Proceedings of SALT 35: 331–348, 2025

Neg-raising, accessibility and propositional anaphora*

Jon Gajewski
University of Connecticut

Abstract Neg-raising predicates show a special behavior with respect to propositional anaphora. When negated, a neg-raising predicate makes a discourse referent available for the negation of the proposition denoted by its embedded clause. Non-neg-raising predicates do not allow such anaphora. The behavior is puzzling since no constituent of the sentence denotes the negation of the embedded clause. I suggest that an excluded middle presupposition-based approach, cf. Bartsch 1973 to neg-raising offers a solution. I propose that one disjunct of the excluded middle presupposition introduces a discourse referent for the negation of the embedded clause. This discourse referent is accessible conditionally on the negation of the other disjunct. Such an interaction between disjunction an anaphora is observed by Rothschild 2017.

Keywords: neg-raising, propositional anaphora, disjunction

1 An Old Observation

Lindholm (1969) gave the sentence (1a) in a discussion of neg-raising as a syntactic rule. The idea is that the first sentence in (1a) contains the constituent (2) and the rule of sentential pronominalization produces (*it*) in the second sentence under identity with that constituent. Conversely, a sentence in which the matrix negation cannot be derived by neg-raising, as in (1b) does not allow such pronominalization.

(1) a. I don't think Bill paid his taxes and Mary is quite sure of **it**.
(Lindholm 1969)
'Mary is quite sure that Bill did **not** pay his taxes'

b. It's not certain/clear that Bill paid his taxes and Mary is quite sure of **it**.
??'Mary is quite sure that Bill did **not** pay his taxes'

(2) Bill didn't pay his taxes.

(3) a. I don't think Bill paid his taxes even though Mary seems quite sure of **it**.
'Mary seems quite sure that Bill paid his taxes.'

* Thanks to participants at the UConn Meaning Group and SALT 35.

b. It's not clear that Bill paid his taxes even though Mary seems quite sure of it.
'Mary seems quite sure that Bill paid his taxes.'

Furthermore, take note that if the neg-raising (NR) predicate is not negated, the negation of its embedded clause is not available as an antecedent for subsequent pronouns.

(4) I think Bill paid his taxes, but Mary is quite convinced of it.
*'I think Bill paid his taxes, but Mary is quite convinced that he did **not** pay his taxes.'

Since then, a number of arguments have been presented against a syntactic account of neg-raising. Consider the argument presented by Jacobson (2018, 2020). If the first sentence in (5) has a representation in which negation is in the lower clause, then ellipsis with the interpretation in (6) should be available. It is not.

(5) *Judy didn't think that Donald would win, and Chris did too.

(6) Judy thought that [Donald didn't win] and Chris did ~~think that Donald didn't win~~.

In a footnote, Jacobson (2018) discusses the cases of anaphora in (1a). She suggests that pronouns involve deep anaphora and are subject only to conditions of salience. Though, consider similar cases of sluicing:

(7) I don't think Bill will pay his taxes and I can guess why ~~he will **not** pay his taxes~~. (cf. Kroll 2019)

Given these observations, I will attempt in this paper to be more precise about how a negated neg-raising predicate makes the negation of its complement clause more salient than a negated non-neg-raising predicate.

2 Theories of Neg-Raising

There are several semantic/pragmatic theories of neg-raising available. One theory, with its roots in Bartsch 1973, argues that neg-raising predicates are distinguished by carrying a disjunctive excluded middle presupposition. For example, *believe* would carry a presupposition that its subject either believes the proposition p denoted by its complement or believes the negation of that proposition. This presupposition is entailed by the assertion in the positive case, but when *believe* is negated, the presupposition is projected and strengthens the asserted lack of belief in p to a belief in the negation of p. A recent alternative criticizes the predictions of the presuppositional account and instead argues that neg-raising can be derived from exhaustification. Building on the implicature-based analysis of Romoli (2013), this

view suggests that neg-raising predicates introduce alternatives that can be exploited by covert exhaustification operators. Jeretič (2022) proposes that NR predicates like *think* have a universal semantics and introduce subdomain alternatives, which are innocently included to strengthen the negation of the NR predicate. See also the account of Staniszewski 2022.

(8) Excluded Middle Presupposition, (Bartsch 1973; Gajewski 2005, 2007)

a. $[\![\text{believe}]\!]$ (p)(a) is defined only if $\text{Dox}_a \subseteq \text{p} \vee \text{Dox}_a \subseteq \bar{p}$

b. When defined, $[\![\text{believe}]\!]$ (p)(a) is true iff $\text{Dox}_a \subseteq \text{p}$

(9) A doesn't believe that p

a. Presupposition: $\text{Dox}_a \subseteq \text{p} \vee \text{Dox}_a \subseteq \bar{p}$

b. Assertion: $\text{Dox}_a \not\subseteq \text{p}$

c. Consequence: $\text{Dox}_a \subseteq \bar{p}$

(10) Exhaustification Accounts, (Jeretič 2022; Staniszewski 2022)

a. Neg-raising predicates are lexically specified for subdomain alternatives.
Alt(think) = $\{ \lambda\text{p}.\lambda\text{z}.\ \text{f}(\mathscr{P}(\text{Dox}_z)) \subseteq \text{p} : \text{f is a choice function} \}$

b. EXH [A thinks p] $\equiv$ A thinks p

c. EXH [A doesn't think p] $\equiv$ A thinks $\bar{p}$

In this paper, I will adopt the excluded middle approach to neg-raising. I argue that it naturally facilitates an analysis of anaphora to the negation of the embedded clause of a negated NR predicate. The excluded middle presupposition is a disjunction and makes explicit reference to the negation of the embedded clause in one of its disjuncts. Snider (2017) suggests that proposition-taking predicates introduce discourse referents for their propositional arguments. For example, attitude verbs introduce propositional discourse referents for their embedded clauses. I propose that a neg-raising predicate conditionally introduces a propositional discourse referent for the negation of the embedded clause within one disjunct of the excluded middle presupposition. The motivation for this proposal comes from recent observations about the interaction of disjunction and anaphora. We will return to exhaustification-based approaches and their prospects for analyzing such anaphora in Section 6.

3 Disjunction and Anaphora

A traditional view on disjunction within dynamic semantics is that it is internally and externally static. That is, an indefinite within the disjunction cannot serve as the antecedent to a pronoun outside the disjunction; nor can an indefinite contained within one disjunct serve as the antecedent of a pronoun within the second disjunct.

These properties follow from a dynamic analysis of disjunctions as NOT[NOTϕ $\wedge$ NOTψ] if negation is externally static.

(11) a. #Either there is a bathroom or it's upstairs.

b. Either Bill left early or he baked a cake. #It has chocolate frosting.

This characterization has of course long been disputed with many instances of external dynamicity attested. Consider the cases below from Groenendijk & Stokhof 1991 and Stone 1992 involving parallel indefinites in two disjuncts jointly serving as antecedent for a singular pronoun.

(12) If a professor or an assistant professor attends the meeting of the university booard, then **he** reports to the faculty. (Groenendijk & Stokhof 1991)

(13) Either a man will bring a comb or a woman will bring a brush. In either case, ask them to leave **it** for me. (Stone 1992)

The core of approaches to external dynamicity in disjunction is to assume that the update performed by a disjunction is formed for the union of two distinct updates of the input context, one involving each of the two disjuncts. The most basic form of this kind of update is already discussed in Groenendijk & Stokhof 1991, namely program disjunction. Program disjunction simpy updates the input context independently with each disjunct and takes the union of the result.

(14) $\sigma[\phi \text{ or } \psi] = \sigma[\phi] \cup \sigma[\psi]$

It is typically assumed that program disjunction is not an adequate model of natural language disjunction. Though several elaborations and alternatives have been proposed we will adopt program disjunction for our proposal. We will return to discussion of alternatives in Section 5.2.

A form of internal dynamicity in disjunction is also observed in the well-known cases of 'bathroom' anaphora. In such sentences, an indefinite within the scope of negation inside one disjunct appears to bind a pronoun in the other disjunct.

(15) Either there isn't a bathroom in this building or **it** is in a strange place.

We will return to the requirements of a full account of bathroom anaphora and its relation to our account.

Now we turn to an observation about the interaction of disjunction and anaphora that is most relevant to us. In particular, it has been claimed that an indefinite that occurs in only one disjunct of a disjunction can serve as antecedent to a subsequent pronoun external to the disjunction in a context where the information that the other disjunct is false has been added. Consider the examples from Rothschild 2017 and Elliott 2020 below. Rothschild introduces cases that he dubs 'Disjunction to Conditional.' A single speaker cannot felicitously utter a disjunction and then in the

same breath contradict one of the disjuncts. The pragmatics of disjunction requires that the speaker considers it possible that each disjunct can be either true or false. So, Rothschild has a single speaker utter a disjunction followed by a conditional. The negation of one of the disjuncts can be safely placed within the antecedent of the conditional. Subsequently, the local context of the consequent of the conditional has been updated with both the disjunction and the negation of its disjunct. This allows for the pronoun in the consequent to be bound by the existential in the other disjunct.

(16) Disjunction to Conditional (Rothschild 2017)

a. Either it's a holiday or a customer will come in. ?He will want to be served.

b. Either it's a holiday or a customer will come in. And if it's not a holiday, **he**'ll want to be served.

Elliott (2020) produces closely related examples which resolve the pragmatic conflict by placing the disjunction and the negation of one of its disjuncts in the mouths of two different speakers. In (17), speaker A utters the disjunction and speaker B contradicts the second disjunct. B successfully uses a pronoun anaphoric to the indefinite in the first disjunct.

(17) a. Either a_1 layperson was in the audience, or we had no press. #They$_1$ enjoyed it.

b. A: Either a_1 layperson was in the audience, or we had no press.
B: We had some press, so I hope they$_1$ enjoyed it! (Elliott 2020)

I take inspiration from these cases in the analysis of our target case of anaphora. I propose to treat the excluded middle presupposition of neg-raising predicates as an actual disjunction that displays the same capacities for facilitating anaphora as overt disjunctions. One disjunct of the excluded middle presupposition says that the subject believes the proposition p denoted by the embedded clause. Note that this disjunct is equivalent to the assertion of the predicate itself. I propose that the second disjunct introduces a discourse referent for the negation of the proposition p and says that the speaker believes that negated proposition. If the neg-raising predicate is not negated, then the discourse referent introduced in the second disjunct will not be available for subsequent reference, cf. (4). However, when the neg-raising predicate is negated, the negated assertion contradicts the information in the first disjunct of the Excluded Middle Presupposition. This makes the discourse referent introduced in the second disjunct of the presupposition accessible to subsequent pronouns.

This kind of account for our target case (1a) assumes that it is possible for a discourse referent to be introduced by a presupposition. Not all dynamic theories allow for such a possibility. In the next section, inspired by the work of Sudo (2012, 2025), I propose a version of update semantics that make such anaphora possible.

4 Update Semantics

To give an analysis of neg-raising and anaphora I will adopt the framework of update semantics (Heim 1982, 1983; Veltman 1996; Groenendijk, Stokhof & Veltman 1996). The distinguishing feature of update semantics among dynamic theories is that sentences denote update functions that maps information states to information states.

4.1 Information States

I will adopt a perspective on which information states are represented by pairs of possible worlds and assignments. I will refer to world-assignment pairs as possibilities. For a possibility i, I use the expression 'w_i' to refer to the world of i and 'g_i' to refer to the assignment. Assignments will be modeled as total functions from natural numbers to the union of the domain of individuals and the undefined individual represented with $\star$.

It will be necessary to define some significant relationships between assignments, possibilities and states in order to formalize our update semantics. These are based on the definitions in Groenendijk et al. 1996. First we define extension. Extension of information states encodes the notion of increase in information, whether adding anaphoric information or eliminating worlds.

(18) Extension

a. An assignment g' extends another assignment g, $g \leq g'$ iff $\{x\text{: } g(x) \neq \star\} \subseteq \{x\text{: } g'(x) \neq \star\}$

b. A possibility i' extends another possibility i, $i \leq i'$ iff $w_i = w'_i$ and $g_i \leq g_{i'}$

c. A state σ' extends another state σ, $\sigma \leq \sigma'$ iff for all $j \in \sigma'$, there is an $i \in \sigma$ s.t. $i \leq j$

Next we define subsistence. In contrast to extension, subsistence of states specifically encodes increase in anaphoric information, without elimination of worlds.

(19) Descent

a. i' is a descendant of i in σ iff $i,i' \in \sigma$ and $i \leq i'$

(20) Subsistence

a. i subsists in σ iff $\exists j[\, i \leq j \wedge j \in \sigma]$ (i.e., i has at least one descendant in σ)

b. σ subsists in σ' iff for all $i \in \sigma$, i subsists in σ'

c. σ supports ϕ iff $\sigma[\phi]$ exists and σ subsists in $\sigma[\phi]$

Finally, we can use subsistence and support to define a useful dynamic notion of entailment for update denoting formulas.

(21) $\phi \models \psi$ iff for all σ, if $\sigma[\phi][\psi]$ exists, then $\sigma[\phi]$ supports ψ.

4.2 Presupposition

In classical update semantics, presuppositions are treated as conditions on the input information state that are required for a successful update. Typically, an update of an information state σ involving a formula ϕ with presupposition ψ is defined just in case updating σ with the presupposition ψ eliminates no worlds from σ, (24). In our language, we will permit predicates and proposition letters to be subscripted with predicates and proposition letters that represent presuppositions.

(22) If ϕ is a formula and ψ is a formula, then ϕ_ψ is a formula.

This standard approach to presupposition in satisfaction based theories will not work for us here. The reason is that if the presupposition ψ contains an existential, the presupposition requires that the input information state contains the information that there exists an individual witnessing the existental, but it does not guarantee that there is a discourse referent for such an individual in the information state. I formalize the classical approach to update presupposition below in terms of an operator W that extracts the set of worlds from an information state (a set of world assignment pairs).

(23) $\mathrm{W}(\sigma) := \{\ \mathrm{w} : \exists \mathrm{g}[\ \langle \mathrm{w},\mathrm{g} \rangle \in \sigma]\ \}$

(24) Presuppositional Definedness (to be revised)
$\sigma[\phi_\psi]$ is defined only if $\mathrm{W}(\sigma[\psi]) = \mathrm{W}(\sigma)$

For our theory to work, we need the presupposition to be able to introduce a discourse referent. One way to approach this would be to allow the subscripted presuppositional formula to have two effects. First, it could impose a definedness condition on the update. Second, the subscripted formula could be included in the update when the definedness condition is met. One natural way to do this would be to sequence the update of the subscripted formula with the main formula. This is similar to, though distinct from, some ideas pursued in Sudo 2025.

(25) Presuppositional Update (to be revised)
When defined,
$\sigma[\phi_\psi] = \sigma[\psi][\phi]$

Since the definedness condition guarantees that the subscripted formula eliminates no worlds from the input information state, the only change the first update can make is to introduce anaphoric information. For example, if ψ were (26), the definedness condition would guarantee that there are cats in every world of the input context. There is no guarantee though that there is any discourse referent that ranges over cats.

(26) $\exists$x[cat(x)]

However, when the first part of the update is processed, no worlds are eliminated, but a discourse referent is added for a cat. This method indeed permits a presupposition to introduce referents under certain circumstances. Still it will not suffice for our purposes. There is a problematic disconnect that remains in this approach. Consider how the update proceeds in the context of negation. We want the presupposition to project through negation, and we want the discourse referent introduction to persist.

(27) $\neg\phi_{\exists x[cat(x)]}$

(28) Negation

a. $\sigma[\neg\phi]$ is defined only if $\sigma[\phi]$ is defined.

b. When, defined, $\sigma[\neg\phi] = \{ i \in \sigma : \neg\exists i'[i \leq i' \wedge i' \in \sigma[\phi]] \}$

(29) a. $\sigma[\neg\phi_{\exists x[cat(x)]}]$ is defined only if for all w in σ, there are cats in w.

b. When defined, $\sigma[\neg\phi_{\exists x[cat(x)]}] = \{ i \in \sigma : \neg\exists i'[i \leq i' \wedge i' \in \sigma[\phi_{\exists x[cat(x)]}]] \}$
$= \{ i \in \sigma : \neg\exists i'[i \leq i' \wedge i' \in \sigma[\exists x[cat(x)]][\phi]] \}$

The important thing to notice here is that while the definition of presuppositional update requires the input info state to entail that there are cats, the output state after processing the negation need not have a discourse referent for any cat. This is because given the way that we have made the presupposition a part of the update, the discourse referent for a cat is only introduced into the local context. The ancestors of these local possibilities are then subtracted from the input context. It is clear though that in some contexts we do need indefinites in presuppositions to introduce discourse referents, cf. Beaver 1992; Elliott & Sudo 2020.

(30) a. Bill doesn't realize that I have a cat.

b. It's staying with my mother.

So, we need a way to incorporate the update with subscripted, presupposed formulas that will guarantee that the output context of an update like (29) includes a discourse referent for a cat. Sudo (2025) proposes a mechanism for this that involves translation of expressions in to two separate dimensions, an assertion tier and a presupposition tier. I would like to propose an alternative that avoids separate tiers. Rather than using two-dimensional translation, I suggest that the definedness condition be stated differently. Instead of requiring the input state σ and the ouput of updating σ with the presupposition ψ to have the same worlds, I suggest we require that there be some context σ' such that σ is the result of updating σ' with ψ. On this view, a presupposition is a prior update to the current context.

(31) Presuppositional Definedness (final)
$\sigma[\phi_\psi]$ is defined only if $\exists\sigma'[\sigma'[\psi] = \sigma]$

If the input info state already has the shape of a state updated with ψ there is no need to include ψ in the actual update:

(32) Presuppositional Update (final)
When defined,
$\sigma[\phi_\psi] = \sigma[\phi]$

With this in mind, let us return to the case of projection through negation. Now our definedness condition requires that the input state being updated with a negation had to have resulted from an update with the presupposition formula, see below.

(33) a. $\sigma[\neg\phi_{\exists x[cat(x)]}]$ is defined only if $\sigma[\phi_{\exists x[cat(x)]}]$ is defined
b. $\sigma[\phi_{\exists x[cat(x)]}]$ is defined only if $\exists\sigma'[\sigma'[\exists x[cat(x)]] = \sigma]$

Now, any state that results from an update with the formula $\exists x[cat(x)]$ will have a discourse referent for a cat.

(34) for any σ', for all i $\in \sigma'[\exists x[cat(x)]]$, $g_i(x) \in I_{w_i}(\text{cat})$ (so, $g_i(x) \neq \star$)

(35) When defined,
$\sigma[\neg\phi_{\exists x[cat(x)]}] = \{\ i \in \sigma : \neg\exists i'[\ i{\leq}i' \wedge i' \in \sigma[\phi_{\exists x[cat(x)]}]\ \}$
$\sigma[\neg\phi_{\exists x[cat(x)]}] = \{\ i \in \sigma : \neg\exists i'[\ i{\leq}i' \wedge i' \in \sigma[\phi]\ \}$

Note that the previous definedness condition follows from the new version. That is, if σ is the result of updating some state σ' with ψ, then $W(\sigma) = W(\sigma[\psi])$.[1] This is so since all updates are either eliminative or add discourse markers to the domains of a possibility. No update can add worlds to the domain of an information state. Because of the definedness condition, σ has x as a discourse referent for cats. The update with negation, will then substract possibilities, but the possibilities that remain will keep x as a discourse referent ranging over cats. So, an existential quantifier in the presupposition of a negated formula can dynamically bind outside the scope of negation.

To illustrate how a presupposition can be filtered given the definition in (32), I turn to a discussion of conjunction. In an update system, conjunction is interpreted as sequential update. If the second conjunct carries a presupposition, then the information state resulting from the update with the first conjunct will have to have resulted from an update with the presupposition.

(36) a. $\sigma[\phi \wedge \psi] = \sigma[\phi][\psi]$
b. $\sigma[\phi \wedge \psi_\chi]$ is defined only if $\sigma[\phi][\psi_\chi]$

1 This follows in any system that Veltman (1996) defines as additive. In any additive system, updates have the property of idempotence: for any information state σ and sentence ϕ, $\sigma[\phi][\phi] = \sigma[\phi]$. Not all dynamic systems have idempotence. Imposition of a novelty condition could interfere in the case at hand.

c. $\sigma[\phi][\psi_\chi]$ is defined only if $\exists\sigma'[\sigma'[\chi] = \sigma[\phi]]$

Of course, if the first conjunct denotes the same update function as the presupposition, i.e. $\phi = \chi$, then this presupposition is satisfied (σ=σ') regardless of the content of σ. Furthermore, if ϕ entails χ (as defined in (21)), the presupposition places no restrictions on the input state σ. The presupposition is satisfied when σ' is instantiated with $\sigma[\phi]$.

4.3 Propositional discourse referents

We will now augment our language with variables for propositional discourse referents. While one could imagine associating these discourse referents with the denotations of sentences (that is context change potentials/update functions), I will follow Stone 1999; AnderBois, Brasoveanu & Henderson 2015; and Hofmann 2025 (among others) in modeling propositional discourse referents with static objects, namely sets of possible worlds.

(37) a. Proposition variables: p, q, r, p′, q′, r′, ...

b. For every assignment g and propositional variable p, g(p) $\in \mathscr{P}(D_s)\cup\{\star_p\}$

I will model attitude predicates as relations between individuals and sets of possible worlds. This means that attitude predicates will not combine directly with the meanings of embedded clauses, which are update functions. Instead embedded clauses will introduce propositional discourse referents, which in turn saturate the argument positions of attitude predicates.

(38) $\sigma[believe(a,p)]$ = { i $\in \sigma$: $\langle$i(a),i(p)$\rangle \in I_{w_i}$(believe) }

Distilling a set of worlds from an update function is a complex matter. One possibility would be to associate the update with the worlds of a state resulting from applying the update to an empty info state.

(39) Empty Assignment
$g_\star$:= { $\langle$x,$\star\rangle$: x is a variable }

(40) Empty Info State
$\varnothing_\sigma := W\times\{g_\star\}$

(41) Propositional dref introduction
$\sigma[\exists p]$ = { i : $\exists j\in\sigma$[j[p]i] }

(42) a. i[p]j iff $w_i = w_j$ and g_i[p]g_j

b. g[p]h iff g and h agree on all variables other than p and h(p) $\neq \star$

The crucial definition is the following definition for a special formula whose function is to set the value of a propositional discourse referent.

(43) Propositional dref assignment
$\sigma[\, p := \phi \,] = \{\, i \in \sigma : i(p) = W(\varnothing_\sigma[\phi]) \,\}$

An obvious concern for this definition is that it makes embedded clauses completely inert for anaphora. It does not permit a pronoun in an embedded clause to pick up a referent from an indefinite outside the embedded clause, nor does it permit an indefinite in an embedded clauses to dynamically bind a pronoun outside the embedded clause. This is so because the empty information state contains no anaphoric information, i.e., every variable is mapped to the undefined object. As it happens, the interpretation of pronouns in the embedded clauses of propositional attitude verbs is a complex issue (cf. Geurts 1998; van Rooy 2000). Though I will not attempt to offer a comprehensive solution here, I make a suggestion for interpreting pronouns within the complement of an attiude verb.

To make a pronoun within a complement clause available to pick up a referent from the broader conversational context, we can add a mechanism that prefixes the variable to the embedded clause. We can then modify the rule for propositional dref assignment to relativize it to the value for the variable in the broader context, as in (44). This will suffice for present purposes.

(44) Propositional dref assignment
$\sigma[\, p := {}_x\phi \,] = \{\, i \in \sigma : i(p) = W(\varnothing_\sigma[x \to i(x)][\phi]) \,\}$

(45) $\sigma[x \to a] = \{\, i : \exists j[\, j \in \sigma \wedge j[x]i \wedge i(x) = a \,] \,\}$

I will not attempt to add an account of indefinites within modal environments beinding pronouns outside the local environment, modal subordination. Instead we turn to condition that is placed on pronouns that will determine when they successfully pick up a value from the input information state.

4.4 Familiarity

Update semantic systems typically include a familiarity condition on pronouns. This condition ensures that the variable associated with the pronoun is defined throughout the information state in which it is being interpreted. In our system, this means that every assignment that occurs in a possibility in the information state assigns the variable a value other than the undefined value $\star$.

(46) Familiarity
A variable x is familiar at an information state σ iff:
$\forall g \in A(\sigma), g(x) \neq \star$ (Elliott & Sudo 2025)

(47) $A(\sigma) = \{g : \exists w[\, \langle w,g \rangle \in \sigma \,] \,\}$

We will make crucial use of familiarity in analyzing the interaction of disjunction and anaphora. If an indefinite introduces a discourse referent only within one disjunct,

then the referent will not necessarily be available for subsequent anaphora. If an externally dynamic meaning is adopted for disjunction, which unites the updates of the two disjuncts, then the variable will receive the value $\star$ in the assignments in the ouput of the other disjunct (i.e., the one that does not contain the indefinite). So, in that case, familiarity is not satisfied in the output information state. Nevertheless, if a subsequent update contradicts the other disjunct and eliminates the possibility it contributed to the output state, then the only remaining assignments will all be defined for the variable introduced by the indefinite in a single disjunct.

5 The Account

Now all the pieces are in place to give a full account of the data in (1), repeated below.

(1) a. I don't think Bill paid his taxes and Mary is quite sure of **it**. (Lindholm 1969)
'Mary is quite sure that Bill did **not** pay his taxes'

b. It's not certain/clear that Bill paid his taxes and Mary is quite sure of **it**.
??'Mary is quite sure that Bill did **not** pay his taxes'

In summary, the ingredients of the analysis are the following.

(48) a. Update semantics

b. Externally dynamic disjunction

c. Discourse referents introduced by presupposition

d. Propositional discourse referents

e. Familiarity

Using these ingredients, we can state the semantics for a neg-raising predicate like *believe* as well as its excluded middle presupposition. As discussed in Section 4.3, I will treat belief in the logical language as a relation between an individual and a proposition, i.e., a set of worlds. The presupposition carried by *believe* will be more complex, involving a crucial dynamic component. We will refashion the traditional Excluded Middle Presupposition into a disjunction with one disjunct that parallels the assertion and one disjunct that introduces a discourse referent for the negation of the embedded clause and says the subject believe that negated proposition.[2]

(49) Traditional Excluded Middle Presupposition
$\text{Bel}(x,p) \vee \text{Bel}(x,\neg p)$

2 Our dynamic use of the excluded middle presupposition is in part inspired by the work of Gotham (2019).

(50) Formalization of Presupposition
$\text{Bel}(x,p) \cup \exists \mathbf{q}[\ \mathbf{q} = \overline{\mathbf{p}} \wedge \text{Bel}(x,q)\ ]$

For simplicity, I will abbreviate the new dynamic excluded middle presupposition with the expression '$\text{EM}^q(x,p)$.'

(51) Abbreviation
$\text{EM}^q(x,p) := \text{Bel}(x,p) \cup \exists q[\ q = \overline{\mathbf{p}} \wedge \text{Bel}(x,q)\ ]$

With these pieces in place, we can fully state the translation and interpretation of a neg-raising predicates such as *believe*.

(52) A believes $p \rightsquigarrow \text{Bel}(a,p)_{EM^q(a,p)}$

For simplicity, I assume that the CP is displaced and introduces a propositional discourse referent that is picked up by the attitude verb.

(53) [[that^p [Bill paid his taxes]] [I believe p]]

This syntactic structure is then translated into the logical form below.

(54) Logical form for *a believes* ϕ
$\exists p[\ p := \phi \wedge \text{Bel}(a,p)_{EM^q(a,p)}\ ]$

We can see already from this structure that the anaphora in (4) will not be licensed. Consider the formalization of this sequence of assertions below.

(55) a. I think Bill paid his taxes. Mary is quite sure of it.
b. $\exists p[\ p := \phi \wedge \text{Bel}(a,p)_{EM^q(a,p)}\ ] \wedge \text{Sure}(m,q)$

The presupposition will introduce a discourse referent for the negation of the embedded clause but only within a portion of the information state. In fact, the subsequent update with the assertion in this case eliminates those possibilities. Consequently, the variable q in the second conjunct in (55b) does not satisfy Familiarity. The variable q is not assigned a non-$\star$ value throughout the information state that is the input to the second conjunct. In contrast, the logical form for a negated NR predicate is as given below.

(56) Logical form for *a doesn't believe* ϕ
$\exists p[\ p := \phi \wedge \neg\text{Bel}(a,p)_{EM^q(a,p)}\ ]$

And the logical form of a sequence with a subsequent prounoun is given in (57b).

(57) a. I **don't** think Bill paid his taxes. Mary is quite sure of it.
b. $\exists p[\ p := \phi \wedge \neg\text{Bel}(a,p)_{EM^q(a,p)}\ ] \wedge \text{Sure}(m,q)$

Here q is successfully bound from the presupposition. The presupposed disjunction requires the input information state to be made up of possibilities in which I believe

that Bill paid his taxes and possibilities in which I believe he didn't and there is a discourse referent for the proposition that Bill did not pay his taxes. The subsequent update with the negated assertion eliminates all the possibilities in which I believe Bill paid his taxes. This leaves only possibilities in which q represents the proposition that Bill did not pay his taxes. Thus, q in the second conjunct can pick up this negated proposition as its value.

Finally notice that in both the negated and non-negated conditions, a propositional discourse referent is available for the unnegated embedded proposition. This comports well with the facts.

(58) a. I think Bill paid his taxes. And Mary believes it too.

b. I don't think Bill paid his taxes, even though Mary believes it.

In this way, we can successfully account for the special behavior of neg-raising predicates with respect to the licensing of propositional anaphora.

5.1 Predictions about accessibility

We must now check some predictions about accessibility of this conditional discourse referent for the negation of the embedded clause to subsequent anaphora. Take for example, the case of a negated NR predicate in the antecedent of a conditional.

(59) If Mary doesn't believe that Bill paid his taxes, then Alex is quite sure of it.

To my ear, it is possible to interpret the pronoun in the antecedent picking up reference to the proposition that Bill did not pay his taxes. This is consistent with our approach to propositional anaphora with NR predicates when combined with a standard dynamic update approach to the conditional. I will adopt the following definition from Groenendijk et al. 1996.

(60) $\sigma[\ \phi \rightarrow \psi\ ] = \{\ i \in \sigma :$ if i subsists in $\sigma[\phi]$, then all descendants of i in $\sigma[\phi]$ subsist in $\sigma[\phi][\psi]\ \}$

According to the semantics for the conditional and the NR predicate, and the principle in (31), the input information state σ must meet the condition below, where 'pay' stands for the variable picking up the propositions that Bill paid his taxes, and '¬pay' stands for the variable introduced to stand for the propositions that Bill did not pay his taxes.

(61) $\exists\sigma'[\ \sigma'[\mathrm{EM}^{\neg\mathbf{pay}}(\mathrm{Mary}, \mathrm{pay})] = \sigma\ ]$

This condition guarantees that a discourse referent for the negation of the embedded clause will be available in the local context of the consequent of the conditional, which has been update with the antecedent which eliminates the non-dynamic

disjunct of the excluded middle presupposition. Thus we predict successful binding of the pronoun in (59).

We furthermore predict that the discourse referent for the negation of the embedded clause should not be accessible outside of the conditional. While the dynamic excluded middle presupposition projects to the input information state, the update with the conditional does not remove all the possibilities consistent only with the first disjunct of the presupposition. So, there will be possibilities in the output information state in which the variable for the negation of the embedded clause is assigned $\star$.

(62) If Mary doesn't believe that Bill paid his taxes, then Alex is quite sure of it. Frida certainly believes it.
?'Frida believe that Bill didn't pay his taxes.'

In this case, I think there is a tendency in the direction of our prediction. But it is not clear that anaphora is not possible here.

5.2 A potential problem

Given the approach we have taken in this paper, we expect anaphora involving the negation of the embedded clause of a negated NR predicate to look like the kind of anaphora involving indefinite antecedents that dynamic update semantics was developed to model. So, we would expect to it exhibit some the behaviors that are trickier to handle, including so-called bathroom anaphora.

(63) Either this house doesn't have **a bathroom**, or **it**'s in a funny place.

Such anaphora is typically handled, in part, by assuming that the second disjunct is interpreted in the context of the negation of the first disjunct. If in addition the dynamic system permits double negation elimination, it is possible to explain how the indefinite *a bathroom* becomes accessible as an antecedent for the pronoun in the second disjunct, cf. Krahmer & Muskens 1995.
Our system is set up so that when a NR predicate is negated, it makes available the negation of its embedded clause as a possible antecedent for further anaphora. So, if disjunction truly involves interpreting the second disjunct in the context of the **negation** of the first disjunct, then we would anticipate that a kind of bathroom anaphora should become possible.

(64) Either Mary thinks that Bill paid his taxes or Alex is quite sure of it.

(65) Either Mary thinks that Bill paid his taxes or (Mary doesn't think that Bill paid his taxes and) Alex is quite sure of it.

I have not been able to construct any natural examples that allow such anaphora with propositional anaphors. As far as I can tell, it is impossible to read the pronoun *it* in (64) as picking up the proposition that Bill did not pay his taxes. (65) shows intuitively why we should predict anaphora to be possible.

As it stands, our theory does not predict bathroom anaphora to be possible with NR predicates. But not for a good reason. It is simply because our system, as we have set it up, does not allow any bathroom anaphora at all. We have been using program disjunction, which does not interpret the second disjunct within the context of the negation of the first. Furthermore, our update semantics does not validate double negation elimination.

It would take us too far afield in this short paper, but I believe that the proposed analysis in this paper could integrate well with a theory with both of these features, one that offers a principled account of bathroom anaphora in disjunctions. A good candidate would be the Bilateral Update Semantics of Elliott & Sudo 2025. They offer a bilateral semantics of disjunction that sequences the negative update of the first disjunct with positive update of the second, cf. (66). And their system validates Double Negation Elimination.

(66) a. $\sigma[\phi \vee \psi]^{+} = \sigma[\phi]^{+}[\psi]^{+} \cup \sigma[\phi]^{+}[\psi]^{-} \cup \sigma[\phi]^{+}[\psi]^{?} \cup \underline{\sigma[\phi]^{-}[\psi]^{+}} \cup \sigma[\phi]^{?}[\psi]^{+}$

b. $\sigma[\phi \vee \psi]^{-} = \sigma[\phi]^{-}[\psi]^{-}$ (Elliott & Sudo 2025)

If such a unification is successful, then our parallel treatment of the negative proposition and indefinites in terms of introducing discourse referents raises questions. We would need to explain why bathroom anaphora is possible in one case but not the other. Further adding to the puzzle is Kroll's 2019 observation that sluicing is possible in a bathroom configuration.

(67) a. Either Bill didn't answer an extra-credit problem or he just didn't mark which one ~~he answered~~. (Kroll 2019)

b. Either Bill paid his taxes or he explained why ~~he didn't pay his taxes~~.

6 Discussion

In this paper, we have given an account of puzzling anaphoric behavior associated with negated neg-raising predicates. The account has relied crucially on the excluded middle approach to neg-raising predicates. In particular we have made use of the properties of the EM presupposition as a disjunction. It is important to note that objections have been raised against the presuppositional approach to the excluded middle, by for example Romoli 2013 and Jeretič 2022. An important next step will be to see whether exhaustification based theories of neg-raising can be integrated with a formal theory of propositional anaphora.

References

AnderBois, Scott, Adrian Brasoveanu & Robert Henderson. 2015. At-issue proposals and appositive impositions in discourse. *Journal of Semantics* 32(1). 93–138. doi:10.1093/jos/fft014.

Bartsch, Renate. 1973. *Negative transportation* gibt es nicht. *Linguistische Berichte* (27). 1–7.

Beaver, David. 1992. *The kinematics of presupposition*. Institute for Language, Logic and Information.

Elliott, Patrick D. 2020. Towards a principled logic of anaphora. *Unpublished manuscript* .

Elliott, Patrick D & Yasutada Sudo. 2020. Generalised crossover. In *Semantics and Linguistic Theory (SALT) 30*, 396–408. doi:10.3765/salt.v30i0.4841.

Elliott, Patrick D. & Yasutada Sudo. 2025. Free choice with anaphora. *Semantics and Pragmatics* 18(2). 1–49. doi:10.3765/sp.18.2.

Gajewski, Jon Robert. 2005. *Neg-raising: Polarity and presupposition*: Massachusetts Institute of Technology PhD dissertation. doi:1721.1/33696.

Gajewski, Jon Robert. 2007. Neg-raising and polarity. *Linguistics and Philosophy* 30(3). 289–328. doi:10.1007/s10988-007-9020-z.

Geurts, Bart. 1998. Presuppositions and anaphors in attitude contexts. *Linguistics and Philosophy* 21(6). 545–601. doi:10.1023/A:1005481821597.

Gotham, Matthew. 2019. Double negation, excluded middle and accessibility in dynamic semantics. In Julian J. Schlöder, Dean McHugh & Floris Roelofsen (eds.), *Amsterdam Colloquium*, vol. 22, 142–151.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 39–100. doi:10.1007/BF00628304.

Groenendijk, Jeroen, Martin Stokhof & Frank Veltman. 1996. Coreference and modality. In Shalom Lappin (ed.), *The handbook of contemporary semantic theory*, 176–216. Blackwell.

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases*. University of Massachusetts Amherst.

Heim, Irene. 1983. On the projection problem for presuppositions. In M. Barlow, D. Flickinger & M. Westcoat (eds.), *West Coast Conference on Formal Linguistics (WCCFL) 2*, 114–125.

Hofmann, Lisa. 2025. Anaphoric accessibility with flat update. *Semantics and Pragmatics* 18(3). 1–69. doi:10.3765/sp.18.3.

Jacobson, Pauline. 2018. Some people think there is neg raising, and some don't: Neg raising meets ellipsis. *Linguistic Inquiry* 49(3). 559–576. doi:10.1162/ling_a_00282.

Jacobson, Pauline. 2020. Neg raising and ellipsis (and related issues) revisited.

Natural Language Semantics 28(2). 111–140. doi:10.1007/s11050-020-09161-z.
Jeretič, Paloma. 2022. Neg-raising with belief predicates as a scaleless implicature. In *Semantics and Linguistic Theory (SALT) 32*, 891–914. doi:10.3765/salt.v1i0.5367.
Krahmer, Emiel & Reinhard Muskens. 1995. Negation and disjunction in discourse representation theory. *Journal of Semantics* 12(4). 357–376. doi:10.1093/jos/12.4.357.
Kroll, Margaret. 2019. Polarity reversals under sluicing. *Semantics and Pragmatics* 12. 18–1. doi:10.3765/sp.12.18.
Lindholm, James M. 1969. Negative-raising and sentence pronominalization. In Robert I. Binnick, Alice Davidson, Georgia M. Green & Jerry L. Morgan (eds.), *Chicago Linguistic Society (CLS)*, vol. 5, 148–158.
Romoli, Jacopo. 2013. A scalar implicature-based approach to neg-raising. *Linguistics and Philosophy* 36(4). 291–353. doi:10.1007/s10988-013-9136-2.
van Rooy, Robert. 2000. *Anaphoric Relations Across Attitude Contexts* 157–181. Dordrecht: Springer Netherlands. doi:10.1007/978-94-011-3947-2_9.
Rothschild, Daniel. 2017. A trivalent account of anaphora and presupposition. In *Amsterdam Colloquium*, 1–13.
Snider, Todd Nathaniel. 2017. *Anaphoric reference to propositions*: Cornell University PhD dissertation. doi:10.7298/X4C24TNB.
Staniszewski, Frank. 2022. *Modality and Time in Logical Context*: Massachusetts Institute of Technology PhD dissertation. doi:1721.1/147577.
Stone, Matthew. 1999. Reference to possible worlds. *RuCCS Report* 49. 302–21.
Stone, Matthew D. 1992. *Or* and anaphora. In *Semantics and Linguistic Theory (SALT) 2*, 367–386. doi:10.3765/salt.v2i0.3037.
Sudo, Yasutada. 2012. *On the semantics of phi features on pronouns*: Massachusetts Institute of Technology PhD dissertation. doi:1721.1/77805.
Sudo, Yasutada. 2025. Specific indefinites and dynamic presuppositions. Talk at UConn Logic Group.
Veltman, Frank. 1996. Defaults in update semantics. *Journal of Philosophical Logic* 25(3). 221–261. doi:10.1007/bf00248150.

Jon Gajewski
University of Connecticut
Department of Linguistics
U-1145
jon.gajewski@uconn.edu