

Free choice and dual prohibition in (non)monotonic contexts*

Paul Marty
Universidade de Lisboa

Patrick D. Elliott
Heinrich Heine University Düsseldorf

Guillermo Del Pinal
University of Massachusetts Amherst

Jacopo Romoli
Heinrich Heine University Düsseldorf

Abstract This paper examines three prominent approaches to Free Choice (FC) and Dual Prohibition (DP): the implicature account relying on *exhaustivity* (EXH; Fox 2007; Bar-Lev & Fox 2017, 2020), the cognitive-bias account based on *Neglect Zero* (NZ; Aloni 2022), and a recent development of the implicature account grounded in *presuppositional exhaustification* (PEX; Bassi, Del Pinal & Sauerland 2021; Del Pinal, Bassi & Sauerland 2024). While all three approaches capture the FC-DP alternation in simple sentences (e.g., *Leo can/cannot buy the cupcake or the pear*), they diverge in their predictions regarding the strength of FC and DP in non-monotonic environments (e.g., *Exactly one child can/cannot buy the cupcake or the pear*). We tested these predictions in two experiments investigating the possible interpretations of simple and non-monotonic sentences varying in polarity. The results align with the predictions of the PEX approach, while posing challenges for the other two. We conclude by discussing potential theoretical refinements for the EXH and NZ accounts and by highlighting remaining open questions for PEX.

Keywords: free choice, dual prohibition, disjunction, possibility modals, non-monotonic contexts, implicature, presupposition, exhaustivity, neglect zero, experimental semantics

1 Introduction

Disjunction in the scope of a possibility modal gives rise to a conjunctive ‘Free Choice’ (FC) reading, which is generally absent under negation (Kamp 1973). To illustrate, consider the singular (SG) sentences in (1)–(2). Intuitively, the POSITIVE sentence in (1) is true only if Leo can buy the cupcake *and* can buy the pear, i.e., if Leo can choose which one to buy. Crucially, its NEGATIVE counterpart in (2) does not

* We thank Maria Aloni, Moysh Bar-Lev, Marco Degano, Émile Enguehard, Diego Feinmann, Alex Kaloimoros, Tomasz Klochowicz, Giorgio Sbardolini, Benjamin Spector and the audiences at SALT 35, Amsterdam, Tel Aviv, and Edinburgh for helpful comments and suggestions. This research was funded by the Deutsche Forschungsgemeinschaft (DFG, Projektnummer: RO6384/1-1) and the Fundação para a Ciência e a Tecnologia (FCT, DOI: 2023.06820.CEECIND/CP2891/CT0008).

merely convey that Leo cannot choose which one to buy (negation of FC), but rather that he cannot buy either—a reading often referred to as ‘Dual Prohibition’ (DP).

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|-----|---|---|
| (1) | Leo can buy the cupcake or the pear. | SG-POS |
| | a. <u>Strong</u> : Leo can choose | <i>with FC</i> : $\diamond\phi \wedge \diamond\psi$ |
| | b. <u>Weak</u> : Leo is not prohibited | <i>w/o FC</i> : $\diamond(\phi \vee \psi)$ |
| (2) | Leo cannot buy the cupcake or the pear. | SG-NEG |
| | a. <u>Strong</u> : Leo is prohibited | <i>with DP</i> : $\neg\diamond\phi \wedge \neg\diamond\psi$ |
| | b. <u>Weak</u> : Leo cannot choose | <i>w/o DP</i> : $\neg(\diamond\phi \wedge \diamond\psi)$ |

In this paper, we examine three approaches to the FC-DP alternation illustrated above: the implicature account relying on *exhaustivity* (EXH; Fox 2007; Bar-Lev & Fox 2017, 2020), the cognitive-bias account based on *Neglect Zero* (NZ; Aloni 2022), and a recent development of the implicature account grounded in *presuppositional exhaustification* (PEX; Bassi et al. 2021; Del Pinal et al. 2024). While all three approaches capture the FC-DP pattern in simple sentences like (1)-(2), their predictions diverge in certain quantificational contexts. To evaluate these differences, we focus here on two types of non-monotonic (NM) environments varying in polarity (POSitive vs. NEGative):¹

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|-----|--|--------|
| (3) | Exactly/Only one child can buy the cupcake or the pear. | NM-POS |
| | a. One child can choose, and <i>all other children are prohibited</i> . | Strong |
| | b. One child can choose, and <i>all other children can't choose</i> . | Weak |
| (4) | Exactly/Only one child cannot buy the cupcake or the pear. | NM-NEG |
| | a. One child is prohibited and <i>all other children can choose</i> . | Strong |
| | b. One child is prohibited and <i>all other children aren't prohibited</i> . | Weak |

Such quantificational statements can be decomposed into two components: (i) a positive component, which states that one child satisfies the scope, and (ii) a negative component, which states that all other children do not. By holding constant the FC/DP interpretation of (i), we can identify potential Strong and Weak readings depending on whether (ii) is also interpreted with FC/DP. For instance, in the positive case, the Strong reading implies that all the other children cannot buy either item (DP), whereas the Weak reading implies only that they cannot choose which one to buy (the negation of FC). Similarly, in the negative case, the Strong reading implies

¹ Gotzner, Romoli & Santorio (2020) explored parts of the paradigm under consideration. Our study provides a more systematic investigation of Weak readings by including NM-NEG cases—crucial for distinguishing between the three approaches—as well as by broadening the range of NM quantifiers.

that all the other children can choose (FC), whereas the Weak reading implies merely that they are not prohibited (the negation of DP).

Section 2 provides an overview of the three approaches and shows that the three systematically differ in their predictions regarding the availability of Weak readings across NM cases: the EXH approach predicts such readings only for the positive case, the NZ approach only for the negative one, and the PEX approach across the board. Section 3 reports on two sentence–picture verification studies designed to test these predictions by investigating the availability of Weak readings for the SG and NM sentences illustrated above, using ‘exactly one’ (Experiment 1) and ‘only one’ (Experiment 2). The results show that Weak readings are readily available in both positive and negative NM cases. These findings align with the predictions of the PEX approach while posing challenges for the other two. Section 4 discusses possible theoretical refinements for the EXH and NZ approaches and highlights some remaining open issues for the PEX approach. Section 5 concludes.

2 Three approaches to the FC-DP pattern

In this section, we present baseline versions of the three approaches and examine their predictions concerning the distribution of Weak and Strong readings for SG and NM sentences. Refinements of these accounts are discussed in Section 4. For concreteness, we illustrate the predictions using the ‘exactly one’ variants of the NM cases, relative to the WEAK contexts shown in Table 1, which correspond to the visual stimuli employed in our experiments (see Section 3.2 for details). In their basic formulations, the three approaches yield analogous predictions for the ‘only one’ variants, aside from potential differences related to the assertion–presupposition divide. A summary of the predictions made by each approach is provided in Table 2.


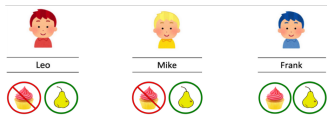
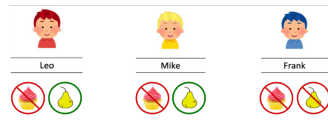
 <p>Leo Mike Frank</p>	 <p>Leo Mike Frank</p>	 <p>Leo Mike Frank</p>
(a) SG-POS and SG-NEG	(b) NM-POS	(c) NM-NEG

Table 1 Example WEAK contexts in which the SG-POS, SG-NEG, NM-POS and NM-NEG sentences illustrated in (1)-(4) are true on their putative Weak reading, but false on their Strong reading.

2.1 The EXH approach

The EXH approach (Fox 2007; Bar-Lev & Fox 2017, 2020, a.o.) rests on three core assumptions. First, disjunction and modals retain their classical existential semantics. Second, disjunction within the scope of an existential modal can be strengthened via an implicature-derivation process, implemented by the covert exhaustivity operator, EXH. Finally, the distribution of EXH is constrained by an economy condition—(5)—which licenses EXH only when its insertion does not lead to vacuity or weakening.

(5) Economy constraint

An occurrence of EXH in a sentence *S* is not licensed if the resulting meaning of *S* with that occurrence of EXH is entailed by *S* without it.

These assumptions jointly account for the singular cases in (1) and (2). The FC-reading of positive sentences such as (1) arises because their literal meaning can—and in fact must—be strengthened to a conjunctive one by applying EXH, as illustrated in (6a). In essence, EXH operates over a sentence and its set of alternatives, by excluding some and including others.² In the case of (1), applying EXH crucially leads to the inclusion of the alternatives *Leo can buy the apple* and *Leo can buy the pear*, whose conjunction yields the FC inference that he can choose between the two. The DP-reading of negative sentences like (2) follows directly from the existential semantics of the modal and disjunction under negation, as show in (6b). Here, the economy constraint in (5) ensures that EXH cannot appear above negation or within its scope, as it would otherwise be vacuous or lead to weakening (see (6c)–(6d)).

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|-----|----|--|----------------------------|
| (6) | a. | EXH[Leo can buy the cupcake or the pear] | ↗ <i>Leo can choose</i> |
| | b. | NEG[Leo can buy the cupcake or the pear] | ↗ <i>Leo is prohibited</i> |
| | c. | *EXH[NEG[Leo can buy the cupcake or the pear]] | ↗ <i>Leo is prohibited</i> |
| | d. | *NEG[EXH[Leo can buy the cupcake or the pear]] | ↗ <i>Leo can't choose</i> |

Thus, for the singular cases, the EXH approach predicts rejection in the relevant WEAK contexts (see Table 1a), since the only interpretations compatible with the economy constraint, (6a) and (6b), are false in those contexts.

Turning to the non-monotonic cases, the EXH approach generally predicts possible parses for these sentences in which exhaustification applies either globally (above the quantifier) or locally (within its scope).³ Some of these parses may

² This sketch follows Bar-Lev & Fox (2017, 2020). In Fox (2007), exhaustification involves only exclusion, possibly applied recursively. The distinction is immaterial for our purposes.

³ In principle, both local and global exhaustification could apply simultaneously. Here, however, this configuration yields the same predictions as one of the others, so we omit it for simplicity.

nonetheless be ruled out by the economy constraint in (5). For the NM-POS case, both global and local parses are available and yield distinct truth conditions. In the relevant WEAK contexts, the sentence is predicted to be true with global EXH but false with local EXH, hence predicted to be ambiguous.

- (3) Exactly one child can buy the cupcake or the pear. NM-POS
- a. EXH [exactly one child_x [x can buy the cupcake or the pear]]
 \rightsquigarrow *one child can **choose** and all others are **prohibited***
- b. [exactly one child_x EXH [x can buy the cupcake or the pear]]
 \rightsquigarrow *one child can **choose** and all others **can't choose***

In the NM-NEG case, both global and local options are also available, but the local one is permitted by economy only when EXH appears within the scope of negation (cf. (6c) is equivalent to an LF without EXH, hence blocked by (5)). Crucially, the interpretations derived from both options are false in the WEAK contexts.⁴

- (4) Exactly one child cannot buy the cupcake or the pear. NM-NEG
- a. EXH [exactly one child_x NEG [x can buy the cupcake or the pear]]
 \rightsquigarrow *one child is **prohibited** and more than one child **can't buy one** and more than one child **can't buy the other***
- b. [exactly one child_x NEG EXH [x can buy the cupcake or the pear]]
 \rightsquigarrow *one child **can't choose** and all others can **choose***
- c. *[exactly one child_x EXH [NEG [x can buy the cupcake or the pear]]]
 \rightsquigarrow *one child is **prohibited** and all others **aren't prohibited***

2.2 The NZ approach

The NZ approach, developed by Aloni (2022), employs a *bilateral, state-based semantics* to model free choice as an interpretive bias. In this framework, a sentence ϕ is always evaluated relative to a state s (a set of possible worlds), which either *asserts* or *rejects* ϕ . Despite this added layer, the underlying logic remains classical. To illustrate, (7) sketches a simplified version of Aloni's fragment, focusing on disjunction and epistemic possibility (the approach extends straightforwardly to other modal flavours). Here, p ranges over atomic sentences and ϕ over formulas.

- (7) a. s **asserts** p iff $\forall w \in s, p(w) = 1$, and **rejects** p iff $\forall w \in s, p(w) = 0$.

⁴ The predicted reading with global EXH seems problematic, as it does not match speakers' intuitive judgments. One might instead expect a stronger reading—e.g., that all other children can choose. For present purposes, however, what matters is that the predicted inference is false in WEAK contexts, as would be the stronger reading.

- b. s **asserts** $\neg\phi$ iff s rejects ϕ , and **rejects** $\neg\phi$ iff s asserts ϕ .
- c. s **asserts** $\phi \vee \psi$ iff $\exists t, t', s = t \cup t', t$ asserts ϕ and t' asserts ψ .
- d. s **rejects** $\phi \vee \psi$ iff s rejects ϕ and s rejects ψ .
- e. s **asserts** $\diamond\phi$ iff $\exists t \subseteq s, t \neq \emptyset, t$ asserts ϕ , and **rejects** $\diamond\phi$ iff s rejects ϕ .

By design, this fragment does not derive FC. To see why, consider the possibility statement $\diamond(a \vee b)$. A non-empty state like $\{w_a\}$ —where a is true and b false—asserts this sentence: $\{w_a\}$ can be split into two sub-states, $\{w_a\} \cup \emptyset$, with $\{w_a\}$ asserting a and the empty state \emptyset vacuously asserting b .⁵ However, $\{w_a\}$ does not assert $\diamond b$, since there is no non-empty substate of $\{w_a\}$ that asserts b . Thus, a state s may assert $\diamond(a \vee b)$ without asserting both $\diamond a$ and $\diamond b$, and so FC does not follow.

On Aloni’s proposal, state-sensitivity is exploited to capture a general interpretive bias to disregard vacuous (empty) states—*Neglect Zero* (NZ)—from which FC emerges. In a nutshell, NZ adds the condition that, for a sentence to be asserted or rejected, its evaluation state must be non-empty. This requirement is assumed to ‘percolate’ down in complex sentences (but see Aloni 2022 for alternative formulations). For ease of exposition, we illustrate the result of NZ on the fragment above by defining more stringent notions of **assertion*** and **rejection***:

- (8) a. s **asserts***/**rejects*** p iff $s \neq \emptyset$ and s asserts/rejects p .
- b. s **asserts***/**rejects*** $\neg\phi$ iff s rejects*/asserts* ϕ .
- c. s **asserts*** $\phi \vee \psi$ iff $\exists t, t', s = t \cup t', t$ asserts* ϕ and t' asserts* ψ .
- d. s **rejects*** $\phi \vee \psi$ iff s rejects* ϕ and s rejects* ψ .
- e. s **asserts*** $\diamond\phi$ iff $\exists t \subseteq s, t \neq \emptyset, t$ asserts* ϕ , and **rejects*** $\diamond\phi$ iff s rejects* ϕ .

With **assertion*** in place, FC now follows for SG-POS sentences: any state asserting* $\diamond(a \vee b)$ also asserts* $\diamond a$ and $\diamond b$. In particular, $\{w_a\}$ does not assert* $\diamond(a \vee b)$, as it cannot be partitioned into *non-empty* substates supporting a and b respectively. On the other hand, even with the enriched notion of **assertion***, negated possibility statements remain appropriately strong. Since a state asserts* $\neg\diamond(a \vee b)$ iff it is non-empty and rejects* both a and b , it follows that such a state also rejects* both $\diamond a$ and $\diamond b$, accounting for the DP reading of SG-NEG sentences.

Aloni (2022) further emphasizes that NZ reflects a *bias*, not a hard constraint, meaning that interpreters can fall back on the non-enriched notion of assertion. We therefore follow Aloni in assuming that both **assertion** and **assertion*** are available. While this does not affect the interpretation of SG-NEG sentences, which are predicted

⁵ Note that disjunction is associated with a ‘split’ assertion condition: for s to support $a \vee b$, it must be possible to partition s into two substates, one supporting a and the other b , cf. (7c). Since the empty state \emptyset asserts every classical proposition, nothing in the fragment in (7) prevents \emptyset from asserting b .

to be false in the WEAK contexts, it predicts that SG-POS sentences should also allow a weaker non-FC reading, which is true in those same WEAK contexts (see Table 1a).

For the non-monotonic cases, the predictions depend on the assertion/rejection clause for the quantificational statement. Aloni (2022) does not spell out the clauses for *exactly one*, but a natural extension is (roughly) as follows: *s* asserts *exactly one child P* iff there is a child *x* such that *s* asserts $P(x)$ and, for every other child $y \neq x$, *s* rejects $P(y)$. NZ-enrichment is thus assumed to enrich both assertion and rejection clauses. For NM-POS sentences, NZ-enrichment yields the Strong reading of interest; what we call the Weak reading, however, cannot be derived, since only the positive component may be weakened.

- (3) Exactly one child can buy the cupcake or the pear.
- a. One child can **choose**; all others are **prohibited**. NZ-enriched
 - b. One child **isn't prohibited**; all others are **prohibited**. w/o enrichment

By contrast, NM-NEG sentences are predicted to be ambiguous between a strong and a weak reading: NZ-enrichment yields the Strong reading of interest, while the Weak reading arises in the absence of enrichment.

- (4) Exactly one child cannot buy the cupcake or the pear.
- a. One child is **prohibited**; all others can **choose**. NZ-enriched
 - b. One child is **prohibited**; all others **aren't prohibited**. w/o enrichment

2.3 The PEX approach

The PEX approach to FC (Del Pinal et al. 2024) combines insights from the EXH approach with insights from Goldstein's (2019) semantic account of FC. Most importantly, on this approach, the content that the exhaustification operator PEX adds to the prejacent is treated as non-asserted, presupposed content. For example, whereas EXH(\exists) outputs both \exists and $\neg\forall$ as asserted contents, PEX(\exists) asserts the prejacent \exists but presupposes $\neg\forall$. In a simple positive sentence like (1), what is presupposed is not simply the FC inference, but rather a *modal homogeneity* inference, following Goldstein (2019). Taken together with the asserted content, FC is entailed. This is illustrated in (9) (note: in the ' p_q ' notation, ' p ' stands for the assertive and ' q ' for the presupposed content):⁶

$$(9) \quad \text{PEX}(\diamond(p \vee q)) = \diamond(p \vee q)_{\diamond p \leftrightarrow \diamond q} \quad \models \diamond p \wedge \diamond q$$

⁶ While on the PEX approach, the modal homogeneity presupposition arises through reasoning over alternatives, on Goldstein's (2019) account, it is built into the lexical semantics of disjunction. See Del Pinal et al. (2024) for detailed discussion of the derivation of this presupposition and for a comparison with Goldstein's account.

For independent reasons, the version of the economy constraint in (5) that best pairs with the PEX approach states, roughly, that PEX applies locally (measured by structural distance to its associated operator), unless doing so would weaken the overall meaning. Importantly, since PEX is a presupposition trigger, we expect the inferences it triggers to behave like other presupposed content, that is, to either project or be locally accommodated. For simplicity, we will model and represent local accommodation here using an A-operator, which ‘flattens’ the outputs of PEX into with the assertive component.

With these assumptions in place, consider first the SG cases. The default parse of (1), shown in (10a), entails FC. For (2), the default parse is as shown in (10b). Recall that, as a result of the embedded instance of PEX, the material in the scope of NEG asserts $\diamond(\text{cupcake} \vee \text{pear})$ and presupposes $\diamond\text{cupcake} \leftrightarrow \diamond\text{pear}$. Following standard projection patterns, the homogeneity presupposition should project out of negation, which then applies to $\diamond(\text{cupcake} \vee \text{pear})$, yielding a strong DP reading (Goldstein 2019). Note, however, that a parse with embedded A is also possible, as in (10c). In that case, the homogeneity presupposition is trapped within the negation’s scope, yielding the weaker reading that Leo cannot choose (negation of FC).

- (10) a. PEX[Leo can buy the cupcake or the pear] SG-POS
 \rightsquigarrow *Leo can **choose***
- b. NEG[PEX[Leo can buy the cupcake or the pear]] SG-NEG
 \rightsquigarrow *Leo is **prohibited***
- c. NEG[A[PEX[Leo can buy the cupcake or the pear]]]
 \rightsquigarrow *Leo **can’t choose***

Consider now the NM cases. On this approach, we get default parses with PEX applied locally to the embedded sentence. In addition, we also expect parses with the A-operator applied to the embedded instance of PEX. For NM-POS sentences, we thus have two main parses. The parse in (a) captures the Strong reading: embedded PEX triggers a homogeneity presupposition over each child’s options; assuming, for now, that presuppositions triggered in the scope of NM quantifiers project universally, we get $\forall x(\diamond\text{cupcake}(x) \leftrightarrow \diamond\text{pear}(x))$, which entails that the one child has FC, while the others have DP. The parse in (b) captures the Weak reading: applying the A-operator over embedded PEX traps the homogeneity presupposition triggered by PEX within the quantifier’s scope; the result entails that one child has FC while the others do not.

- (3) Exactly one child can buy the cupcake or the pear. NM-POS
- a. [exactly one child_x PEX[x can buy the cupcake or the pear]]
 \rightsquigarrow *one child can **choose** and all others are **prohibited***
- b. [exactly one child_x A[PEX[x can buy the cupcake or the pear]]]
 \rightsquigarrow *one child can **choose** and all others **can’t choose***

For the NM-NEG case, we also have two main parses. The parse in (a) captures the Strong reading. On this parse, the embedded instance of PEX triggers a modal homogeneity presupposition which projects out of both the negation and the NM quantifier. The negation then applies directly to the classical content $\diamond(\textit{cupcake} \vee \textit{pear})$, yielding that exactly one child has DP. Since the others lack DP but are modally homogeneous, they have FC. The parse in (b) captures the Weak reading, since accommodation traps the homogeneity presupposition within the quantifier's scope; accordingly, one child is prohibited, while the others are not.

- (4) Exactly one child cannot buy the cupcake or the pear. NM-NEG
- a. [exactly one child_x NEG[PEX[x can buy the cupcake or the pear]]]
 \rightsquigarrow *one child is **prohibited** and all others can **choose***
- b. [exactly one child_x A[NEG[PEX[x can buy the cupcake or the pear]]]]
 \rightsquigarrow *one child is **prohibited** and all others **aren't prohibited***

2.4 Summary

A salient difference between the three approaches is the extent to which they allow for Weak readings. Given the economy constraint on the distribution of the EXH-operator, the EXH approach is the least flexible in this regard: it predicts Weak readings only in the MM-POS cases. By contrast, to the extent that enrichments via neglect-zero are non-mandatory, the NZ approach predicts Weak readings in the SG-POS cases and in negative MM cases, but not in positive ones. Finally, the predictions of the PEX approach depend on whether the modal homogeneity presupposition projects or is locally accommodated. In the NM cases, if it projects, a Strong reading arises; if it is locally accommodated below *exactly one child* (or below negation in the SG-NEG cases), a Weak reading results. Table 2 summarizes the core assumptions and predictions of each approach. These predictions, illustrated for *exactly*-sentences, extend to their *only*-variants as well.

3 Experiments

3.1 Participants

A total of 148 adult native speakers of UK/US English (72 female; mean age 42 years) were recruited online via the Prolific platform to participate in Experiment 1 ($n = 75$) and Experiment 2 ($n = 73$). Eligibility criteria included nationality, country of birth and country of residence (UK/US), first and primary language (English), and a minimum approval rate of 99% on the platform. Participants were compensated £1.35 for their participation, with an average completion time of about 8 minutes. Written informed consent was obtained from all participants.

	EXH	NZ	PEX
Assumptions	Regular boolean \vee Obligatory EXH Economy constraint	Analogous to boolean \vee NZ or other bias [.]* Meaning of QPs & Neg	Presuppositional EXH \forall -projection through QPs Local accommodation
Interpretations	① Global EXH EXH [DP can A \vee B] ② Local EXH DP EXH [can A \vee B]	① Literal [DP can A \vee B] ② Enriched [DP can A \vee B]*	① Projection DP PEX [can A \vee B] ② Accommodation DP [A [PEX [can A \vee B]]]
Predictions			
SG-POS	False	Ambiguous	False
SG-NEG	False	False	Ambiguous
MM-POS	Ambiguous	False	Ambiguous
NM-NEG	False	Ambiguous	Ambiguous

Table 2 Summary of the main assumptions and interpretative options available on each approach, along with their predictions for the four sentence types in the WEAK contexts illustrated in Table 1.

3.2 Materials and design

Both studies build on the materials of Marty, Romoli, Sudo & Breheny (2021, 2024b) and the design of Marty, Amiraz, Elliott, Del Pinal & Romoli (2024a). Each item consisted of a sentence displayed above a picture. Each picture depicted three children aligned horizontally, each positioned in front of two distinct food items (see Table 3 for examples). Each food item was enclosed within one of two symbols: a green circle or a red circle. Participants were instructed and trained to interpret these symbols as representing what the relevant child can buy (green circle) and cannot buy (red circle). The food items depicted in the pictures varied across trials but were always identical for all three children within a given trial. The positions of the children in the pictures were fully randomized. Food items were selected from a list of 20 options in a pseudo-randomized manner, ensuring that for each child, the items that this child can and cannot buy remained consistent throughout the study.

The test sentences corresponded to the four types illustrated in examples (1) through (4): SG-POS, SG-NEG, NM-POS and NM-NEG. SG sentences were of the form ‘⟨NAME⟩ {can, cannot} buy the ⟨FOOD₁⟩ or the ⟨FOOD₂⟩’, where ⟨NAME⟩ was the name of one of the three children (*Frank*, *Mike* or *Leo*). NM sentences were of the form ‘⟨NM⟩ child {can, cannot} buy the ⟨FOOD₁⟩ or the ⟨FOOD₂⟩’, where ⟨NM⟩ was the NM quantifier tested in the experiment—*exactly one* in Exp.1 and *only one* in Exp.2. ⟨FOOD₁⟩ and ⟨FOOD₂⟩ were food names corresponding to the two types of food items depicted below each child on the picture. Each test sentence was

	FALSE	WEAK	TRUE
SG-POS Frank can buy the ⟨FOOD ₁ ⟩ or the ⟨FOOD ₂ ⟩.			
SG-NEG Frank cannot buy the ⟨FOOD ₁ ⟩ or the ⟨FOOD ₂ ⟩.			
NM-POS ⟨NM⟩ child can buy the ⟨FOOD ₁ ⟩ or the ⟨FOOD ₂ ⟩.			
NM-NEG ⟨NM⟩ child cannot buy the ⟨FOOD ₁ ⟩ or the ⟨FOOD ₂ ⟩.			

Table 3 Summary and illustration of the test conditions in Exp.1 and Exp.2, where ⟨FOOD⟩ correspond to the two different food items depicted on the picture and ⟨NM⟩ is a placeholder for the NM quantifier tested in the experiment, i.e., *exactly one* in Exp.1 and *only one* in Exp.2.

paired with one of three picture types: FALSE pictures, which made the sentence false on both the Weak and Strong readings; TRUE pictures, which made it true on both readings; and WEAK pictures, which made it true only on the Weak reading.

In addition to the test items, each survey included control items designed to verify that participants understood the instructions, had no difficulty with the visual layout and correctly interpreted the symbols. These controls involved four sentence types of the form ‘{⟨NAME⟩, ⟨NM⟩ child} {can, cannot} buy the ⟨FOOD₁⟩’, obtained from the test sentences by removing the second disjunct (e.g., *Frank can buy the burger*). Control sentences were paired with TRUE and FALSE pictures, visually similar to those used in the test trials, that made these sentences true or false, respectively. Each test and control condition was instantiated three times, resulting in 36 test items and 24 controls, for a total of 60 items.

3.3 Procedure

Experiments 1 and 2 followed the same procedure. In the instructions, participants were introduced to three boys and presented with a brief background story:

The three boys are going to the supermarket to buy food. The parents of each child have different rules about what their child is allowed

and not allowed to buy. Here is how we will represent these rules:



A **red circle** with a line through the food item means the boy is **not allowed** to buy that item.



A **green circle** around the food item means the boy **is allowed** to buy that item.

Participants were informed that they would see various pictures depicting, for each of the three boys, what he can or cannot buy and that each picture would be accompanied by a sentence describing it. Their task was to decide whether the sentence provided a good description of the picture.

Following the instructions, participants completed 8 training trials designed to familiarize them with the task display and the symbols introduced in the instructions. These trials featured control sentences paired with pictures where only one food item was associated with each child. Each control sentence appeared twice: once with a picture that made it true and once with a picture that made it false. Participants received feedback on the correctness of their response; if their response was incorrect, they were required to redo the trial. Participants could not proceed to the test phase until all 8 training trials were answered correctly. Once the training was completed, participants were informed that they would now see pictures where each child is associated with two food items. They were also informed that they would no longer receive feedback on their responses. The test trials were presented in random order. In each trial, participants responded by clicking one of two buttons labeled ‘Yes’ and ‘No’. Each item remained on the screen until participants provided their response.

3.4 Data availability

Data files and analysis output associated with the experiments are available open access on the Open Science Framework platform at <https://osf.io/rs952>.

3.5 Data software

Data preparation and analyses were carried out in the R statistical environment (R Core Team 2023) using the following packages: DHARMa (Hartig 2022), dplyr (Wickham, François, Henry, Müller & Vaughan 2023), fmsb (Nakazawa 2024), ggplot2 (Wickham 2016), Hmisc (Harrell 2023), lme4 (Bates, Mächler, Bolker & Walker 2015), performance (Lüdtke, Ben-Shachar, Patil, Waggoner & Makowski 2021a), see (Lüdtke, Patil, Ben-Shachar, Wiernik, Waggoner & Makowski 2021b).

	V1	V2	GLMM Formula
model.EXH	① Global Exh	② Local Exh	Exp.1-2: $\text{resp} \sim V1+V2+(0+V1+V2 \text{subject})$
model.NZ	① Literal	② NZ-Enrichment	Exp.1: $\text{resp} \sim V1+V2+(0+V1+V2 \text{subject})$ Exp.2: $\text{resp} \sim V1+V2+(0+V2 \text{subject})$
model.PEX	① Projection	② Accommodation	Exp.1-2: $\text{resp} \sim V1+V2+(0+V1 \text{subject})$

Table 4 Predictors (V1 and V2) corresponding to theory-specific interpretive options, and the corresponding GLMM specifications.

3.6 Data preparation

One participant in Exp.1 and one in Exp.2 were excluded prior to data analyses for failing to meet the pre-established accuracy threshold of 80% on the control items (i.e., fewer than 20 out of 24 correct responses). All remaining participants ($n = 74$ in Exp.1, $n = 72$ in Exp.2) performed at ceiling on these items, indicating no difficulty in interpreting the pictures or the relevant quantifiers.

3.7 Data analyses

To evaluate the predictions of the three theoretical approaches, we built theory-driven models that simultaneously encoded all predictions and tested their fit to the data. Each model included two binary predictors, V1 and V2, corresponding to the two main interpretive options posited under the respective approach; see Table 4 for an overview. V1 was coded as 1 for conditions expected to yield acceptance under option ① and 0 otherwise; V2 was coded in the same way for option ②.

We implemented generalized linear mixed-effects models (GLMMs) to predict responses to the test sentences ('Yes' = 1, 'No' = 0) based on these predictors. Models were fitted with the `glmer` function from the `lme4` package in R, using a binomial logit link function and the `bobyqa` optimizer. Each model included V1 and V2 as fixed effects and, where appropriate, by-subject random slopes for both. When the variance estimate of one random slope was zero, the model was simplified to include only the other. None of the resulting models were singular at tolerance levels $\geq 1e-4$. Model performance indices were then computed and compared to determine the best-fitting theory, providing a multifaceted evaluation of how well each theoretical model accounts for the observed data.

3.8 Results

Mean acceptance rates for the test sentences are shown in Figure 1. Overall, response patterns were highly similar across the two experiments. SG sentences were generally

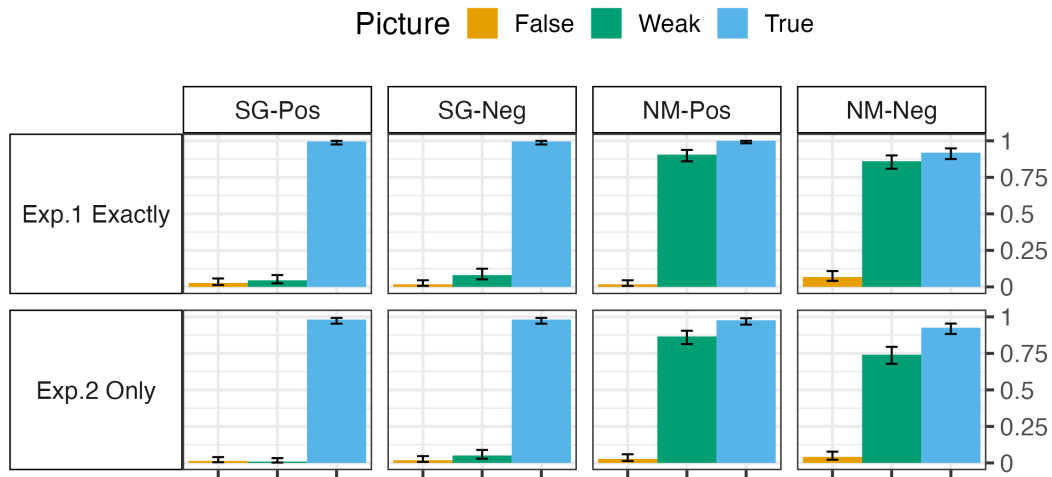


Figure 1 Mean acceptance rate (proportion of ‘Yes’-responses) by Sentence and Picture type in Exp.1 (top) and Exp.2 (bottom). Error bars represent 95% confidence intervals (CIs) estimated from binomial distributions.

rejected with WEAK pictures (all $M_s < 9\%$), indicating that they generally gave rise to strong, with-FC readings in positive forms and strong, with-DP readings in negative forms. By contrast, both NM-POS and NM-NEG sentences yielded high acceptance rates with WEAK pictures ($74\% < M_s < 91\%$), close to those obtained with TRUE pictures (all $M_s > 91\%$), and much higher than those with FALSE pictures (all $M_s < 7\%$). These results support the view that both positive and negative NM sentences are ambiguous between strong and weak readings, with the weak reading being true with WEAK pictures and readily available to participants.

Each approach was linked to the data through a GLMER model, with theory-specific predictors encoding its central assumptions (see Marty et al. 2024a for a similar method). Table 5 reports the performance indices of the three models. Across both datasets, `model.PEX` consistently outperformed `model.EXH` and `model.NZ` on all indices, indicating that PEX provides the best fit. This advantage primarily reflects the fact that, unlike the other two approaches, PEX predicts weak readings for NM sentences in both positive and negative forms, in line with our observations. However, we note that the data offer little evidence for weak readings of SG-NEG sentences, with only small differences in mean acceptance rates between their FALSE and WEAK conditions in Exp. 1 (1.8%, 95% CI [0.7, 4.5] vs. 8.1%, 95% CI [4.5, 7]) and Exp. 2 (1.8%, 95% CI [0.7, 4.6] vs. 5%, 95% CI [2.8, 8.8]).

	npar	AIC	BIC	R^2_{cond}	R^2_{marg}	
Exp.1 Exactly						
model.PEX	4	1421	1444	0.77	0.74	
model.EXH	6	1694	1729	0.72	0.67	
model.NZ	6	2040	2075	0.70	0.65	
Exp.2 Only						
model.PEX	4	1378	1402	0.77	0.74	
model.EXH	6	1615	1444	0.70	0.65	
model.NZ	4	1935	1444	0.67	0.63	

Table 5 AIC, BIC, marginal and conditional R^2 for each model in Exp.1 and Exp.2. Best scores are highlighted in grey. Information-theoretic measures are depicted by AIC weights (AIC_{wt} , $AICc_{wt}$) and BIC weights (BIC_{wt}), which reflect the relative likelihood of each model given the data under different penalty schemes for complexity. Explanatory power is shown through conditional R^2 (variance explained by fixed and random effects) and marginal R^2 (variance explained by fixed effects alone).

4 Discussion

Our findings pose a major challenge for the EXH approach, which predicts the availability of Weak readings only for NM-POS sentences. A parallel issue arises for the NZ approach, which restricts Weak readings to NM-NEG sentences. By contrast, the PEX approach offers greater flexibility: Strong readings follow from universal projection, while Weak readings can be derived for both NM-POS and NM-NEG sentences through local accommodation. Before concluding, we outline possible adjustments to the EXH and NZ approaches that could bring their predictions closer to our results, and highlight some remaining challenges for the PEX account.

4.1 The EXH approach

Consider again the NM-NEG sentence in (4), repeated below. As discussed, the EXH approach fails to predict the availability of the Weak reading in such cases due to the combined effects of obligatoriness and economy. We see two potential routes to address this limitation. One is to relax the obligatoriness of EXH, allowing an EXH-free LF such as (11a), which is true in the target WEAK contexts. Alternatively, one could maintain obligatoriness but relax the economy constraint to permit an LF like (11b)—with a vacuous instance of EXH—yielding the same interpretive result.

(11) Exactly one child cannot buy the cupcake or the pear. NM-NEG

- a. [exactly one child_x NEG [x can buy the cupcake or the pear]]
 b. [exactly one child_x EXH [NEG [x can buy the cupcake or the pear]]]
 \rightsquigarrow *one child can buy **neither** and all others can buy **either***

However, relaxing obligatoriness predicts simple positive cases like (1) to easily allow for weak, non-FC readings, which we found no evidence for. Relaxing the economy constraint conversely predicts the availability of a weak, non-DP reading for negated cases like (2), which would not explain the large difference we found between simple negated and non-monotonic sentences with respect to weak readings. Further work is thus needed to determine if there is a version of the economy constraint which allows local EXH in negative NM environments, but not under negation for FC and other kinds of scalar implicatures (see Enguehard & Chemla 2021 for relevant discussion).

4.2 The NZ approach

The main problem for the NZ approach lies in its inability to derive Weak readings for NM-POS sentences.⁷ This limitation follows from the *assertion* clause we assumed for NM quantifiers. In more detail, a natural way to provide a semantics for *exactly one child P* in Aloni’s framework is to conjoin $P(x)$ with $\neg P(y)$ for each child y distinct from x , and to take the grand disjunction of the resulting conjunctions. Given children $child_1$ and $child_2$, and taking the scope of *exactly one child* to be an FC disjunction $\diamond(P(x) \vee Q(x))$, the resulting LF can be schematized as in (12). Enrichment of the *assert* clause derives FC in the positive component, but regardless of enrichment, the negative component is always interpreted as DP.

$$(12) \quad (\diamond(P(child_1) \vee Q(child_1)) \wedge \neg \diamond(P(child_2) \vee Q(child_2))) \\ \vee (\diamond(P(child_2) \vee Q(child_2)) \wedge \neg \diamond(P(child_1) \vee Q(child_1)))$$

One potential fix would be to introduce a ‘weak’ negation operator \sim , with the semantics in (13). The key idea here is to interpret asserting $\sim \phi$ as *not asserting* ϕ , rather than as rejecting ϕ . Enrichment-via-NZ is then assumed to target the weak notion of non-assertion* rather than rejection*. On this view, an ambiguity in the semantics of *exactly one child* becomes conceivable: the Strong reading applies \neg , while the Weak reading applies \sim instead. To see how this yields the Weak reading for NM-POS, consider the conditions under which a state s asserts* $\diamond(P(child_1) \vee Q(child_1)) \wedge \sim \diamond(P(child_2) \vee Q(child_2))$. This obtains just in case s

⁷ Another potential issue for this approach is that the presumed optionality of NZ enrichment allows for weak, non-FC readings of simple positive sentences. Our data do not support this prediction, though they do not formally disconfirm it either. For the time being, we set this issue aside.

asserts* $\diamond(P(\text{child}_1) \vee Q(\text{child}_1))$, deriving FC, and s does not assert* $\diamond(P(\text{child}_2) \vee Q(\text{child}_2))$. The latter is consistent with a state that asserts $\diamond(P(\text{child}_2))$ but not $\diamond(Q(\text{child}_2))$, thus deriving the Weak reading of interest.⁸

- (13) a. s **asserts** $\sim \phi$ iff s does not **assert** ϕ .
 b. s **asserts*** $\sim \phi$ iff s does not **assert*** ϕ .

Our main concern with this solution is one of explanatory adequacy: positing a lexical ambiguity in the semantics of NM quantifiers raises the question of whether there is a principled way to predict how quantifiers behave in a bilateral, state-based framework. Furthermore, if negation in general is taken to be ambiguous between \neg and \sim , this would further complicate the predictions for SG-NEG sentences by predicting that these sentences should also allow weaker non-DP reading just as easily as in the non-monotonic case.

4.3 The PEX approach

The PEX approach aligns most closely with our results. Two key features are primarily responsible for this: (i) the projective nature of modal homogeneity inferences, and (ii) the availability of local accommodation.⁹ That said, it is not without challenges. First, in the absence of constraints on local accommodation, this approach predicts that SG-NEG sentences should allow for weak readings just as easily as the NM cases; it remains unclear why local accommodation would be freely available in quantificational NM sentences but more costly in singular negative ones. In a sense, this parallels the problem we observed above for our suggested fix to the NZ approach, namely why should weak negation be freely available for NM sentences but available only at a greater cost for simple negative ones.

Second, the PEX-account of the Strong reading in NM environments relies on the assumption of universal projection from the scope of quantifiers such as *exactly one*. While universal projection has been observed for quantifiers like universals and negatives (Heim 1983), there is little evidence for it with, for instance, existential quantifiers (Beaver 2001). The empirical and theoretical status of projection for non-monotonic quantifiers remains largely unexplored, with a few notable exceptions:

⁸ Marty et al. (2024a) note a parallel problem in the ability of Sbardolini's (2023) NZ-based account of homogeneity effects to capture Weak readings in NM environments. The fix we consider here for Aloni (2022) is identical to the one we proposed for Sbardolini (2023), which in turn was independently suggested to us by Maria Aloni and Giorgio Sbardolini (p.c.).

⁹ Our results do not discriminate between the PEX approach and a more lexicalist approach along the lines of Goldstein (2019), so long as the modal homogeneity inference is treated as a *presupposition*. There may however, be independent reasons to prefer the PEX approach, such as its ability to derive so-called 'negative free choice' (Marty et al. 2021). See Del Pinal et al. (2024) for discussion.

Chemla (2009) reports suggestive evidence for weaker-than-universal projection, and George (2008) and Sudo (2014) share the intuition that the resulting presupposition is weaker than universal, each developing distinct accounts to capture this result.

Although this may appear problematic for the PEX approach, we believe that weaker-than-universal projection may in fact offer an alternative route to weak readings in NM environments—one that does not rely on local accommodation, which, as noted, creates difficulties for simple sentences.¹⁰ This raises a key question for the PEX framework: are weak readings in NM environments due to local accommodation or weaker-than-universal projection? The apparent difficulty of accessing weak readings in simple negative sentences lends some support to the latter hypothesis. However, to fully evaluate the weaker-than-universal option for presuppositional approaches, we must compare the behaviour of those inferences with that of more standard presupposition triggers in the same environments. For initial work in this direction and relevant discussion, see Marty, Del Pinal, Elliott, Kalomoiros & Romoli 2025.

5 Conclusion

Quantificational sentences—and non-monotonic ones in particular—provide a rich testing ground for theories of a variety of related inferences. Beyond free choice, Marty et al. (2024a) recently investigated homogeneity inferences of plural definites in quantificational environments. Their results show that plural definites also permit weak readings when embedded under non-monotonic quantifiers, both with and without negation, lending further support to presuppositional accounts of homogeneity. Another closely related domain that naturally invites similar investigation is that of multiplicity inferences with bare plurals—e.g., the inference from *Frank opened presents* that *Frank opened more than one present*. Here again, we find a comparable constellation of theoretical approaches, yielding parallel predictions that can be tested using the same experimental methodology. Such studies would help distinguish between competing accounts within that domain and, more broadly, assess the potential for a unified theory across these linguistic phenomena.

¹⁰ This begs the question of how to derive strong readings in NM contexts, but see Sudo (2014) for an account where both universal and weaker-than-universal projection are possibilities for *exactly one*.

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Free choice and dual prohibition in (non)monotonic contexts

Paul Marty
Universidade de Lisboa
Centro de Linguística da Universidade de Lisboa
Alameda da Universidad, Lisboa, Portugal
pmarty@edu.ulisboa.pt

Guillermo del Pinal
University of Massachusetts Amherst
E305 South College
150 Hicks Way, Amherst, MA 01003, USA
gdelpinal@umass.edu

Patrick D. Elliott
Heinrich Heine University Düsseldorf
Building: 23.21
Universitätsstraße 1, Düsseldorf, Germany
patrick.d.elliott@gmail.com

Jacopo Romoli
Heinrich Heine University Düsseldorf
Building: 23.21
Universitätsstraße 1, Düsseldorf, Germany
jacopo.romoli@hhu.de