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Probabilistic non-maximality*

Diego Feinmann
Institute of Computer Science
Polish Academy of Sciences

Abstract In this paper, I present a probabilistic reconstruction of Križ's (2015, 2016) account of non-maximal interpretation. The proposal addresses a general problem noted by Križ (2015: 85–6), which is shared by other QUD-based approaches to non-maximality (e.g. Bar-Lev 2021; Križ & Spector 2021): for these accounts to work, one needs to posit a fundamental disconnect between the question that is asked and the QUD that is addressed. This is problematic, especially in the absence of a non-stipulative method for identifying the QUD that is being addressed in a given context. In the proposed framework, the overt question alone determines the relevant QUD, allowing non-maximal interpretations to be generated in a principled manner.

Keywords: homogeneity, non-maximality, imprecision, pragmatic slack, Bayesian pragmatics

1 Introduction

Unquantified plural sentences, as opposed to universally quantified ones, can be used to describe non-maximal situations, at least under certain QUDs (see Brisson 1998; Lasersohn 1999; Malamud 2012 a. o.). An illustration of this phenomenon is given in (1).

(1) [Context: Jane is at a friend's party with her nine children.]

a. *John*: Will you stay at the party for a bit longer?

b. *Jane*: The kids are tired.
$\approx$ At least some of the kids are tired.

c. *Jane*: *All* the kids are tired.
$\not\approx$ At least some of the kids are tired.

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By uttering (1b), Jane conveys at least two things: that her kids are tired, and that she is unlikely to stay at the party for much longer. Notably, whether all or only some of Jane's kids are tired is something that (1b) does not appear to settle. Indeed, while (1b) is acceptable as a description of a situation in which all of them are, it also seems acceptable as a description of a situation in which only some of them are (namely, a non-maximal situation). This contrasts with (1c), which is acceptable *only* as a description of a situation in which all of Jane's kids are tired. Interpretations of the kind illustrated by (1b) are known in the literature as *non-maximal interpretations*.

2 The hidden costs of the partial answer assumption

Theories such as Križ 2015, 2016, Bar-Lev 2021, and Križ & Spector 2021 are all built on the assumption that non-maximal interpretations are *partial answers*, viz. the union of some, but not all, QUD cells.[1] In Bar-Lev 2021 and Križ & Spector 2021, this is a consequence of adopting the precepts of classical relevance (Groenendijk & Stokhof 1984; Roberts 1996/2012). In Križ's (2015, 2016) account, in turn, it stems from a relevance-related constraint he calls *Addressing*.

(1b), however, does not provide a partial answer to the QUD induced by (1a)—it merely raises the probability of the NO-cell, as the contrast between (2) and (3) exposes:

(2) [Context: Jane is at a friend's party with her nine children.]

a. *John*: Will you stay at the party for a bit longer?

b. *Jane*: The kids are tired. *So I probably won't.*

(3) [Context: Jane is at a friend's party with her nine children.]

a. *John*: Will you stay at the party for a bit longer?

b. *Jane*: #No. *So I probably won't.*

To generate the attested interpretation of (1b) in current frameworks, two basic strategies are available:

- **Strategy 1 (common ground idealisation):** To assume a 'boutique' common ground according to which if none of Jane's children are tired, then she is staying at the party, and if some or all are tired, then she is leaving.
- **Strategy 2 (QUD accommodation):** To claim that (1b) is not interpreted via the QUD induced by (1a), the overt question, but rather via the QUD induced by 'Are any of the kids tired?'

1 Theoretical contributions predating Križ 2015—such as Brisson 1998, Burnett 2011, and Malamud 2012—are not discussed here. For an overview of these approaches, see Križ 2015: §3.6.

Strategy 1 is rather unappealing, at least insofar as it involves sweeping the clear contrast between (2) and (3) under the rug. Indeed, (2b), although interpreted non-maximally, does not—unlike (3b)—rule out the possibility that Jane may be staying at the party.

Strategy 2 is no less problematic. For it to count as an explanation, it would need to be coupled with a non-stipulative account of QUD accommodation, which, to my knowledge, has not yet been specified. The issue is this: by stipulating that (1b) is not interpreted via the QUD induced by (1a) (the overt question) but via the QUD induced by 'Are any of the kids tired?', what is effectively being stipulated is that (1b) ought to be assigned a non-maximal interpretation (relative to that QUD, all current frameworks predict an existential—and hence non-maximal—interpretation). But this is precisely what needs to be explained, not stipulated. In other words, why can (1b) not be taken to address the QUD induced by 'Are all of the kids tired?' Relative to that QUD, current frameworks all predict a maximal rather than a non-maximal interpretation.

Could Strategy 2 be rescued by articulating a substantive, non-stipulative theory of QUD accommodation—one that selects the *Are any…?* rather than the *Are all…?* QUD? The answer, I will argue, is no: even with such an account in place, Strategy 2 still faces a problem—one that is linked to a well-known limitation of current accounts.

As mentioned, if the operative QUD is the one induced by 'Are any of the kids tired?', the accounts in Križ 2015, 2016, Bar-Lev 2021, and Križ & Spector 2021 predict (1b) to give rise to an existential interpretation. However, when 'Are any of the kids tired?' is posed overtly, the expected interpretation fails to surface, as Križ (2015: 85) is quick to observe and acknowledge as a problem. This is illustrated in (4) below.

(4) a. *John*: Are any of the kids tired?

b. *Jane*: ??Yes, the kids are tired.
$\not\approx$ Yes, at least some of the kids are tired.

It is worth noting that, apart from failing to give rise to the expected interpretation, (4b) does not constitute a natural answer to (4a) (hence the ??-marking, which is absent in Križ's example[2]). This is interesting: *prima facie*, (4b) could fail to give rise to the expected non-maximal interpretation and yet be perfectly natural as an answer. That this judgment of unnaturalness arises merits further attention.

The issue for Strategy 2 is thus this: (4), which constitutes a challenge to current frameworks in its own right, also undermines the idea that QUD accommodation is at work in exchanges such as (1a)–(1b). Indeed, according to this strategy, (1b)

2 See Križ 2015: 85, example (33).

doesn't address the QUD induced by (1a), the overt question, but the one induced by 'Are any of the kids tired?'. Yet when the overt question *is* 'Are any of the kids tired?', the expected interpretation does not arise.

How might an advocate of Strategy 2 respond to this challenge? A natural reaction, compatible with the observation that (4b) seems like a bad answer, is the following: (4b) violates an alignment constraint of sorts, one that prevents an unquantified sentence of the form 'The *X*s are *blah*' from serving as an answer to a quantified question of the form 'Are *QUANT* of the *X*s *blah*?' (I won't attempt to spell out such a constraint here, but let's assume, for the sake of the argument, that it can be formulated.) From this perspective, the reason why (4b) does not give rise to the predicted non-maximal interpretation is that, due to the violation of an independent form-level constraint, the natural course of interpretation is disrupted. This constraint, crucially, is not violated in (1a)–(1b)—hence, in such a case, once the right QUD is accommodated (the one induced by 'Are any of the kids tired?'), (1b) can proceed to be interpreted non-maximally.

There is something I agree with here: (4b) does seem to violate *something*—otherwise, it would be hard to make sense of its perceived deviance. I am less convinced, however, that (4b)'s deviance can be explained by reference to the formal mismatch between (4b) and (4a). Consider, for example, (5) below, which exhibits the same mismatch between question and answer as (4) does:

(5) a. *John*: Are all of the kids tired?
b. *Jane*: ?Yes, the kids are tired.

Though not perfect, (5b) is better than (4b). In fact, as far as I can tell, the slight deviance of (5b) is due to the repetition of 'the kids' (failure to reduce a given referent; cf. Gundel, Hedberg & Zacharski 1993) and 'tired' (failure to perform VP ellipsis; cf. Schäfer, Lemke, Drenhaus & Reich 2021). Indeed, as shown below, if one replaces 'the kids' by 'they', and elides 'tired', (7b) sounds just fine, whereas (6b) continues to sound bad.

(6) a. *John*: Are any of the kids tired?
b. *Jane*: ??Yes, they are.

(7) a. *John*: Are all of the kids tired?
b. *Jane*: Yes, they are.

The acceptability of (7b) undermines the idea that the oddness of (4b)/(6b) is due to a violation of a form-level constraint: in terms of surface configuration, there is no difference between (6a) and (7a); thus, if what prevents (6b) from addressing (6a) is a form-level constraint, that constraint should also prevent (7b) from addressing (7a). As mentioned, however, (7b) seems perfectly acceptable as an answer to (7a).

The problem noted, therefore, remains: it is hard to accept that the QUD at play in cases like (1a)–(1b) is the one induced by 'Are any of the kids tired?': (1b) is odd under that question, and, as shown, this oddness cannot be explained away by reference to form-level factors.

✺

In the next section, I shall present a probabilistic reconstruction of Križ's (2015, 2016) account. Here, the attested interpretation of (1b) can be derived without positing an idealised common ground, as in Strategy 1, a move that makes it hard to account for the contrast between (2) and (3). Nor is it necessary to invoke QUD accommodation, as in Strategy 2, which comes with its own set of problems. Rather, the attested interpretation can be derived in a straightforward manner via the QUD induced by the overt question.

3 Towards a probabilistic reconstruction of Križ's (2015, 2016) account

Since the account to be advanced here is—or can at least be seen as—a probabilistic reconstruction of Križ's (2015, 2016) account, a brief overview of the latter is in order. Križ's account has a semantic as well as a pragmatic component:

» *Križ's semantics*

Unquantified plural sentences are known, at least since Fodor 1970, to exhibit a truth-value gap. For example, it is clear that 'The kids are tired' is true in a situation where all the kids are tired; it is also clear that it is false in a situation where none of the kids are tired. But what about situations in which some, but not all, of the kids are tired? As it happens, 'The kids are tired' is judged to be neither true nor false in such cases (see Križ & Chemla 2015 for experimental work in this domain). This phenomenon goes by the name of *homogeneity* or *polarity*.

Importantly, Križ (2015, 2016) treats homogeneity as a purely semantic phenomenon; distinct from, though not unrelated to, the phenomenon of non-maximality (more on this to come). On his approach, homogeneity is encoded via an additional (third) truth value, as illustrated below:

$$[\![\text{the kids are tired}]\!] = \begin{cases} 1 & \text{if all the kids are tired} \\ 0 & \text{if none of the kids are tired} \\ \# & \text{otherwise} \end{cases}$$

Following Križ (2015, 2016), I shall call the set of worlds where S is true the *positive extension* of S (S^+); the set where it is false, the *negative extension* of S (S^-); and the set where it is undefined (neither true nor false), the *extension gap* of S ($S^\#$). Questions pertaining to the nature of the homogeneity gap and its compositional status will not be discussed here. For present purposes, it suffices to assume that the extension gap of plural sentences is represented as shown above.

» *Križ's pragmatics*

Križ's pragmatics is built upon a revision of the maxim of Quality and a principle he calls *Addressing*, which severely constrains the kind of QUDs that a sentence can address. Here I won't discuss these principles; for our purposes, what matters is the effect that these principles have. The effect—what Križ (2015: 177–8) refers to as his theory's core generalisation—is the following: a sentence S can be used as long as it is *true enough*, where 'true enough' means: (i) S is not false; (ii) the actual world is, for current purposes, equivalent to a world in which S is true, i.e. the actual world 'lives' in the same QUD cell as a world in which S is true.

This generalisation induces a procedure to update C (the common ground, modelled as a set of worlds) with a sentence S via a QUD Q (a partition of C). This procedure can be conceptualised as having two levels of filtering—one targeting individual words, and one targeting whole QUD cells:

(8) **Križ's update procedure**

a. *World-filtering level* — Remove from C the worlds that would be excluded upon learning that S is not false.

b. *Cell-filtering level* — Remove from Q the cells that would be excluded upon learning that S is true.

Together, (8a) and (8b) predict that the (pragmatic) interpretation of S—i.e. the state that results from revising C with S, henceforth C^S—is the union of Q-cells that S^+ intersects, with S^- subtracted. Namely:

$$C^S = \left(\bigcup\{q \in Q \mid q \cap S^+ \neq \varnothing\}\right) \setminus S^-$$

This reduces to $S^+ \cap C$ when $S^\# = \varnothing$; in such cases, it is structurally impossible for S to receive a non-maximal interpretation. When $S^\# \neq \varnothing$, a non-maximal interpretation may or may not arise, depending on the nature of QUD at play. According to this account, a sentence S receives a non-maximal interpretation if and only if $C^S \cap S^\# \neq \varnothing$.

3.1 SFK update

Križ's update procedure is defined in a non-probabilistic setting, where the shared background state is represented as a set of worlds and QUDs are identified with partitions of that set. By contrast, the procedure I advance here is cast in a probabilistic framework. Here, the shared background state is represented by a probability distribution P (the so-called *common prior*), and QUDs are modelled as partitions of Logical Space—denoted by Ω. I shall refer to this procedure as *SFK update* (*Spector–Feinmann–Križ*). The name reflects the procedure's intellectual lineage: it was developed during my PhD, in collaboration with Benjamin Spector, my advisor at the ENS, and was inspired by the work of Manuel Križ.

With these preliminaries in place, I proceed to the formal details. The formal challenge lies in giving a probabilistic formulation of (8). (8a), the world-filtering level, is straightforward: conditioning the prior on $\Omega \setminus S^-$ (on the information that S is either true or undefined) does the trick. Indeed, this operation assigns zero probability to those worlds where S is false, thereby excluding them as live possibilities. (8b), the cell-filtering level, is more challenging. One cannot simply condition on S^+: doing so would exclude the worlds in which S is false as well as those in which it is undefined, and that is not what (8b) does. (8b) filters out *entire* cells (those cells that are incompatible with S^+) and *nothing else*. What (8b) appears to call for is an operation that assigns to each QUD cell the probability it would have been assigned had one conditioned on S^+, without actually conditioning on S^+. This is what SFK update, defined below, accomplishes.

Definition I (SFK update). Let Q be a partition of Ω. Let S be a declarative sentence, with S^+ and S^- its positive and negative extensions, respectively. Let P_0 be the common prior (a probability distribution over Ω). The interpretation of S via Q proceeds as follows: for any $p \in \mathscr{P}(\Omega)$,

$$\begin{aligned}
&\text{[i]} \quad P_1(p) = P_0(p \mid \Omega \setminus S^-);\\
&\text{[ii]} \quad P_2(p) = P_1(p \mid S^+);\\
&\text{[iii]} \quad P_3(p) = \sum_{q \in Q} P_1(p \mid q) \times P_2(q).
\end{aligned}$$

P_3 is the final interpretation.

Let's unpack this. Step [i] rules out the possibility that S is false by conditioning the prior P_0 on $\Omega \setminus S^-$. That is, it updates the prior so that all the probability mass is redistributed over worlds where S is either true or undefined. The resulting distribution is P_1. Step [ii] conditions P_1 on S^+, producing the distribution P_2. Since $S^+ \subseteq \Omega \setminus S^-$, this is equivalent to directly conditioning the prior P_0 on S^+; that

is, $P_2 = P_1(\cdot \mid S^+) = P_0(\cdot \mid S^+)$. Thus, at this point, two distributions are in play: $P_1 = P_0(\cdot \mid \Omega \setminus S^-)$ and $P_2 = P_1(\cdot \mid S^+) = P_0(\cdot \mid S^+)$. Step [iii] 'combines' these two distributions: it does so by producing a new distribution, P_3, that assigns to each cell the same probability mass as P_2—for every cell $q \in Q$, $P_3(q) = P_2(q)$—while preserving the within-cell probabilities from P_1—for every cell $q \in Q$, $P_3(\cdot \mid q) = P_1(\cdot \mid q)$. This is accomplished by determining what fraction of each cell $q \in Q$ is taken up by p under P_1, and then weighting that fraction by the probability of q under P_2—an operation that amounts to an application of what is known as *Jeffrey conditionalisation* (Jeffrey 1965).[3]

SFK update mirrors the behaviour of Križ's update procedure. When S is not homogeneous—that is, when $S^\# = \varnothing$—a non-maximal interpretation cannot arise: in such cases, SFK update simply reduces to conditioning P_0 on S^+ ($P_3 = P_2 = P_1$). When $S^\# \neq \varnothing$, a non-maximal interpretation may or may not arise, depending on the context—namely, on which QUD is at play and on the background information encoded in the prior.

4 SFK update and non-maximal interpretation

In what follows, I will show how SFK update can be applied to (1a)–(1b), repeated below as (9).

(9) [Context: Jane is at a friend's party with her nine children.]

a. *John*: Will you stay at the party for a bit longer?

b. *Jane*: The kids are tired.
$\approx$ At least some of the kids are tired.

As discussed in §2, the accounts in Križ 2015, 2016, Bar-Lev 2021, and Križ & Spector 2021 must, in order to handle cases such as (9), make non-trivial assumptions to ensure their respective built-in relevance criteria are met—assumptions that become problematic once a broader range of data is taken into consideration. Unlike these accounts, SFK update is couched in a probabilistic framework, which can handle such cases in a straightforward manner. Indeed, in probabilistic frameworks, the conception of relevance that is typically adopted—dating back at least to Keynes 1921[4]—is less demanding than the one operative in non-probabilistic QUD frameworks: in particular, answers are not required to rule out a QUD cell—merely shifting

3 Jeffrey conditionalisation is defined as follows. Let $\mathscr{I} = \{c_1, \ldots, c_n\}$ be a partition of Ω, and let P be a prior over Ω. Suppose the updated probabilities $P'(c_i)$ for each cell $c_i \in \mathscr{I}$ are given. The Jeffrey update of P over the partition $\mathscr{I}$ is the distribution P' such that, for any $p \subseteq \Omega$, $P'(p) = \sum_{i=1}^{n} P(p \mid c_i) \times P'(c_i)$. This reduces to standard Bayesian conditioning in the special case where P' assigns probability 1 to a single cell of the partition.

4 See also Carnap 1950 and Büring 2003.

the probabilities assigned to the cells is enough. The formal definition is given below.

Definition II (Probabilistic Relevance). Let P_0 be the common prior, Q a partition of Ω, S a sentence, and S^+ the positive extension of S. Then S is relevant to Q under P_0 iff:

(a) $P_0(S^+) > 0$; and

(b) there is a $q \in Q$ such that $P_0(q) \neq P_0(q \mid S^+)$.

Under this definition, and given any reasonable prior P_0, (9b) qualifies as relevant to the QUD induced by (9a)—indeed, conditioning P_0 on $(9b)^+$ shifts the probabilities of the cells of that QUD. Thus, none of the complications discussed in §2 arise here: the QUD being addressed can be identified with the QUD induced by the overt question.

To make the workings of SFK update explicit, let's assume that the prior P_0 makes the following assignments:

YES (0.4)	NO (0.6)
(∀) 0.01	(∀) 0.1
(∃¬∀) 0.1	(∃¬∀) 0.4
(¬∃) 0.29	(¬∃) 0.1

Figure 1 P_0 (the prior distribution). YES and NO label the two cells of the (9a)–induced QUD. The dotted lines partition Ω into three rows, corresponding to the three extensions of (9b): the positive extension (top), the gap (middle), and the negative extension (bottom). The weights for each QUD cell and each intersection region appear in red.

The sentence that is uttered is (9b); thus, at step [i] of SFK update, P_0, the prior, is conditioned on $\Omega \backslash (9b)^-$ (on the information «S is either true or undefined»). The resulting distribution, P_1, is shown in the left panel of Fig. 2. At step [ii], P_1 is conditioned on $(9b)^+$ (on the information «S is true»), producing P_2, shown in the middle panel. At step [iii], SFK update combines these two distributions: the internal distribution of each QUD cell is inherited from P_1, while the weight of each individual cell is given by P_2. The final interpretation, P_3, is displayed in the right panel.

YES (0.18)	NO (0.82)
(∀) 0.016	(∀) 0.164
(∃¬∀) 0.164	(∃¬∀) 0.656
(¬∃) 0	(¬∃) 0

P_1

YES (0.09)	NO (0.91)
(∀) 0.09	(∀) 0.91
(∃¬∀) 0	(∃¬∀) 0
(¬∃) 0	(¬∃) 0

P_2

YES (0.09)	NO (0.91)
(∀) 0.008	(∀) 0.182
(∃¬∀) 0.082	(∃¬∀) 0.728
(¬∃) 0	(¬∃) 0

P_3

Figure 2 The three distributions produced by SFK update for (9b) given the QUD induced by (9a) and the prior P_0 in Fig. 1.

Does this interpretation conform to intuitive judgment? I believe it does. First, $P_3\big((9b)^{\#}\big) > 0$, which indicates that the final interpretation is non-maximal.[5] Moreover, while $P_3(\text{NO}) > P_3(\text{YES})$, $P_3(\text{YES}) > 0$. This is also a good prediction—after hearing (9b), one tends to infer that Jane is more likely to leave than to stay, and not that her departure is certain. SFK update, it appears, performs well in this case.

It is worth noting that P_2 and P_3, as shown in Fig. 2, assign identical probabilities to the QUD cells. This is entirely expected: in SFK update, the final probabilities of the QUD cells are fixed under the assumption that S, the uttered sentence, is true. (This is a reflection of Križ's original account, according to which the decision about which cells to keep and which to eliminate is likewise made under that assumption.) P_1 and P_3, by contrast, do not assign identical probabilities to the QUD cells in our toy example: as shown in Fig. 2, both P_1 and P_3 exhibit a bias towards the NO-cell, but this bias is stronger in P_3. This is also as expected: P_1 fixes QUD-cell probabilities on the basis of the information that S is either true or undefined (in the present case, that at least some of the kids are tired), whereas P_3 fixes them on the basis of the information that S is true (that all the kids are tired). It is thus unsurprising that the bias towards the NO-cell is stronger in P_3 (that all the kids are tired is an even stronger reason to leave the party than that at least some of them are).

SFK update, therefore, makes two interesting predictions: it predicts that if S were '*All* the kids are tired', with everything else held constant, P_3 (the final interpretation) would assign the same probabilities to the QUD cells as when S is 'The kids are tired'. By contrast, if S were '(At least) some of the kids are tired', P_3 would be expected to assign more weight to the NO-cell, though not as much as with

5 On Križ's original account, a sentence S receives a non-maximal interpretation iff at least one world in which S is undefined survives the update procedure. To interpret SFK's output, I adopt the natural probabilistic analogue of this condition—namely, that a sentence S receives a non-maximal interpretation iff $P_3(S^{\#}) > 0$. This assumption may, however, be too strong: if $P_3(S^{\#}) = 0.00001$, should we still count the interpretation as non-maximal? The question of how close to 0 $P_3(S^{\#})$ should be in order to count as 0 is not one I am in a position to answer—in fact, it is not even clear that the question makes sense: there may simply be no sharp cut-off point distinguishing maximal from non-maximal interpretations.

'The kids are tired' and 'All the kids are tired'. If these predictions are borne out, they may help to explain why, intuitively, it is not quite the same to respond to (9a) with 'The kids are tired' vis-à-vis an existential statement: 'The kids are tired' partly does what an existential statement does (communicating that some or all of the kids are tired), yet it is not quite the same as an existential statement—and this is because it induces the distribution over QUD cells associated with a universal statement.

5 Over-informative answers

In §2, I discussed some of the complications that come with having to enforce partial-answer compliance—either by idealising the common ground (Strategy 1) or by assuming that the operative QUD is not the one provided by the overt question (Strategy 2). SFK update avoids these complications: as shown in the previous section, partial-answer compliance does not need to be enforced on this account, and hence nothing needs to be idealised or accommodated.

There is, however, an empirical issue that remains on the table. As was also discussed in §2, when 'The kids are tired' is offered as a response to the overt question 'Are any of the kids tired?', it does not give rise to a non-maximal interpretation; instead, it sounds odd or infelicitous, as shown in (4) (repeated below in (10)). This observation, alongside other considerations, was part of the case against Strategy 2.

(10) a. *John*: Are any of the kids tired?

b. *Jane*: ??Yes, the kids are tired.
$\not\approx$ Yes, at least some of the kids are tired.

A general theory of non-maximal interpretation must be able to explain both (i) why, under (9a), 'The kids are tired' receives a non-maximal interpretation, and (ii) why, under (10a), it sounds odd. SFK update does (i) but not (ii). Indeed, just like Križ's (2015, 2016) account and subsequent proposals, SFK update predicts (10b) to be interpreted existentially (and hence non-maximally).[6] This is because, in such a case, P_3 is equal to P_1,[7] and P_1 corresponds to an existential interpretation—i.e. it is the result of updating the prior with the information that S is either true or undefined.

6 Križ (2015) attempts to deflect the challenge posed by the data point in (10) by making a strong methodological claim, namely: 'It is not possible to just ask a single overt question and pretend that the meaning of this question just is the current issue [the QUD], not even as a hypothesis in a thought experiment' (p. 86). He recognises, however, that such a move comes at a substantial cost:'[it] weakens the predictive power of the theory, since one cannot set up a context so precisely as to fully constrain the current issue and put a prediction to the test. The current issue is not directly accessible and cannot be so easily manipulated. One is therefore forced to restrict oneself to considerations of plausibility' (p. 86). I do not endorse Križ's claim: in the general case, overt questions strike me as reliable indicators of the underlying QUD.

7 To see this, it is enough to realise that, in a question-answer pair such as (10), P_1 and P_2 cannot differ

Now, if one accepts Križ's semantics for plural sentences, there is something clearly amiss in offering (10b) as an answer to (10a): the positive extension of (10b) is a strict subset of the YES-cell. This is problematic in the following sense: identifying the partition cell in which the actual world lives is, arguably, the ultimate goal of any inquiry; hence, providing more information than that is doing more than is required—in other words, it is over-informative. (10b) can therefore be regarded as a bad answer—'bad' in the sense that it is not admissible given the following ban:

Definition III (Ban on over-informative answers). Let Q be a partition of Ω, S a declarative sentence, S^+ the positive extension of S, and P_0 the common prior (a probability distribution over Ω). S *cannot* be used to address Q under P_0 if it is *over-informative*—that is, if there exists a $q \in Q$ such that

$$P_0(q \mid S^+) = 1 \quad \text{and} \quad P_0(S^+ \mid q) < 1.$$

This simple ban, motivated by general pragmatic considerations,[8] accounts for the oddness of (10b) in a straightforward manner: (10b) is over-informative; hence the update procedure stalls; hence what the listener is left with is the bare semantics of the uttered sentence (true if all, false if none, undefined otherwise).

Importantly, this ban does not prevent (9b) from addressing the QUD induced by (9a) under the P_0 in Fig. 1—(9b) is compatible with both cells of the QUD under such a prior, and thus not over-informative. Nor does it prevent (10b) from addressing the QUD induced by the question 'Are all of the kids tired?'—(10b), irrespective of what the prior is, is bound not to be over-informative in this case. I take this to be a good result, given the observations made in §2 (compare (6) with (7)).

At this point, one might ask why Križ (2015, 2016) could not adopt an analogous ban. The answer is that doing so would make it impossible for his account to generate non-maximal interpretations relative to a two-cell QUD. Indeed, on Križ's (2015, 2016) account, a sentence S is interpreted non-maximally under a two-cell QUD Q just in case S^+ is a proper subset of one of the two cells of Q—i.e. precisely in cases in which S is over-informative. Banning over-informative answers, therefore, would eliminate the only configuration that yields non-maximality.

SFK's comparative advantage is thus this: non-maximal interpretations can arise in cross-cutting configurations—i.e. cases in which the sentence's positive extension is compatible with every QUD cell, and hence does not rule out any of them. In fact, given a two-cell QUD Q, SFK update, together with the over-informativity ban in Def. III, predicts that an unquantified plural sentence S with $P_0(S^\#) > 0$

with respect to the weights that they assign to the QUD cells: under either distribution, the YES-cell is assigned probability 1, whereas the NO-cell is assigned probability 0.

8 Recall Grice's (1975: 45) second sub-maxim of Quantity: 'Do not make your contribution more informative than is required.'

can be interpreted non-maximally *only if* its positive extension is compatible with both cells of Q. This prediction can be tested by identifying cases of non-maximal interpretation and checking, using the *So probably...* test exemplified in (2)–(3), whether cell elimination does or does not take place.

5.1 A note on the proposed ban

The over-informativity ban imposed in Def. III does seem to make bad predictions in some cases, at least on first inspection. For example, if (11b-i) is bad due to over-informativity, why is (11b-ii) acceptable? Indeed, according to Križ's semantic architecture, the positive extensions of (11b-i) and (11b-ii) are one and the same. Thus, for the same reason the proposed ban rules out (11b-i), it also rules out (11b-ii).

(11) a. *John*: Are any of the kids tired?
b. *Jane*: [i] ??The kids are tired.
[ii] ✓ ALL the kids are tired.

Note, however, that (11b-ii) is acceptable only if 'all' bears focal stress (cf. Bar-Lev & Bassi 2025). This is suggestive: it appears the speaker is compelled to actively signal—by means of stress placement—that she is addressing a QUD other than the one induced by the overt question, viz. *How many kids are tired?*

A similar contrast, also involving two truth-conditionally equivalent sentences, is noted in Umbach 2005:

(12) a. *John*: What did the small children do today?
b. *Jane*: [i] ??The small children stayed at home, and the bigger ones went to the zoo.
[ii] ✓ The small children stayed at home, but the bigger ones went to the zoo.

According to Def. III, both (12b-i) and (12b-ii) should be infelicitous—however, (12b-ii) does seem like an acceptable discourse move. Umbach (2005) takes this contrast to indicate that 'but', as opposed to 'and', is capable of inducing a QUD shift—in the case at hand, from *What did the small children do today?* to *What did the small children do today, and did the bigger ones do the same?*

It is clear that there are cases which, at least *prima facie*, constitute counterexamples to the proposed ban. To make sense of these apparent counterexamples, it seems reasonable to claim that, in such cases, speakers are not actually being over-informative, because—arguably—the QUD associated with the overt question has been shifted. I am partial to such an account; that said, in the absence of a robust understanding of how this QUD-shifting mechanism works, the explanation offered remains incomplete.

6 SFK update: limitations and shortcomings

Empirically, given a two-cell QUD, there are two configurations in which *maximal* interpretations are observed: (i) complete-answers configurations, as in (13); and (ii) cross-cutting configurations, i.e. configurations in which the sentence's positive extension is compatible with both cells, as in (14).

(13) a. *John*: Are all the kids tired?

b. *Jane*: They are. (**So probably yes.*)[9]
$\approx$ All the kids are tired.

(14) [Context: Jane is at a friend's party with her nine children.]

a. *John*: Can any of your kids help me move the table?

b. *Jane*: The kids are tired. (✓*So probably not.*)
$\approx$ All the kids are tired.

SFK update, as discussed, has no issues deriving maximal interpretations in the first kind of configuration: in such cases, S^+ is identical to one of the cells, while $S^\#$ is a (proper) subset of the other. Since the final probabilities of the cells are given by P_2, and $P_2 = P_1(\cdot \mid S^+) = P_0(\cdot \mid S^+)$, the cell containing $S^\#$ is bound to be assigned probability 0—and thus so is $S^\#$.

However, in the cross-cutting configuration—i.e. when $P_0(\text{YES} \wedge S^+) > 0$ and $P_0(\text{NO} \wedge S^+) > 0$—SFK fails to yield maximality as soon as $P_0(S^\#) > 0$. *Proof:* Suppose $P_0(S^\#) > 0$. By step [i] of SFK update (Def. I), $P_1(\cdot) = P_0(\cdot \mid S^+ \cup S^\#)$, hence $P_1(S^\#) > 0$. Step [ii] produces $P_2(\cdot) = P_1(\cdot \mid S^+) = P_0(\cdot \mid S^+)$; because of cross-cutting, $P_2(\text{YES}) > 0$ and $P_2(\text{NO}) > 0$. Finally, by step [iii],

$$P_3(S^\#) \quad = \quad \underbrace{\sum_{q \in \{\text{YES},\text{NO}\}} P_1(S^\# \mid q) \times P_2(q)}_{>0}$$

This follows since $P_2(\text{YES}) > 0$ and $P_2(\text{NO}) > 0$, and $P_1(S^\#) > 0$ implies $\exists q \in \{\text{YES}, \text{NO}\}$ with $P_1(S^\# \mid q) > 0$. Hence, in the cross-cutting configuration, SFK fails to derive a maximal interpretation whenever $P_0(S^\#) > 0$.

The formal question therefore is this: how can the pragmatic system tell apart cases like (9), in which the answer is interpreted non-maximally, from cases like (14), in which the answer is interpreted maximally. On Križ's framework, it is clear how—maximal and non-maximal interpretations are assumed to be generated in different formal configurations: maximality obtains when no QUD cell overlaps

9 I am using the 'reduced' version, as opposed to the longer 'The kids are tired', for the reasons discussed in §2.

with both S^+ and $S^\#$, while non-maximality obtains when at least one cell overlaps with both. However, as soon as one refrains from working with 'boutique' epistemic states or accommodating unasked questions, this structural assumption at the heart of Križ's account comes under pressure: as the exchange in (14) indicates, a plural sentence S can be interpreted maximally despite there being a QUD cell that overlaps (is compatible) with both S^+ and $S^\#$.

How, then, can we distinguish between (9) and (14)? In Feinmann 2020, I proposed a method to tell them apart based on the following observation. In the former (non-maximal) case, S^+ and $S^\#$ are equivalent with respect to the QUD in a *qualitative* sense: both support the conclusion (make it more likely) that Jane will leave the party soon.[10] In the latter (maximal) case, by contrast, S^+ and $S^\#$ are not equivalent in this sense: S^+ supports the conclusion that no child can help move the table, while $S^\#$ arguably supports the opposite conclusion.[11]

The following generalisation thus suggests itself: a non-maximal interpretation arises only if S^+ and $S^\#$ are qualitatively equivalent with respect to the QUD; if they are not, a maximal interpretation ensues. To make this generalisation precise, however, the notion of qualitative equivalence must be made explicit. The following definition does just that.

Definition IV (Qualitative equivalence of two propositions under P and Q). Let Q be a partition of Ω and P a probability distribution over Ω. For any two propositions $p, r \subseteq \Omega$ with $P(p) > 0$ and $P(r) > 0$, we say

$$p \approx_{Q,P} r$$

if and only if the following two conditions hold:

(i) **Direction-Preservation:** For every cell $q \in Q$,

$$\big[(P(q \mid p) - P(q)) \times (P(q \mid r) - P(q))\big] > 0 \quad \vee \quad P(q) = P(q \mid p) = P(q \mid r).$$

10 Both (9b)$^+$ and (9b)$^\#$ entail that at least one of Jane's children is tired, and a single tired child is a good enough reason to leave a party.

11 Learning that (14b)$^+$ is the case is expected to favour the NO-cell—the proposition that none of Jane's children can help—given ordinary background assumptions. By contrast, learning that (14b)$^\#$ is the case is expected to favour the YES-cell. Indeed, (14b)$^\#$ entails both that at least one child is tired and that at least one child is not tired. Given reasonable assumptions about the prior—in particular, that the presence of a non-tired child is more informative for the YES-cell than the presence of a tired child is for the NO-cell (the YES-cell is existential, whereas the NO-cell requires that *no* child can help)—conditioning on (14b)$^\#$ is expected to have this effect.

(ii) **Order-Preservation:** For any two distinct cells $q, q' \in Q$,

$$\begin{aligned} P(q \mid p) > P(q' \mid p) &\implies P(q \mid r) > P(q' \mid r), \\ P(q \mid p) = P(q' \mid p) &\implies P(q \mid r) = P(q' \mid r), \\ P(q \mid p) < P(q' \mid p) &\implies P(q \mid r) < P(q' \mid r). \end{aligned}$$

To better grasp the content of the definition above, here is a plain-language breakdown of its two conditions:

- **Direction-Preservation**: for every cell $q \in Q$, it requires that p and r affect q's initial probability in the same way: whenever p raises (or lowers) q's probability, r must raise (or lower) it as well; and whenever p leaves q's probability unchanged, so must r.
- **Order-Preservation:** For any two distinct cells $q, q' \in Q$, whenever q is more likely than q' conditional on p, q must also be more likely than q' conditional on r; and conversely, if q' is more likely than q conditional on p, then it must also be more likely conditional on r. Moreover, if q and q' are equally likely conditional on p, then they must be equally likely conditional on r as well.

These two conditions make precise the notion of *qualitative equivalence* appealed to earlier: even if two distinct propositions p and r yield different probabilities for the cells of a given QUD Q, they may nonetheless induce the same qualitative profile on Q—namely, the same direction of change relative to the prior for each cell and the same posterior weak order on the partition.

With Def. IV in place, a stronger version of SFK update can be formulated ('stronger' in the sense that it raises the threshold needed for a homogeneous sentence to be interpreted non-maximally). In this version, step [iii] of SFK update is executed only if $S^+ \approx_{Q,P_0} S^\#$; if it is not, then the final interpretation is given by P_2. For this reason, I will refer to it as *QE-gated SFK*, abbreviated SFK^{QE}, where 'QE' stands for qualitative equivalence (as defined in Def. IV).

Definition V (SFK$^{\mathbf{QE}}$ update). Let Q be a partition of Ω. Let S be a declarative sentence, with S^+ and S^- its positive and negative extensions, respectively, and $S^\#$ its extension gap. Let P_0 be the common prior (a probability distribution over Ω).

The interpretation of S via Q proceeds as follows: for any $p \in \mathscr{P}(\Omega)$,

$$\begin{aligned}
&\text{[i]} \quad P_1(p) = P_0(p \mid \Omega \setminus S^-);\\
&\text{[ii]} \quad P_2(p) = P_1(p \mid S^+);\\
&\text{[iii]} \quad \text{if } S^+ \approx_{Q,P_0} S^{\#} \text{ then}\\
&\qquad P_3(p) = \sum_{q \in Q} P_1(p \mid q) \times P_2(q);\\
&\text{otherwise,} \quad P_3(p) = P_2(p).
\end{aligned}$$

Due to space constraints, I will not provide a full derivation using SFK^{QE} update; it should be clear, however, how this procedure accounts for the contrast in interpretation between (9b) and (14b). Under a reasonable prior, $S^+ \approx_{Q,P_0} S^{\#}$ should hold in the former case, and the resulting interpretation is therefore expected to be non-maximal. In the latter case, by contrast, $S^+ \approx_{Q,P_0} S^{\#}$ is not expected to hold, and the resulting interpretation is thus expected to be maximal—i.e. in such a case, $P_2 = P_3$.

Unfortunately, the proposed update procedure—whether in its original form or in the QE-gated version—faces an additional issue.

(15) [Context: Lea, a newly trained teacher, had to teach a large class—40 children—for the first time.]

a. *Paul*: Did the class go well?

b. *Lea*: The kids looked happy. (✓*So probably yes.*)
$\approx$ Most (or all) of the kids looked happy.
$\not\approx$ Some (or all) of the kids looked happy.

(15b) is clearly interpreted non-maximally: it does not entail that all the kids looked happy. However, although non-maximal, (15b) is not interpreted existentially—for instance, it seems incompatible with a situation in which only two kids looked happy.

Neither version of the proposed update procedure—neither SFK update (Def. I) nor SFK^{QE} update (Def. V)—can deliver (15b)'s attested interpretation. Consider first SFK update. Since the positive extension of (15b) is compatible with both QUD cells, SFK update can only rule out worlds in which S is false. Thus, if...

$$P_0(\text{only two kids looked happy}) > 0,$$

then

$$P_3(\text{only two kids looked happy}) > 0;$$

and this is a bad prediction: such worlds should have probability 0 under P_3.

SFK^{QE} update doesn't fare any better. If P_0(only two kids looked happy) > 0, then two outcomes are possible—either $P_3 = P_2$, which means the sentence has been assigned a maximal interpretation (a bad outcome), or P_3(only two kids looked happy) > 0 (the same outcome as in the non-gated version).

It is worth noting that the earlier problem that I have discussed—which led me to the formulation of SFK^{QE} update—and the problem just discussed are, at a certain level of abstraction, one and the same. Both, at least, can be characterised as manifestations of a general issue: when the positive extension of the sentence is compatible with every QUD cell—SFK update, given a realistic prior, generates an existential interpretation despite the fact that, as we have seen, the attested interpretation is often stronger. SFK^{QE} update (a refinement of the original system) helps in cases such as (14)—where the attested interpretation is maximal or universal—but fails in cases such as (15)—where the attested interpretation is stronger than an existential but weaker than a universal. From this perspective, QE-gating merely narrows the scope of the failure: though it succeeds in generating universal interpretations in certain cross-cutting configurations, it fails to provide a comprehensive solution.

7 Concluding remarks

SFK update does some interesting things. First, it generates the correct interpretation for (1b)/(9b), and it does so without idealising the common ground or making use of unasked questions. Second, it affords a new perspective on cases such as (10)—indeed, it permits (10b) to be treated as an over-informative answer, an analysis that is not available on Križ's original account. Third, it makes a novel prediction: SFK makes no distinction, at the level of QUD-cell probabilities, between 'The *X*s are *blah*' and '*All* the *X*s are *blah*'—both end up assigning the same probability to each QUD cell.

SFK update also exposes a rather serious problem that, arguably, was latent though not visible in Križ's framework: if one refuses to indulge in idealisations and takes the empirical facts at face value, what Križ's update procedure tells us to do (namely, to remove the worlds in which S is false, and to discard the cells that are not compatible with S^+) often fails to exclude all the worlds that should be excluded.

Now, the fact that what Križ tells us to do is not enough does not mean that it should not be done. Assuming that it should be, the following question arises: *what else* needs to be done, and at which level? Should we strengthen the pragmatics, as we attempted in Def. V with limited success, or should we perhaps reconsider the semantics of homogeneous sentences? These questions will need to be addressed elsewhere.

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Diego Feinmann
Jana Kazimierza 5, 01-248 Warsaw, Poland
diego.feinmann@ipipan.waw.pl