right| = 2
\end{aligned}
\end{equation}

While (15) captures the meaning of third highest mountain correctly, it does so by giving no semantic contribution to -est. The only components of third highest mountain that play a semantic role in (15) are third, high, mountain, and the comparison class. Yee (2010, 2011) presents no arguments or discussion to support the idea that -est is vacuous in ordinal superlatives. But -est has to be vacuous in order for his analysis to work; (14), his proposed lexical entry for ordinals, is so similar to...
(11a) that if -est were treated as non-vacuous, (14) would face the same problems with ordinal superlatives as (11a) did (section 2.2). Yee (2011: 274) acknowledges this problem with his analysis in a footnote, noting that “if we strictly follow the principle of compositionality” and give -est a contribution in ordinal superlatives, a compositional puzzle akin to the one discussed for (13a) in section 2.2 arises. Yee (2011) states that he lacks an answer to this puzzle.

Not only does Yee (2010, 2011) treat -est in ordinal superlatives as vacuous out of necessity rather than out of principle, but semantic contrasts like the one shown in (16) strongly suggest that -est is not vacuous in ordinal superlatives.

(16) a. In April 2022, Judge hit the ninetieth longest home run of his career.
   b. In April 2022, Judge hit the ninetieth long home run of his career.

For one thing, it is difficult to read (16a) as ordering home runs along the same dimension as (16b). While the ninetieth longest home run denotes the home run that ranks ninetieth in length, the ninetieth long home run is more naturally understood as denoting the ninetieth in a temporal ordering of “long home runs.” On Yee’s analysis, it is not at all clear why the definite description in (16a) is strongly preferred over the one in (16b) as a way of referring to the home run that ranks ninetieth in length, since long is presumably available as ninetieth’s G argument in both sentences.

(16a) and (16b) also differ in their entailments in a way that strongly suggests a semantic contribution of -est in the former. In particular, (16b) entails that Judge hit a long home run in April 2022, while (16a) does not. To see this, suppose that we live in a possible world (very distant from our own, of course) in which all of Judge’s April 2022 home runs were short by every metric. On the basis of this information alone, we cannot draw a conclusion about whether (16a) is true or false, but we can conclude that (16b) is not true. This difference between ninetieth long vs. ninetieth longest replicates a parallel difference between long and longest, suggesting that -est in (16a) has a semantic contribution. For example, X’s long home run is necessarily long, while Y’s longest home run may not count as long in certain contexts.

3 The proposal

I have argued that existing accounts for ordinals struggle with ordinal superlatives when -est in this construction is treated as non-vacuous (section 2.2), and I have further argued that treating -est in ordinal superlatives as vacuous is not empirically tenable (section 2.3). Given these results, I suggest that ordinal superlatives warrant a significant revision to previous proposals about ordinals, one that takes ordinal

Interestingly, Yee’s analysis may work quite well for languages like Japanese, in which the semantic equivalent of the second highest mountain literally translates to the second high mountain.
Ordinals as modifiers of superlatives

superlatives as a starting point. In that spirit, I propose a semantics according to which ordinals are exceptive modifiers of overt or covert superlatives. One can encapsulate the idea behind this proposal via the following sort of intuitive paraphrase for ordinal superlatives: the n-th highest mountain is the mountain that, with n - 1 exceptions, is the highest. I formalize this intuition via the entry in (17).

(17) \[ n \text{-th} \text{[-est]} = \lambda C. (\lambda x. x \in C \land |C| \geq n \land \forall y \ [y \in C \rightarrow \exists d \ [G(d)(y) = 1]]. \]

\[ |\{z \in C: \forall Q [[Q \subseteq C \land [\text{-est}](Q)(G)(x) = 1] \rightarrow z \notin Q]\}| = n - 1 \]

In (17), \( n \text{-th} \) takes the entry for \(-est\) from (4b) as its first argument and returns, so to speak, a modified version of (4b). Recall that (4b) takes a comparison class \( C \), a degree predicate \( G \), and an individual \( x \) as arguments and returns true iff \( x \) is the member of \( C \) with the highest degree of \( G \). In (17), the constituent \([n \text{-th} \text{-est}]\) takes the same three arguments but instead returns true iff one must subtract exactly \( n - 1 \) other individuals from \( C \) before \( x \) is the element with the highest degree of \( G \). For example, consider a set of mountains \( C \) where \( m_1 = 4,000\text{ft}, m_2 = 3,000\text{ft}, m_3 = 2,000\text{ft}, \) and \( m_4 = 1,000\text{ft} \). The idea is that \( m_2 \), for instance, counts as the second highest mountain because there is exactly one mountain in \( C \) (namely, \( m_1 \)) that is absent from every subset of \( C \) in which \( m_2 \) is the highest.

Using (17), one can give LFs for the absolute and relative readings of ordinal superlatives by minimally modifying Heim’s (1999) LFs for the absolute and relative readings of non-ordinal superlatives. Heim’s (1999) LFs for non-ordinal superlatives (see 5a and 6a) involve movement of the constituent \([-est C]\), and my proposed LFs for ordinal superlatives involve movement of the constituent \([n \text{-th} \text{-est}] C\). For example, (18a) shows the proposed LF for the absolute reading of (12).

(18) a. John climbed \[\text{the \{eighth \text{-est} C\}}[[\lambda d. \text{d-high mountain}]]\]

b. \[\text{\{eighth \text{-est} C\}}[[\lambda d. \text{d-high mountain}]]\] = \( \lambda x. |\{z \in C: \forall Q [[Q \subseteq C \land \text{-est}](Q)(\text{G})(x) = 1] \rightarrow z \notin Q\}| = 7 \]

As shown in (18b), the LF constituent corresponding to eighth highest mountain denotes a predicate that is true of a mountain in the comparison class \( x \) iff one must subtract exactly seven other mountains from \( C \) before \( x \) counts as the highest mountain. (18a) as a whole is thus true if and only if John climbed the relevant mountain that is shorter than exactly seven other relevant mountains, as desired.

As in Heim 1999, one can derive the relative reading of a sentence like (12) via in situ domain restriction or via DP-external movement. An in situ version of my proposal would derive (12)’s relative reading by restricting \( C \) to include only mountains climbed by a (relevant) person. A movement-based version of my proposal for ordinal superlatives would assign (12)’s relative reading the LF in (19).
(19) John [[[eighth -est] C][λd [climbed A d-high mountain]]]

John’s sister denotes the property that holds of a mountain-climber x iff one must subtract exactly seven other mountain-climbers from C before x counts as climbing a higher mountain than anyone else. (19) asserts that this property holds of John.

The theory of ordinal superlatives just proposed necessitates no changes in our semantic treatment of non-ordinal superlatives like the highest mountain. After all, the entry for -est used in LFs (18a) and (19) is the same as Heim’s (1999), so there is no need to change anything about LFs (5a) and (6a). In this way, my proposal interferes minimally with the vast literature on non-ordinal superlatives.

By contrast, the above theory of ordinal superlatives restricts us to a very particular semantic treatment of bare ordinals like the second train. Since the entry for n-th given in (17) requires a degree predicate and something of -est’s type as two of an ordinal’s arguments, adopting (17) leads us to conclude that that even though there is no overt superlative adjective in the second train, there is a covert one. In addition to being required by the semantics in (17), the covert superlative adjective in the second train plays the important role of establishing the contextually-given dimension along which the trains are ordered. The idea is that the second train can come to denote the train that ranks second in earliness with the help of a covert earliest (the second earliest train), while it can come to denote the train that is second to the left in a line with the help of a covert leftmost (the second leftmost train). And so on.

While positing a null superlative adjective in bare ordinals may seem unappealing, all existing semantics for ordinals posit some covert element that, for example, orders trains in the second train. Yee (2010, 2011) gives this role to a covert gradable adjective, while other existing proposals give this role to an ordering variable like o in (11a). Furthermore, positing a null superlative adjective in bare ordinals provides a natural explanation of the superlative-like properties of expressions like the second train; what gives rise to such properties is not second, but rather the covert superlative required as second’s argument.

I conclude this section with two remarks on the syntactic assumptions made in LFs (18a) and (19). Note first that prior to movement, I treat eighth highest as a constituent. In other words, I treat eighth highest as a complex AP rather than treating eighth as an adjective modifying highest mountain. Strong evidence for the complex-AP treatment of ordinal superlatives comes from German. Stacked adjectives in German must each show concord morphology (21a). But in a complex AP, only the head adjective bears concord (21b). Ordinal superlatives in German exhibit the complex-AP pattern rather than the stacked-adjective pattern: the ordinal obligatorily lacks concord, while the superlative carries it (Kai von Fintel, p.c.).

(20) a. Hans ist der groß-e alt-e Mann.
    Hans is the tall-MASC.NOM old-MASC.NOM man.
Ordinals as modifiers of superlatives

'Hans is the tall old man.'

b. Hans ist der wirklich groß-e Mann.
   Hans is the really tall-MASC.NOM man.

'Hans is the really tall man.'

c. Hans ist der dritt größt-e Mann.
   Hans is the third tallest-MASC.NOM man.

'Hans is the third tallest man.'

Second, treating [n-th -est] as a constituent is admittedly strange for English since the order is n-th highest mountain and not *n-th-est high mountain. However, semantic equivalents of n-th and -est do form a surface constituent in languages like Farsi (Amir Anvari, p.c.). Given this variation, a proponent of LFs (18a) and (19) could potentially posit a bracketing paradox in English ordinal superlatives that is absent in Farsi. I leave further examination of this aspect of my LFs to future work.

4 A core prediction

Section 3 addressed the puzzle of ordinal superlatives via a novel semantics for ordinals where n-th takes an overt or covert -est as argument. One might wonder why I did not instead adopt a revised semantics for superlatives where -est takes an overt or covert n-th as argument. This sort of alternative would be a perfect mirror of the proposal advanced in section 3. The general idea would be to treat (17) not as the result of applying n-th to its first argument -est, but rather as the result of applying -est to its first argument n-th. On this alternative proposal, the entry for -est would require an ordinal as an argument, which would in turn necessitate positing a covert ordinal in bare superlatives like the highest mountain (presumably a covert one-th).

This alternative proposal has no need of a covert superlative in the second train, just like section 3’s proposal had no need of a covert ordinal in the highest mountain.

The proposal from section 3 and this alternative are similar in many ways, even though the details of the alternative remain to be worked out. Nonetheless, it is clear that the two approaches make different predictions about ordinal superlatives. The proposal from section 3 predicts that ordinal superlatives will always semantically pattern with bare ordinals, as the only difference concerns the overt vs. covert status of the superlative adjective. By contrast, the alternative proposal just mentioned predicts that ordinal superlatives will always semantically pattern with bare superlatives, as the only difference concerns the overt vs. covert status of the ordinal. As such, we can adjudicate between the two types of proposal if we can find a way in which

8 It is natural to think of first as a suppletive form meaning “one-th.” However, first (and last) may be better analyzed as superlatives. See section 6.
bare ordinals semantically behave differently from bare superlatives; the proposal in section 3 predicts that ordinal superlatives will behave like bare ordinals, while our alternative predicts that they will behave like bare superlatives.

5 Plurality

Bare ordinals and bare superlatives exhibit different behavior under plurality, making the behavior of plural ordinal superlatives a fruitful testing ground for the competing predictions of section 3’s proposal and the alternative sketched in section 4. In section 5.1, I lay out the relevant plural data and show that the prediction of section 3’s proposal is borne out. In sections 5.2 and 5.3, I sketch a preliminary analysis of the data discussed in section 5.1, with section 5.2 introducing some assumptions about plural superlatives and section 5.3 tackling plural ordinals and ordinal superlatives.

5.1 The data

Characteristic examples of plural superlatives, plural ordinals, and plural ordinal superlatives are given in (21a-c):

(21) a. A and B were the earliest trains.
    b. A and B were the eleventh trains.
    c. A and B were the eleventh earliest trains.

  Plural superlatives and plural ordinals exhibit a distributivity-related contrast. Plural superlatives are non-distributive in the following sense: (21a) does not require that the predicate earliest train hold of A and B individually. Instead, (21a) is true so long as A and B are the only trains to meet or exceed the contextual threshold of earliness (Stateva 2005; Fitzgibbon, Sharvit & Gajewski 2008; Bylinina et al. 2014). By contrast, plural ordinals are only felicitous when understood as distributive: (21b) requires that A and B are tied for eleventh in the order. For example, if the relevant order involves arrival time, (21b) requires that A and B arrived at the same time and after exactly ten other trains (Bylinina et al. 2014). If A and B arrived at different times, referring to them with a plural ordinal is less natural than referring to them via a conjunction like the eleventh and twelfth trains.

9 In addition to this absolute or “individual” reading (Fitzgibbons et al. 2008), plural superlatives can also have a relative or “group” reading. The group reading is salient in a sentence like Nepal has the highest mountains, whose most natural reading expresses that Nepal’s mountains are, on average, higher than those anywhere else. Plural ordinals and ordinal superlatives have group readings as well and do not differ from plural superlatives in this regard. As such, the group reading is less useful for testing the predictions of our two approaches, so I leave it aside. For more on the group reading, see Fitzgibbon et al. 2008 and Scontras 2008.
With regards to this contrast, plural ordinal superlatives pattern with bare ordinals rather than bare superlatives. Similarly to (21b), (21c) requires that A and B are tied for “eleventh earliest:” that is, they arrived at the same time and after ten other trains. The similar properties of (21b) and (21c) are predicted under section 3’s proposal, which draws no distinction between the two sentences at LF. Furthermore, section 3’s proposal distinguishes (21a) from (21b-c) in terms of the absence vs. presence of an ordinal at LF, making it possible to reduce the distributivity contrast in (21a) vs. (21b-c) to a presupposition of n-th not shared by -est (see sections 5.2 and 5.3). By contrast, the alternative proposal mentioned in section 4 would struggle to derive the contrast between (21a) and (21c) given that this proposal posits a covert ordinal in (21a). If (21a) and (21c) each contain an ordinal and a superlative, it is not obvious how the distributivity contrast between them arises.

5.2 Plural superlatives

Having argued that my framework for thinking about singular ordinals and ordinal superlatives is well-equipped to handle the data in (21), I now sketch a “proof of concept” for an analysis of (21a-c) within my framework, starting with some assumptions about plural superlatives like (21a). There are a small handful of analyses of plural superlatives already on the market (e.g. Stateva 2005; Fitzgibbons et al. 2008; Scontras 2008). Here, I choose to utilize an approach discussed by Fitzgibbons et al. (2008: 312–313) that is quite similar to but simpler than their final proposal. I opt for a set of fairly barebones assumptions about plural superlatives for expository clarity; I believe my proposal for plural ordinals and ordinal superlatives (section 5.3) is compatible with more involved ideas about plural superlatives as well.

The approach to plural superlatives I utilize here hinges on a revision to the entries for gradable adjectives like early that encodes a particular idea of what it means for a plural individual X to be d early: (i) X is d early if all of its atomic parts are; (ii) X is not d early if none of its atomic parts are; (iii) if some of X’s atomic parts are d early but others are not, [early](d)(X) is undefined. When looking at (22), note that I use an approach to plurality where individuals are identified with sets (Schwarzschild 1996): a singular individual like Imogen is {Imogen}, while the plural individual consisting of Imogen and Laudna is {Imogen, Laudna}. Nothing hinges on choosing this approach to plurality over, e.g. Link’s (1983) mereology.

\[
\text{[early]} = \lambda d. \lambda X. \begin{cases} 
1 & \text{if } \forall x [x \in X \rightarrow x’s earliness \geq d] \\
0 & \text{if } \neg \exists x [x \in X \land x’s earliness \geq d] \\
# & \text{otherwise}
\end{cases}
\]
With (22) in mind, consider the proposed LF for (21a). I assume the lexical entry for -est from (4b). ‘∗’ is Link’s (1983) distributivity operator, which takes a one-place predicate P (e.g. train) and returns a one-place predicate that is true of all the Ps and true of every plural every individual all of whose atomic parts are Ps.

\[(23)\]

\begin{enumerate}
\item a. A and B are [the [[-est C][d-early ∗train]]]
\item b. \[∗P = λX. \forall x ∈ X \ [P(x) = 1]\]
\item c. \[[-est C][d-early ∗train]]((A,B)), when defined, is true iff \[∃d \ [∗train]((A,B)) = 1 \text{ and } \text{early}(d)((A,B)) = 1 \text{ and } \forall Y [Y ∈ C \text{ and } Y ≠ X] → [\text{early}(d)(Y) = 0]\]
\end{enumerate}

As shown in (23c), (23a) is true iff there is a degree of earliness \(d\) with the following two properties: (i) every atomic part of \(\{A,B\}\) is a train that is at least \(d\) early; (ii) for every other (plural or singular) train in the comparison class, none of their atomic parts are at least \(d\)-early. In other words, (23a) is predicted to be true iff the latest train in the set \(\{A,B\}\) is earlier than every other train. These are quite close to the intuitive truth-conditions, thanks largely to how we defined the truth, falsity, and definedness conditions for early. However, the truth-conditions in (23c) are not quite right for (21a), as Fitzgibbons et al. (2008) note: (21a) requires not just that A and B are earlier than every other train but also that A and B are the only trains to meet or exceed the contextual cut-off of earliness. I set aside the issue of cut-offs in superlatives here, as it introduces a layer of (here unnecessary) complexity.

The approach to plural superlatives sketched in (22-23) necessitates no changes to our Heim (1999)-style approach to singular superlatives (section 2.1): the entry for -est used in this section is the same as Heim’s (1999), and (22) only differs from Heim’s (1999) entries for gradable adjectives once pluralities enter the picture.

### 5.3 Plural ordinals and ordinal superlatives

Armed with a simple theory of superlatives that allows us to account for plural cases like (21a), I now extend section 3’s treatment of ordinals and ordinal superlatives to plural cases like (21b-c). The only previous analysis of plural ordinals and ordinal superlatives I know of comes from Yee (2010, 2011). Bylinina et al. (2014) mention plural ordinals as well, but they do not provide an analysis. Having already argued that Yee’s analysis is inadequate for singular ordinal superlatives (section 2.3), I do not thoroughly compare our analyses of the plural cases here.\(^\text{11}\)

My approach to plural ordinals and ordinal superlatives involves one revision to section 3’s semantics for ordinals: namely, a new presupposition for \(n-th\) that is not shared by -est. This new presupposition is given in (24):

\[11\] Moreover, I am unsure whether our analyses deal with the same data: Yee (2010, 2011) seems concerned with something akin to the “group” reading of plural ordinals (fn. 9), while I am not.
Ordinals as modifiers of superlatives

(24) \([n\text{-th}][\text{[-est]}](G)(C)(x)\) is defined only if \(\forall X \in C: |X| > 1 \rightarrow\)

(i) \(\forall x \forall x' \in X: \forall d [G(d)(x) = G(d)(x')]\) and

(ii) \(\neg \exists Y [Y \in C \text{ and } Y \neq X \text{ and } \forall d [G(d)(X) = G(d)(Y)]]\)

Informally, (24) imposes a strong restriction on the kind of plural individual permitted in an ordinal or ordinal superlative’s comparison class. Such a plural individual X must have the following two properties: (i) X must consist of atomic parts that are all tied on the G dimension; (ii) no other individual in the comparison class may be tied with the atoms of X on the G dimension. For example, suppose that Mt. Ebott is 4,000 feet, Mountains 2, 3, 4, and 5 are each 3,000 feet, and Mt. 6 is 2,000 ft. The only plural individual built from these atoms that is allowed in an ordinal or ordinal superlative’s comparison class is \{Mt.2,Mt.3,Mt.4,Mt.5\} (assuming that the G dimension in (24) is height). A plural individual like \{Mt.Ebott,Mt.2\} is ruled out by (24.i), while plural individuals like \{Mt.2,Mt.3\} are ruled out by (24.ii) under the assumption that Mt. 4 or Mt. 5 are also in the comparison class.

With this revision in mind, consider the proposed LF for sentences (21b-c), which is given in (25). I utilize the entry for \(n\text{-th}\) given in (17), modulo our new presupposition; I also use the entries for \(\text{-est}\) and \(\text{early}\) given in (4b) and (22).

(25) A and B are \([\{\text{eleventh -est} \text{ C}\}[\lambda d. d\text{-early *train}]])

Once we incorporate (24) into our entry for \(n\text{-th}\), LF (25) can only be true and felicitous if A and B arrived at the same time and after ten other members of the comparison class, matching our intuitions about (21b-c). The truth-conditions and presuppositions of (25) both play a role in deriving this correct meaning. (25) asserts that with exactly ten exceptions, the plural individual \{A,B\} is the earliest member of the comparison class (akin to 18a). Meanwhile, the presupposition in (24) guarantees that for \{A,B\} to be in (25)’s comparison class at all, A and B must be equally early. Since the presupposition which enforces that A and B are equally early is present only when \(n\text{-th}\) is present, we correctly predict that a plural superlative like (21a) lacks the requirement that A and B arrive at the same time.

Plural ordinals and ordinal superlatives deserve further scrutiny, but the foregoing discussion suggests that an analysis within section 3’s framework for ordinals is likely feasible. The above approach to plural ordinals and ordinal superlatives requires no changes to section 3’s approach to singular ordinals and ordinal superlatives: the only difference between the entry for ordinals used in section 3 and the one used here concerns (24), which is vacuously satisfied unless pluralities enter the picture.

6 Conclusion

I have argued that ordinal superlatives pose intractable problems for existing theories of ordinals and proposed an alternative theory that takes ordinal superlatives as a
starting point. My proposal allows for the possibility of ordinal superlatives behaving differently from bare superlatives but predicts that ordinal superlatives and bare ordinals will have the same semantic properties. I have argued that these predictions of my theory are borne out in the realm of plurality, but there is another area in which these predictions may not hold: the (un)availability of Heim’s (1999) “upstairs de dicto” readings. Heim (1999) notes that sentences like (26a) are acceptable in “upstairs de dicto” scenarios like (26), a fact which (for reasons I lack space to discuss) she construes as an argument for a movement theory of superlatives. In my judgment, ordinal superlatives are also acceptable in such scenarios. By contrast, bare ordinals have been claimed by Bylinina et al. (2014) to lack upstairs de dicto readings, although the relevant judgments are fairly murky in my estimation.

(26) Context: John’s desire is to take some train or other between 1pm and 2pm. Bill’s desire is to take some train or other between 3pm and 4pm. Mary’s desire is to take some train or other between 5pm and 6pm.
   a. John wants to take the earliest train.
   b. Bill wants to take the second earliest train.
   c. (?)Bill wants to take the second train.

Since my proposal draws no distinction between ordinal superlatives and bare ordinals at LF, my proposal would require revision if the contrast in (26b-c) is a genuine one. Such a revision, if necessary, should draw a more substantive distinction between ordinal superlatives and bare ordinals while not ignoring the ways in which ordinal superlatives and bare ordinals are alike and behave differently from bare superlatives (section 5). I leave a more nuanced proposal of this sort to future work.

Another direction for future research concerns the status of first and last, which differ from other ordinals in a number of ways. For one thing, first and last cannot appear in “the ordinal slot” of ordinal superlatives (#first highest, #last highest). Furthermore, plurals with first and last act more like plural superlatives than plural ordinals: A and B were the first/last trains to arrive triggers no inference that A and B arrived at the same time (cf. 2b-c). Finally, first and last behave like superlatives and unlike other ordinals with respect to Bhatt’s (2002) “intensional superlative” ambiguity, as discussed in Heycock 2005 and Charnavel 2022. These facts suggest that first and last may be better analyzed as superlatives than ordinals.

Yet another direction for future research concerns how ordinal superlatives slot into the broader cross-linguistic picture of ordinals. Although ordinals and superlatives cohabit freely in languages like English and German, languages such as Russian and Japanese (see fn. 7) lack ordinal superlatives altogether and express similar meanings without (overt) superlative morphology. Given this variation, future work should identify parameters for the expression of ordinal meaning.
Ordinals as modifiers of superlatives

References


Marty, Paul. 2017. Implicatures in the DP domain: Massachusetts Institute of
Technology PhD dissertation.


Johanna Alstott
Department of Linguistics and Philosophy
Massachusetts Institute of Technology
32 Vassar Street
Cambridge, MA 02139
jalstott@mit.edu