Telescoping in incremental quantification*

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Abstract Bumford (2015) argues that universal quantification in dynamic semantics should be analyzed as generalized dynamic conjunction for empirical benefits. However, this analysis is incompatible with the existing telescoping analyses, which use a pluralized dynamic system (van den Berg 1996; Nouwen 2003; Brasoveanu 2007: a.o.). This study aims to resolve this conflict. It is proposed that quantification over events and their participants allows us to account for telescoping without the pluralized dynamic system.

Keywords: dynamic semantics, incremental quantification, generalized conjunction, telescoping

1 Incremental quantification in dynamic semantics: Bumford (2015)

Bumford (2015) analyzes universal quantification in dynamic semantics as generalized dynamic conjunction. Assuming our model contains three relevant students, John, Bill, and Fred, sentence (1a) is analyzed as (1b), where ; is dynamic conjunction in the sense of Groenendijk & Stokhof (1991).

(1) a. Every student read a book.
   b. [[John read a book]] ; [[Bill read a book]] ; [[Fred read a book]]

This incremental analysis of universal quantification accounts for various "pair-list" phenomena, two of which are described here (see Bumford 2015 for the others). One is sentence-internal readings of different, exemplified in (2). In the sentence-internal reading, the sentence is true iff: for every pair of different boys x and y, x recited a different poem than the one y recited.

(2) Every boy recited a different poem. (Brasoveanu 2011: 94)

Under the incremental analysis, the sentence subsumes three conjuncts, as in (3). Each conjunct introduces a discourse referent (dref) for a boy and a poem. Then

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different requires that the dref for a poem newly introduced to the discourse is distinct from any other drefs previously introduced to the discourse.

(3)  [[John recited a different poem]] ;
     [[Bill recited a different poem]] ;
     [[Fred recited a different poem]] ;

Suppose that a discourse is represented as a numbered list. Then the way the proposal works is depicted as follows. The initial, empty discourse \(i_\star\) is updated to \(i_1\) by the first conjunct. Different requires \(\text{poem1}\) not to be in the previous discourse, \(i_\star\), which is trivially satisfied for \(i_\star\) being vacuous. Then \(i_1\) is further updated to \(i_2\) by the second conjunct, which requires the new referent of a poem – \(\text{poem2}\) – to be distinct from anything in the previous discourse, \(i_1\). The third conjunct updates \(i_2\) to \(i_3\) in the same manner.\(^1\)

\[
\begin{array}{c|ccc}
\text{sentence} & \text{John recited a different poem} & \text{Bill recited a different poem} & \text{Fred recited a different poem} \\
\hline
\text{discourse} & i_\star & i_1 & i_2 & i_3 \\
\text{update} & 0 & 0 & 0 & 0 \\
\text{drefs} & \text{john poem1} & \text{john poem1 bill poem2} & \text{john poem1 bill poem2 fred poem3} \\
\text{new drefs} & \text{poem1} & \text{poem2} & \text{poem3} \\
\end{array}
\]

The benefit of the incremental analysis is that each conjunct has access to the drefs introduced by previous conjuncts. It enables different to require a new dref to be distinct from any other drefs in the last discourse and allows us to account for sentence-internal readings of different without further complexities.

The second pair-list phenomenon Bumford discusses is sentence-internal readings of comparative, exemplified in (5a). The sentence is true in the sentence-internal reading iff for each year \(y\), Mary wrote a more interesting book in \(y\) than the book(s) she wrote before \(y\). The sentence-internal comparative differs from the sentence-internal different in that the former is asymmetric while the other is symmetric. In

\(^1\) It turns out, however, that the analysis makes a wrong prediction for other cases. Consider (4a) with the model under which John, Bill, and Fred are students.

(i)  a. Each student wrote to a different person.
    b.  [[John wrote to a different person]] ;
        [[Bill wrote to a different person]] ;
        [[Fred wrote to a different person]] ;

The sentence is true in the sentence-internal reading if John wrote to Bill, Bill to Fred, and Fred to John. This is not predicted by the analysis described here. The update by the third conjunct in (3) cannot introduce John to discourse as the person Fred wrote to because the previous discourse already contains John. This problem seems inherent to Bumford’s analysis and does not show up in another dynamic analysis of sentence-internal readings proposed by Brasoveanu (2011).
the symmetric sentence-internal reading in (2), for any poem x and y, x is distinct from y and vice versa. This is not the case in the asymmetric reading, under which one of the two books is better than the other. The asymmetric sentence-internal comparative is accounted for by the incremental analysis by letting *more interesting* require a book newly introduced to the discourse to be more interesting than the book(s) previously introduced.2

(5) a. Every year Mary wrote a more interesting book. (Bumford 2015: 6)  
   b. [[In 2019 Mary wrote a more interesting book]] ;  
      [[In 2020 Mary wrote a more interesting book]] ;  
      [[In 2021 Mary wrote a more interesting book]]

The incremental analysis also has a theoretical benefit. It predicts the close connection between the "pair-list" phenomena and universal quantification observed by Beck (2000) and Brasoveanu (2011): sentence-internal readings of *different* and comparatives are licensed by universal quantification like *each* and *every*, but not by other quantifiers, conjunction, or plurals. Compare (2) with (6) and (7a) with (7b).

(6) {All (of) the , Both} boys read a different book. (Brasoveanu 2011: 99)

(7) a. Each generation inhabits a progressively more Orwellian world. (Brasoveanu 2011: 144)  
   b. #{ Those, Several, Most, 0} generations inhabit a progressively more Orwellian world. (Bumford 2015: 9)

Bumford hypothesized that the unavailability of a sentence-internal reading in (6) or in (7) is because of the lack of incremental quantification. It is only a universal quantifier that induces incremental dynamic conjunction.

Despite the empirical and theoretical benefits, the incremental analysis faces an issue in accounting for *telescoping.*3 I propose that this issue be avoided by incorporating event discourse referents into the system. By quantifying over events and their participants, we can retrieve the dependency between drefs lost in the incremental approach. The rest of the paper is organized as follows. The issue is more closely investigated in section 2. Section 3 is devoted to the proposal. As I did in the current section, I abstract away from formal details in the discussion

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2 As Bumford (2015) points out, the parallel accounts of the symmetric and the asymmetric readings are one of the empirical advantages of the incremental analysis over Brasoveanu (2011).
3 As discussed in section 4, the incremental analysis also loses the standard analysis of plural anaphora and quantificational subordination (Karttunen 1976). I informally suggest that the analysis to be proposed can be extended at least to quantificational subordination.
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until section 3.1. Section 4 compares the proposal with the standard analysis based on a pluralized system and discusses other phenomena for which the incremental analysis seems to lose an account. I argue that the current proposal has an empirical advantage, despite some remaining issues. Section 5 concludes.

2 An issue: telescoping

A loss by adopting the incremental analysis is the standard analysis of telescoping, exemplified in (8) from Groenendijk & Stokhof 1991. It is interpreted as if the quantifier every took scope over the entire sequence of sentences, that is, beyond the sentence boundary, to bind the personal pronoun he. Furthermore, it does not refer to any specific pawn, and its reference depends on the reference of he. Thus, the sequence of sentences is interpreted as For every player x: x chooses a pawn y and x puts y on square one.

(8) Every player chooses a pawn. He puts it on square one.

The example has posed two theoretical challenges for dynamic theories like DPL (Groenendijk & Stokhof 1991) and DRT (Kamp & Reyle 1993). These theories define universal quantification to be externally static. Groenendijk & Stokhof (1991), for example, define it as (9). Crucially, the update by the universal quantification does not introduce any new discourse referent for the resultant assignment function (hence $f = g$): universal quantification carries out a test. The drefs introduced by the scope of the quantification are discarded (in the DPL terms) or become inaccessible (in the DRT terms) for the following discourse. More specifically, the result of the update by the first sentence in (8) is equivalent to its input, and the resultant discourse does not contain any referential information for players and pawns. The apparent scope of every beyond the sentence boundary is not predicted in any way.

(9) $\forall x \phi \leadsto \{ (f, g) \mid f = g \land \forall i : g[i]i \rightarrow \exists j : (i, j) \in \llbracket \phi \rrbracket \}$

The second issue is related to the first one. As noted below, the reference of it in the telescoping reading depends on the reference of he. Roughly, the second sentence means He puts the pawn he chooses. Any theory must capture this dependency between pronouns.

Both of these issues are resolved in systems that represent a context as a matrix (more formally, a set of ‘lists’ of the kind we had in the previous section), as proposed by van den Berg (1996), Nouwen (2003), and Brasoveanu (2007). Below I call such a system a pluralized dynamic system.\(^4\) There, the analysis of the telescoping sentence in (8) goes informally as follows. Every player updates the initial, empty

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\(^4\) Therefore, by pluralize I do not mean to augment a system with pluralized individuals in the sense of Link (1983).
matrix $I_*$ to $I_1$ by storing one and only one player in each row in a column, this time in column 0. Then chooses a pawn updates $I_1$ to $I_2$ by storing in column 1 the pawn that the player in that row chooses. Crucially, the second sentence he puts it in square one update each row of $I_2$ separately: it updates each $i \in I_2$ with he picking up the player and it picking up the pawn in that row. It checks if the player puts the pawn on square one. This way of updating is called distributive update.\footnote{The distributive update is argued to be induced by a covert operator or the overt presence of a quantifier such as each.}

\begin{align*}
\begin{array}{ccc}
I_1 & 0 & \ldots \\
\hline
i_1 & \text{player1} & \ldots \\
i_2 & \text{player2} & \ldots \\
i_3 & \text{player3} & \ldots \\
\end{array}
\end{align*}
\begin{align*}
\begin{array}{ccc}
I_2 & 0 & 1 & \ldots \\
\hline
i_1 & \text{player1} & \text{pawn1} & \ldots & \Leftarrow \text{he}_0 \text{ puts } i_1 \text{ on square one.} \\
i_2 & \text{player2} & \text{pawn2} & \ldots & \Leftarrow \text{he}_0 \text{ puts } i_1 \text{ on square one.} \\
i_3 & \text{player3} & \text{pawn3} & \ldots & \Leftarrow \text{he}_0 \text{ puts } i_1 \text{ on square one.} \\
\end{array}
\end{align*}

Under the pluralized system, universal quantification does pass information of the referents introduced by the quantification to the following discourse. That is, universal quantification is no longer a test. The dependency between players and pawns is represented in each row of the matrix, which contains a player and the pawn he chooses. The apparent extension of the scope of every beyond the sentence is now analyzed as a distributive update of the matrix. The singular pronouns he and it pick up the player and the pawn in each row, respectively.

The incremental analysis does not face the first issue: since dynamic conjunction is externally dynamic, so is the incrementant universal quantification. However, it faces the second issue: it loses the analysis of telescoping described above. The analysis of telescoping in the pluralized dynamic system appeals to the apparatus lost in the incremental analysis, namely a matrix, to capture the dependency between players and pawns. On the other hand, the incremental analysis produces a flat structure as in (11) as the output by universal quantification, in which all the players and the pawns are contained in one single list (or equivalently, one single row in the matrix).

\begin{align*}
(11) \quad \text{Every player chooses a pawn} \rightarrow \\
& \llbracket \text{Player 1 chooses a pawn} \rrbracket ; \\
& \llbracket \text{Player 2 chooses a pawn} \rrbracket ; \\
& \llbracket \text{Player 3 chooses a pawn} \rrbracket
\end{align*}
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\[
\begin{array}{c|cccccc}
& 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{player} & \text{pawn} & \text{player} & \text{pawn} & \text{player} & \text{pawn} \\
\text{1} & \text{1} & \text{2} & \text{3} & \text{4} & \text{5} \\
\end{array}
\]

The issue is more apparent when telescoping occurs after a sentence-internal use, as in (12).

(12) Every boy chose a different poem. He recited it.

Thus, we are in a conflict. The incremental analysis nicely accounts for the sentence-internal \textit{different} and comparatives but loses an analysis for telescoping. In the next section, I propose that this conflict can be resolved – telescoping can be accounted for with a flat structure like (11).

3 Resolving the conflict

The issue is how the dependency between players and pawns is retrieved from the flat representation of a context. I propose achieving this with the incremental system augmented with a representation of \textit{events}. Suppose, following Kamp (1979, 1981), that a dynamic system contains discourse referents for events as well as individuals. Suppose further that an event dref is introduced by a verb. Then, each conjunct in the incremental analysis in (8) introduces three drefs: two individual drefs for a player and a pawn, and one event dref for a choosing event. The resultant discourse is now represented as (13), where \textit{e1}, \textit{e2}, \textit{e3} are event drefs for choosing events. Following the Neo-Davidsonian event semantics (Parsons 1990), I assume the current system keeps track of thematic relations w.r.t. each event. Thus, \textit{player1} and \textit{pawn1} (p11 and pa1 for short in the table, respectively) are participants (the agent and the patient, respectively) of \textit{e1}, and \textit{player2} and \textit{pawn2} are of \textit{e2}, etc.

\begin{align*}
\text{(13)} & \quad \begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\text{p11} & \text{pa1} & \text{e1} & \text{p12} & \text{pa2} & \text{e2} & \text{p13} & \text{pa3} & \text{e3} \\
\end{array}
\end{align*}

Now we can think of a subpart of the list in (13) w.r.t. each event. That is, we can retrieve a list w.r.t. an event \textit{e} so that the list contains \textit{e} and all and only the participant(s) of \textit{e}. From (13), we can obtain three such lists as shown in (14). Notice that each list retrieves the dependency between players and pawns by making use of thematic relations. Now suppose the second sentence of (8) updates each of these lists. Then we obtain the intended telescoping reading.
I call this entire operation event distribution because it can be conceptualized as distributive quantification over events and their participants. The remaining task is to formalize event distribution, to which I will turn next.

### 3.1 Formalization
#### 3.1.1 Setting up

I follow the incremental dynamics proposed by van Eijck (2001) and Nouwen (2003), as Bumford (2015) does. A characteristic feature of incremental dynamics is that it takes a context as a numbered list. The growth of context by introducing a new referent is formalized as ‘appending’ it at the end of the list. I first formalize incremental quantification in such a system.

There are three basic types, $t$ for truth values, $e$ for individuals, and $s$ for lists. (The fourth one, type $v$ for events, is added later.) I use $i, j, k, h...$ as variables of type $s$. A list $i$ has its length, $|i|$. Note that since each list starts numbering with 0, a list $i$ such that $|i| = 4$ has an element in 0th position to 3rd position.

\[
\begin{array}{c|cccc}
0 & 1 & 2 & 3 & 4 \\
\hline
0 & 1 & 2 & | & \\
\end{array}
\]

(A list with the length 4)

I say $i(n)$ is the element in $n$th position in the list $i$.\(^7\) For the list in (15), $i(2) = c$.

---

6 The incremental dynamics has a technical advantage in formalizing incremental quantification. To see this, consider (i) under the incremental universal quantification, but with a more standard definition of indefinites, as in (ii).

(i) Every player chooses a\(^x\) pawn.

(ii) $a^x(P)(Q) \rightarrow \lambda f. \lambda g. f[x] g \land P(g(x)) \land Q(g(x))$

A\(^x\) pawn updates the input $f$ into $g$ so that $g$ differs from $f$ at most w.r.t. the referent it stores under the variable $x$. This definition does not let us formalize incremental quantification. Successive dynamic conjunction [[Player 1 choose a\(^x\) pawn] ; [[Player 2 chooses a\(^x\) pawn] ... keeps replacing the element stored under $x$, not keep adding a new pawn to the discourse. Thus, the resultant context only contains the pawn introduced by the last conjunct. As verified below, the incremental dynamics does not face this problem.

7 This is a sloppy notation because I do not formalize lists as functions. I stick to this notation for its
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(16) \( i(n) := \text{\textit{n}th element in } i \), defined only if \( n < |i| \)

Following Muskens (1996), I define drefs (for individuals) as of type \( se \), a function from lists to individuals. I use \( u_1, u_2, \ldots \) for variables of type \( se \). Taking list \( i \), \( u_n \) returns the element \( i(n) \).

(17) \( u_n := \lambda. i(n) \), defined only if \( n < |i| \)

Introducing a new dref is equivalent to appending a new element to the list. It is useful to note that if \( \exists e(i)(j) \), then \( u_{|i|}(j) \) is the element just appended. Thus, \( \exists e \) extends an input \( i \) to an output \( j \) by appending a new referent at the end of \( i \).

(18) New dref introduction (for individuals)
\[
\[\exists e\] := \lambda. i. \lambda. j. \exists d_e : j = i \cdot d, \\
\text{where } j = i \cdot d \text{ iff } |j| = |i| + 1 \land j(|i|) = d \land \forall n < |i| : i(n) = j(n)
\]

Some useful abbreviations and definitions are laid out in (20) the notation of which is borrowed from Brasoveanu (2008). Dynamic lexical relation (19a) and equivalence (19b) calls for a test. They check if a relation \( R \) holds for \( u_{n_1}, \ldots, u_{n_m} \) and if the two referents \( u_n \) and \( u_m \) are equivalent, respectively.

(19) a. Dynamic lexical relation
\[
[R\{u_{n_1}, \ldots, u_{n_m}\}] := \lambda. i. \lambda. j. i = j \land R(u_{n_1} i, \ldots, u_{n_m} i)
\]

b. Equivalence
\[
[u_n = u_m] := \lambda. i. \lambda. j. i = j \land u_n j = u_m j
\]

c. Dynamic conjunction
\[
D_1; D_2 := \lambda. i. \lambda. j. \exists k : D_1(i)(k) \land D_2(k)(j)
\]

d. Truth
\[
D \text{ of type } \langle s, \langle s, t \rangle \rangle \text{ is true w.r.t. list } i \text{ iff there is list } j \text{ such that } D(i)(j) = 1.
\]

A new dref is introduced by indefinites and names. In the object language notation in (20), indefinites carry the superscript \( u \) to indicate that they introduce a dref (for individuals). Pronouns carry a numerical number \( n \) as a subscript and pick up the \( n \)th element from the list. Now a simple discourse in (20a) is represented as (20b). The compositional analysis follows shortly.

(20) a. \( A^u \text{ man jumped. } A^u \text{ woman laughed. } \text{She}_{u_1} \text{ jumped, too.} \)

b. \( \exists e ; [\text{man}\{u_0\}] ; [\text{jumped}\{u_0\}] ; \\
\exists e ; [\text{woman}\{u_1\}] ; [\text{laughed}\{u_1\}] ; [\text{jumped}\{u_1\}] \)

Suppose the sentences are uttered against the empty list \( i \) with length 0. The first sentence appends a new referent to the 0th position of the list and tests if the new

intuitive clarity.

8 Note that in the definition of dynamic lexical relation, \( n_1 \ldots n_m \) are some numerical numbers.
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referent is a man and if he jumped. Then the second sentence appends another referent to the 1st position of the list and checks if it is a woman and if it laughed. The third sentence does not introduce a new referent but checks if the referent in the 1st position (i.e., the woman) jumped. The growth of the discourse is represented as (21).

\[
\begin{align*}
&\exists e; \text{man}\{u_0\}; \text{jumped}\{u_0\} \quad 0 \quad \text{a is a man, a jumped} \\
&\exists e; \text{woman}\{u_1\}; \text{laughed}\{u_1\} \quad 0 \quad 1 \quad \text{a} \quad \text{b} \quad \text{b is a woman, b laughed} \\
&\text{jumped}\{u_1\} \quad 0 \quad 1 \quad \text{a} \quad \text{b} \quad \text{b jumped}
\end{align*}
\]

The intended update by (20) is obtained compositionally with the translation of English in (22). Note that e for the abbreviation of type se, t for s, st.

\[
\begin{align*}
&\text{a. } a \mapsto \lambda P_{et} \cdot \lambda Q_{et} \cdot \lambda i_s . (\exists e) ; P(u_{ij}) ; Q(u_{ij})(i) \\
&\text{b. } \text{man} \mapsto \lambda u_e . \text{man}\{u\} \\
&\text{c. } \text{jumped} \mapsto \lambda u_e . \text{jumped}\{u\} \\
&\text{d. } s/he_{u_n} \mapsto \lambda i_s . i(n)
\end{align*}
\]

\[
\begin{align*}
&\text{A man jumped } \mapsto \lambda i_s . (\exists e) ; \text{man}\{u_{ij}\} ; \text{jumped}\{u_{ij}\}(i) \\
&\mapsto \lambda i_s . (\lambda k . \lambda j . \exists d : j = k \cdot d \land \text{man} (u_{k|j}) \land \text{jumped}(u_{k|j})(i)) \\
&\mapsto \lambda i_s . \lambda j . \exists d : j = i \cdot d \land \text{man} (u_{i|j}) \land \text{jumped}(u_{i|j})
\end{align*}
\]

3.1.2 Incremental Quantification

In defining incremental quantification, it is helpful to have set \( \mathbb{D} \) of drefs that correspond to specific discourse referents in Muskens 1996.\(^9\) Specific drefs of type se have a fixed value regardless of the input list. For example, specific discourse referent John returns the person John in \( D_e \) for any list. I suppose there is a corresponding specific discourse referent for any individual \( x \in D_e \). That is, for every \( x \in D_e \), there is a dref \( u \) such that \( u(i) = x \) for any \( i \). Using the notion of specific drefs, we can convert a dynamic property \( P_{et} \) to a set of specific drefs:

\[
\forall \{\forall: \lambda u . u \in \mathbb{D} \land \forall i : P(u)(i) = 1 \}
\]

\(^9\) The use of the adjective specific in here has nothing to do with the one in specific indefinites. One may want to call specific drefs here as rigid drefs.
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For example, \(^\exists \text{man}\) returns a set of specific drefs that makes \([\text{man}]\(u\))(i)(i) true for any \(i\). It effectively returns a set of specific drefs mapped to a man.

\[ (25) \quad \forall i. u \in D \land \forall i : ([\text{man}]\(u\))(i)(i) = 1 \]

Now universal quantifiers every and \(\text{each} \), which I suppose to be semantically equivalent in this paper, are defined as (26).

\[ (26) \quad \text{every/each} \rightsquigarrow \lambda P_{\text{et}}. \lambda Q_{\text{et}}. \lambda i. \lambda j. \]
\[
\left( \left\{ \lambda k. \left( \exists e. [u|k| = u'] : Q(u|k|) \right) (k) \mid u' \in [\text{man}] \right\} \right) (i)(j)
\]

The large semicolon is generalized dynamic conjunction, defined in (28).\(^{10}\)

\[ (27) \quad \left\{ D_1, D_2, \ldots, D_n \right\} := \lambda i. \lambda j. \exists k : i = k_0 \land j = k_n \land D_1(k_0)(k_1) \land D_2(k_1)(k_2) \land \ldots \land D_n(k_{n-1})(k_n) \]

Every man jumped, with the relevant individuals John, Bill, and Fred, are now analyzed as (28). As shown in the last line, the update is equivalent to the successive updates by \([\text{John jumped}], [\text{Bill jumped}], \) and \([\text{Fred jumped}]\)

\[ (28) \quad \text{Every man jumped} \]
\[
\rightsquigarrow \lambda i. \lambda j. \left( \left\{ \lambda k. \left( \exists e. [u|k| = u'] : \text{jumped}(u|k|) \right) (k) \mid u' \in [\text{man}] \right\} \right) (i)(j)
\]

(Lexical Entry)

\[
\rightsquigarrow \lambda i. \lambda j. \left( \left\{ \begin{array}{l}
\lambda k. \left( \exists e. [u|k| = \text{John}] : \text{jumped}(u|k|) \right) (k), \\
\lambda k. \left( \exists e. [u|k| = \text{Bill}] : \text{jumped}(u|k|) \right) (k), \\
\lambda k. \left( \exists e. [u|k| = \text{Fred}] : \text{jumped}(u|k|) \right) (k)
\end{array} \right\} \right) (i)(j)
\]

(Unpacking the set)

\[
\rightsquigarrow \lambda i. \lambda j. \left( \exists k : i = k_0 \land j = k_3 \land \begin{array}{l}
\exists d_e : k_1 = k_0 \cdot d \land u|k_0| k_1 = \text{john} \land \text{jumped}(u|k_0| k_1) \land \\
\exists d_e : k_2 = k_1 \cdot d \land u|k_1| k_2 = \text{bill} \land \text{jumped}(u|k_1| k_2) \land \\
\exists d_e : k_3 = k_2 \cdot d \land u|k_2| k_3 = \text{fred} \land \text{jumped}(u|k_2| k_3)
\end{array} \right)
\]

(Definition of \(\cdot, \beta\)-reduction)

\(^{10}\) Technically, since there is no order in a set, a result of applying dynamic conjunction to a set is not unique. I do not see any theoretical or empirical issue raised by this, at least within the scope of this paper.
3.1.3 Adding Events

To formalize event distribution, I add to the ontology events of type \( v \). Drefs for events are of type \( sv \) abbreviated as \( v \). I use \( \varepsilon_1, \varepsilon_2, \ldots \) for variables of type \( v \). I assume verbs introduce an event discourse referent. In static terms, it means that verbs existentially quantify over events, the line of analysis proposed and justified by Champollion (2015). Although nothing hinges on this choice, it lets us avoid theoretical problems caused by scope interactions between the existential quantifier over events and other scope-taking elements (see Champollion 2015 for more details).

With the event components, the lexical translation for jumped and \( a \) in (22) are revised as (30), where the definition of jumped makes use of continuation (the variable \( V \)). The entry for \( \text{man} \) is kept the same. I use subject and object in order to represent thematic relations instead of agent, patient, or experiencer to be as theory-neutral as possible.

\[
(29) \quad \text{New dref introduction (for events)}
\]

\[
\exists v. \lambda i. \lambda j. \exists d_v : j = i \cdot d
\]

\[
(30)\quad a. \quad \text{jumped}^e
\quad \leadsto \lambda u_e. \lambda V_{vt}. \lambda i. \left( [\exists v] ; [\text{sleep}\{\varepsilon_{[i]}\}] ; [\text{subject}\{\varepsilon_{[i]}, u\}] ; V(\varepsilon_{[i]})(i) \right)
\]

\[
\quad b. \quad \text{man} \leadsto \lambda u_e. [\text{man}\{u\}]
\]

\[
\quad c. \quad a^u \leadsto \lambda P_{vt}. \lambda Q_{(\varepsilon_{(vt,i)})}. \lambda V_{vt}. \lambda i. \left( [\exists v] ; P(u_{[i]}) ; Q(u_{[i]})(V) \right)(i)
\]

Composing the lexical entries in (30) results in (31) for sentence Every man jumped.

\[
(31) \quad \lambda V. \lambda i. \left( \left[ \exists v \right] ; [\text{man}\{u_{[i]}\}] ; \lambda k. \left( [\exists v] ; [\text{sleep}\{\varepsilon_{[k]}\}] ; [\text{subject}\{\varepsilon_{[k]}, u_{[i]}\}] ; V(\varepsilon_{[k]})(k) \right) \right)(i)
\]

The remaining variable \( V \) is saturated by the closure \( \text{true} \) that is true of any event. (32) is a dynamicized definition of Champollion’s (2015).

\[
(32) \quad \text{true} \leadsto \lambda v. [\text{true}\{e\}]
\]

Applying \( \text{true} \) to (31) results in (33). Suppose that the update takes an empty list \( i \) as its input. Then the update appends an individual \( d_e \) to \( i \), resulting in \( k \). Note that \( |i| = 0 \), thus \( u_{[i]}k \) picks up the 0th element in \( k \), namely the referent just introduced. It tests if \( u_{[i]}k \) is a man. Then an event referent \( d_v \) is appended to \( k \), producing \( j. \varepsilon_{[k]}j \) is this new event referent. It is tested if this event is a jumping event, and \( u_{[i]}j \) is the subject of \( \varepsilon_{[k]}j \).
Telescoping in incremental quantification

(33) true \( a^\alpha \) man jumped

\[ \sim \lambda i. \left( \exists e; [\text{man}(u_{ij})]; \lambda k. ([\exists v; [\text{jumped}(e_k)]]; [\text{subject}(e_{kij}, u_{ij})]; \text{true}(e_{kij})(k) \right) (i) \]

\[ \sim \lambda j. \left( \exists e; [\text{sleep}(e_{ij})]; [\text{subject}(e_{ij}, u')]; [\text{object}(e_{ij}, u)]; V(e_{ij})(V) \right) \]

\[ \sim \lambda j. \exists k : \exists d_e : k = i \cdot d \land \text{man}(u_{ij}k) \land \exists d_v : j = k \cdot d \land \text{jumped}(e_{kij}) \land \text{subject}(e_{kij}, u_{ij}j) \]

Transitive sentences are composed accordingly. To avoid further complexities, I assume the indefinite article \( a \) has two types, the one defined above and the higher type definition in (34b). The latter is used when an indefinite occurs in the object position in a sentence, and it can be derived by a dynamic counterpart of a generalized type-shifting rule such as the one defined in Jacobson (2014). The verb phrase chooses a pawn is translated as (35).

(34) a. choose \( \sim \)

\[ \lambda u_e, \lambda u'_e, \lambda V_{vt}, \lambda i. \left( \exists e; [\text{sleep}(e_{ij})]; [\text{subject}(e_{ij}, u')]; [\text{object}(e_{ij}, u)]; V(e_{ij})(V) \right) \]

b. \( a_{obj}^\alpha \sim \lambda P_{et}, \lambda T(e(e_{vt}t)); \lambda u'_e, \lambda V_{vt}, \lambda i. \left( \exists e; P(u_{ij}); T(u_{ij})(u')(V) \right) \]

(35) choose \( \alpha^\rho \) a\( \alpha^\rho \) pawn \( \sim \)

\[ \lambda u_e, \lambda V_{vt}, \lambda i. \left( \exists e; [\text{pawn}(u_{ij})]; \lambda k. ([\exists v; [\text{choose}(e_k)]]; [\text{object}(e_{kij}, u_{ij})]; [\text{subject}(e_{kij}, u)]; V(k) \right) \]

The (incremental) universal quantifier is redefined as (36). The only difference from the above is that the variable \( V \) in each conjunct is saturated by true so that each conjunct is of type \( t \).

(36) every/each \( \sim \lambda P_{et}, \lambda Q_{et}; \lambda i. \lambda j. \left( ; \left\{ \lambda k. ([\exists e; [u_{kij} = u']]; Q(u_{kij})(\text{true}) (k) | u' \in \{P\} \right\} (i)(j) \right) \]

Thus, every player chooses a pawn produces list \( j \) as in (37).
3.1.4 Event distribution

Now, for each of those events \( e \in \{e_1, e_2, e_3\} \), we can define a smallest sublist \( j_e \) of \( j \) w.r.t. \( e \), defined as (38). \( j_e \) is a smallest sublist that contains \( e \), all of its participants, and nothing else. (Read \( \text{Ran}(j) \) as the range of \( j \), the set of elements contained in the list \( j \).)

(38) \( j_e \) is a smallest sublist of \( j \) w.r.t. \( e \) such that:
\[
e \in \text{Ran}(j_e) \land \forall d \in D_e : d \in \text{Ran}(j_e) \leftrightarrow \text{participant}(d, e) \land d \in \text{Ran}(j)
\]

The relevant smallest sublists \( j_{e_1}, j_{e_2}, j_{e_3} \) are visualized as (39). Incremental dynamics guarantee that each smallest sublist starts numbering with 0. We can appeal to a variant of the distributive update at this point. The second sentence of the telescoping sentence, \( \text{He}_0 \text{ puts it}_1 \text{ on square one} \), updates each of these smallest sublists, correctly capturing the player-pawn dependency.

The distributive operator, which induces an update of smallest sublists, is defined as (40) (to be revised shortly). \( \delta \) obtains numerical subscripts \( n_1, \ldots, n_m \), which are the numbers for the relevant events. \( \phi \) is a dynamic proposition. The operator calls for a test: it tests if each relevant smallest sublist can be updated into some list \( k \) (i.e., \( \phi \) is true w.r.t. the smallest sublist).

(40) \( \delta_{n_1, \ldots, n_m}(\phi) \leadsto \lambda i. \lambda j. i = j \land \forall n \in \{n_1, \ldots, n_m\} : \exists k : \phi(i(i(n)))(k) \)
\[
\text{(Defined iff } i(n_1), \ldots, i(n_m) \in D_e)\]

The intended result is derived in the following way. Suppose that the first sentence in the telescoping sentence produces the list in (41) and that the second sentence is represented as (42a). Then the event distribution is unpacked as (42b).

---

11 I assume here that \( \delta \) applies after \( \text{true} \) just for simplicity of the definition and composition. The application of \( \text{true} \) can also be incorporated within the definition of \( \delta \).
Telescoping in incremental quantification

(41) \[
\begin{array}{cccccccc}
j & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
p11 & pa1 & e1 & p12 & pa2 & e2 & p13 & pa3 & e3 \\
\end{array}
\]

(42) a. \[\delta_{2,5,8} (\text{true \ he}_0 \ \text{puts}^e \ it_1 )\]

b. \[\Rightarrow \lambda \cdot \lambda \cdot j \cdot i = j \land \forall n \in \{2, 5, 8\} : k:\]
\[\left( \lambda h. ([\exists v]; [\text{put}(e_{|h|})]; [\text{subject}(e_{|h|}, u_0)]; [\text{object}(e_{|h|}, u_1)]; ) (h)) (i_{(n)}) (k) \right.\]
\[\Rightarrow \lambda \cdot \lambda \cdot j \cdot i = j \land \forall n \in \{2, 5, 8\} : k:\]
\[\exists d_v : k = i_{(n)} \cdot d \land \text{put}(e_{|i(n)|} k) \land \text{subject}(e_{|i(n)|} k, u_0 k) \land \text{object}(e_{|i(n)|} k, u_1 k)\]
\[\Rightarrow \lambda \cdot \lambda \cdot j \cdot i = j \land \forall n \in \{2, 5, 8\} : k:\]
\[\exists k : \exists d_v : k = i_e \cdot d \land \text{put}(e_{|i(e)|} k) \land \text{subject}(e_{|i(e)|} k, u_0 k) \land \text{object}(e_{|i(e)|} k, u_1 k)\]
\[\exists k : \exists d_v : k = i_{e1} \cdot d \land \text{put}(e_{|i(e1)|} k) \land \text{subject}(e_{|i(e1)|} k, u_0 k) \land \text{object}(e_{|i(e1)|} k, u_1 k)\]
\[\exists k : \exists d_v : k = i_{e2} \cdot d \land \text{put}(e_{|i(e2)|} k) \land \text{subject}(e_{|i(e2)|} k, u_0 k) \land \text{object}(e_{|i(e2)|} k, u_1 k)\]
\[\exists k : \exists d_v : k = i_{e3} \cdot d \land \text{put}(e_{|i(e3)|} k) \land \text{subject}(e_{|i(e3)|} k, u_0 k) \land \text{object}(e_{|i(e3)|} k, u_1 k)\]

As desired, it tests if each smallest sublist can be updated into some k by \(\text{he}_0 \ \text{puts}^e \ \text{it}_1\), that is, it tests if the sentence is true w.r.t. each smallest sublist, deriving the telescoping reading.

(43) \[
\begin{array}{cccc}
\lambda e_1 \cdot 0 & 1 & 2 \\
p11 & pa1 & e1 \\
\cdot \leftarrow \text{he}_0 \ \text{puts}^e \ \text{it}_1 \\
\lambda e_2 \cdot 0 & 1 & 2 \\
p12 & pa2 & e2 \\
\cdot \leftarrow \text{he}_0 \ \text{puts}^e \ \text{it}_1 \\
\lambda e_3 \cdot 0 & 1 & 2 \\
p13 & pa3 & e3 \\
\cdot \leftarrow \text{he}_0 \ \text{puts}^e \ \text{it}_1 \\
\end{array}
\]

A technical revision is in order. Notice that the definition of the smallest sublist in (38) does not define the unique smallest sublist of j w.r.t. e. In fact, there are six possible smallest sublists of j w.r.t. e:

(44) \[
\begin{array}{cccc}
\lambda e_1 \cdot 0 & 1 & 2 \\
p11 & pa1 & e1 \\
\cdot \leftarrow \text{he}_0 \ \text{puts}^e \ \text{it}_1 \\
\lambda e_2 \cdot 0 & 1 & 2 \\
p11 & e1 & pa1 \\
\cdot \leftarrow \text{he}_0 \ \text{puts}^e \ \text{it}_1 \\
\lambda e_3 \cdot 0 & 1 & 2 \\
p11 & pl1 & e1 \\
\cdot \leftarrow \text{he}_0 \ \text{puts}^e \ \text{it}_1 \\
\end{array}
\]

Let \(J_e\) be a set of smallest sublists \(j_e\) of j w.r.t. e. The definition of \(\delta\) is now revised as follows.
One may wonder then if, in the case of (8), the scope of \( \delta \) targets a ‘wrong’ smallest sublist, say \( j \neq e_1 \). Although this is possible in principle, the update results in false because of the gender/person presuppositions of pronouns. I suggest that these presuppositions force \( \varphi \) to target the predicted smallest sublists to derive the correct meaning.

\[
\begin{array}{c|ccc}
\delta & 0 & 1 & 2 \\
e_1 & e & pa_1 & pl_1 \\
\end{array} \leq \text{he}_0 \text{puts}^e \text{it}_1
\]

Summarizing this section, I proposed and formalized event distribution. The proposed operation successfully resolves the conflict between incremental quantification and telescoping.

4 Discussion

In this section, I discuss issues around the current proposal by comparing it with pluralized dynamic systems, although formalizations of the idea presented here will be left for future occasions. Firstly, I point out the robustness of the issue raised in section 2. It is not only the telescoping configuration for which incremental quantification would lose an account. It also loses (at least) analyses of plural anaphora and quantificational subordination/dependent anaphora.

(47) Plural anaphora
   a. Every student wrote an abstract.
   b. They got exhausted.

(48) Quantificational subordination / Dependent anaphora
   a. Every student wrote an abstract.
   b. They each submitted it to SALT.

Under an analysis with a matrix, every student wrote an abstract creates the discourse represented as (49). It stores one student and one abstract in each row such that the former wrote the latter. Then the plural pronoun they in (47), with index 0, picks up the (sum of) referents in the column 0. Each in (48b) induces a distributive update of each row, and it picks up the abstract in each row.
Telescoping in incremental quantification

\[
\begin{array}{c|cccc}
  & I_1 & 0 & 1 & \ldots \\
  i_1 & \text{st1} & \text{ab1} & \ldots \\
  i_2 & \text{st2} & \text{ab2} & \ldots \\
  i_3 & \text{st3} & \text{ab3} & \ldots 
\end{array}
\]

(49)

Since the incremental system proposed above does not have the matrix discourse structure, it is not immediately clear how plural anaphora are resolved. One thing we could do is to assume that plural pronouns like they carry more than one index \(n_1, \ldots, n_m\) and collect every referent in these positions. For instance, after obtaining the discourse in (49) by incremental quantification (ignoring events), they might carry indices 0, 2, 4 to collect the reference to students 1–3 and refers to the sum of these referents, in the sense of Link (1983).

\[
\begin{array}{c|cccccc}
  & 0 & 1 & 2 & 3 & 4 & 5 \\
  i & \text{st1} & \text{ab1} & \text{st2} & \text{ab2} & \text{st3} & \text{ab3} 
\end{array}
\]

(50)

Although this is undoubtedly a viable hypothesis, it is evident that this should not be the only way to resolve plural anaphora. That is, we need a matrix structure anyway. As briefly mentioned at the end of section 1, Bumford (2015) claims that incremental quantification is unavailable to non-universal quantification to capture the close connection between the pair-list phenomena and universal quantification. Most, for instance, does not license the sentence-internal comparative:

(51) #Most generations inhabit a progressively more Orwellian world.

(Bumford 2015: 9)

If most does not quantify incrementally, it is reasonable to assume that it does so with a matrix structure. Then the anaphoric possibility to the witness set of the quantification by most, as in (52), should be resolved based on a discourse represented as a matrix.\(^{12}\)

(52) a. Most students wrote an abstract.
    b. They got exhausted.

Thus, we cannot discard a pluralized dynamic system to resolve plural anaphora. At this point, one may wonder if the pair-list phenomena discussed by Bumford (2015) can be analyzed under pluralized dynamic semantics without incremental quantification. In fact, Brasoveanu (2011) offers such an analysis, the details of which are not discussed in this paper. Then the question is if we still need incremental quantification and event distribution. It is not the purpose of this paper to justify incremental quantification, and I refer readers to Bumford 2015. Instead, I point out an empirical advantage of the analysis with event distribution over other approaches.

\(^{12}\) A possible alternative is proposed in Kamp & Reyle 1993.
Although there is still an issue around the plural anaphora resolution mentioned above, it is clear that the gist of event quantification can be extended to the quantificational subordination in (48). We would obtain the discourse as (53) with events and distribute over the events and their participants in the way proposed in section 3.

\[
\begin{array}{cccccccc}
 i & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 st1 & ab1 & e1 & st2 & ab2 & e2 & st3 & ab3 & e3 \\
\end{array}
\]

There is a configuration where we must appeal to event quantification for anaphoric resolution. Consider the following sentences.\(^\text{13}\)

13 (54) is brought to my attention by a SALT reviewer. The same reviewer observes that (i) is also felicitous in the relevant reading:

(i)  
 a. Candidate 1 wrote a paper. Candidate 2 also wrote a paper. A third paper was written by Candidate 3.
   b. They each submitted it to SALT.

The same reviewer wonders if the proposed analysis works for (i), highlighting that the third conjunct orders an abstract and its author differently from the first and the second conjuncts. The answer is yes, thanks to the flexibility of order in smallest sublists as we discussed at the end of section 3.

(54)  
 a. Candidate 1 wrote a paper. Candidate 2 wrote a paper. And candidate 3 also wrote a paper.
   b. They each submitted it to SALT.

(55)  
 a. Alex saw a donkey, and Bill saw a monkey.
   b. Each of them caught it.

(56)  
 a. Alex was in the station at 5, and Bill was in the park at 6.
   b. Each of them got a phone call at that time / then.

(57)  
 a. Alex was in the park at five, and Bill was in the station at six.
   b. Each of them saw a weird animal there.

(58)  
 a. Alex caught a monkey, and Bill caught a donkey.
   b. Each of them did it quickly.

In the b-sentences, a singular pronoun refers to a referent introduced in the a-sentences. Moreover, the referent of the pronoun depends on the referent of the subject. For instance, it in (55) refers to the donkey for Alex, and the monkey for Bill, given that the sentence is true iff Alex caught the donkey and Bill caught the monkey. Thus, the pronoun exhibits the same dependency observed in telescoping
and quantificational subordination.

It is clear by now how event distribution accounts for these sentences. Each conjunct of the a-sentences introduces an event, which the operator $\delta$ defined above distributes over. For example, smallest sublists created by $\delta$ retrieves the dependency between persons and animals in (55). However, the way the pluralized system captures the dependency in the case of telescoping does not work in these cases. This is because the relevant referents are introduced to the context by conjunction, not by quantification. Thus, by the definition of dynamic conjunction, the resultant context after (55a) is a flat structure as in (59) even under the pluralized system (with or without events). The row-wise dependency between people and animals is not available here.

$$\begin{array}{c|cccc}
i & 0 & 1 & 2 & 3 \\
\hline
\text{alex} & \text{donkey} & \text{bill} & \text{monkey}
\end{array}$$

One may avoid the problem by positing a different definition of dynamic conjunction by which the update by each conjunct is summed up, as in (60).

$$D_1; D_2 := \lambda I. \lambda J. \exists K \exists H : D_1(I)(K) \land D_2(I)(H) \land J = K \cup H$$

The definition is internally static, however. Thus, it does not extend to configurations that need dynamicity between conjuncts, as in (61). See Yagi (To appear) for more discussions on these examples.

$$\begin{array}{c}
a. \text{A man saw a donkey, and his son saw a monkey.} \\
b. \text{Each of them caught it.}
\end{array}$$

Summarizing, I discussed issues for incremental quantification caused by the resolution of plural anaphora and pointed out an empirical advantage of event distribution.

5 Conclusion

I proposed that incorporating event discourse referents paves the way to retrieve the dependency of drefs lost in the incremental analysis. The current proposal, event quantification, also has an empirical advantage. I conclude this paper by mentioning one theoretical point. The proposal, if correct, implies that an analysis telescoping (and possibly, quantificational subordination) does not require a pluralized dynamic system. It would be interesting to consider if we can stick to the non-pluralized system in other domains, and/or if the current analysis of retrieving dependency between drefs and the one in a pluralized dynamic system are ultimately isomorphic.\(^{14}\)

\(^{14}\)I thank a SALT reviewer for bringing this latter possibility to my attention.
References


Telescoping in incremental quantification


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